Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

JUNE 2022

MATHEMATICS P2

MARKS: **150**

TIME: **3 hours**

This question paper consists of 12 pages and an answer book of 19 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of ELEVEN questions. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.

2. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answer.

3. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

4. Answers only will not necessarily be awarded full marks.

5. If necessary, round off answers to TWO decimal places, unless stated otherwise.

6. Diagrams are NOT necessarily drawn to scale.

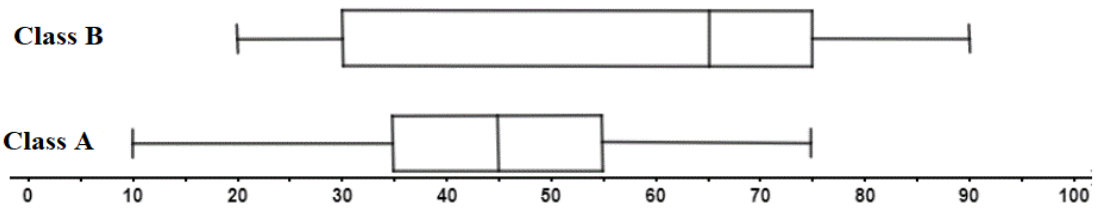
7. Number the answers correctly according to the numbering system used in this question paper.

8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

The box and whisker diagrams below show the Mathematics results of class A and class B in the June Examination. It is also given that class B has a Median of 65%.



1.1 Which class had the top learners? (1)

1.2 Determine which class had the greatest Inter Quartile Range (IQR). (1)

1.3 What percentage of class A scored less than 60%? (1)

1.4 If all the learners in class A were given an extra 5%, what would happen to the standard deviation of the marks in class A? (1)

1.5 Determine the semi-interquartile range of class B. (1)
[5]

QUESTION 2

A group of 30 pupils was asked to complete an obstacle course at their Grade 11 camp. The times (in seconds) taken by the pupils to complete the obstacle course are given in the table below.

Time taken	$60 \leq t < 90$	$90 \leq t < 120$	$120 \leq t < 150$	$150 \leq t < 180$	$180 \leq t < 210$
No. of pupils	3	6	7	8	6

2.1 Complete the cumulative frequency table for above data in the SPECIAL ANSWER BOOK. (1)

2.2 Draw a cumulative frequency curve for the above data on the grid provided. (4)

2.3 Indicate on your graph where you would read off:

2.3.1 The number of pupils that took 135 seconds to complete the course (Use the letter A) (1)

2.3.2 The value of t if 60% of the pupils took less than t seconds to complete the obstacle course. (Use the letter B) (1)

2.3.3 The 75th percentile. (Use the letter C) (1)
[8]

QUESTION 3

Consider the following set of four positive whole numbers and their frequency.

Scores	$x+3$	$2x$	$x-1$	6
Frequency	4	3	2	2

3.1 Determine the median score. (1)

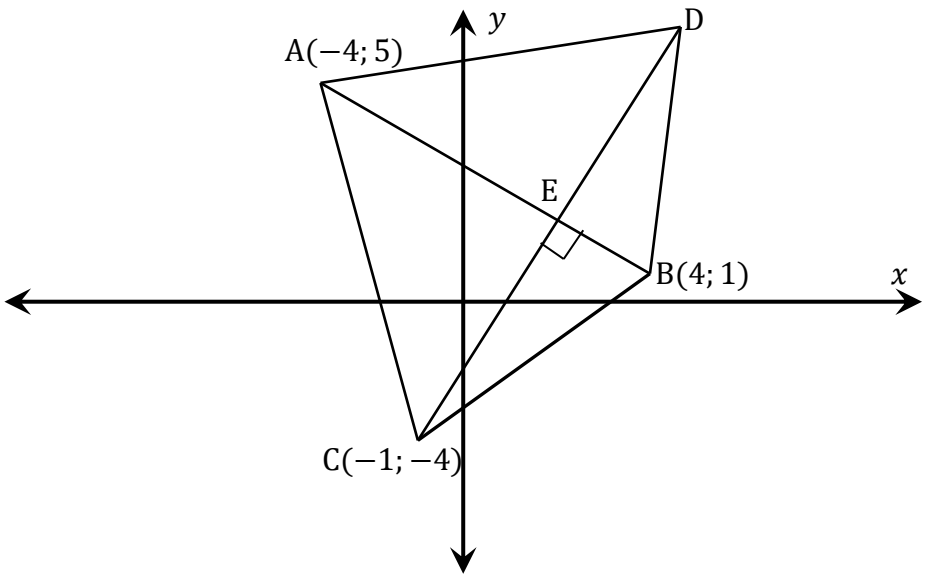
3.2 Determine the mean in terms of x. (3)

3.3 If only the scores are taken into consideration (without frequency), determine the standard deviation if it is given that $x = 5$. (2)

[6]

QUESTION 4

In the diagram below, the coordinates of $A(-4;5), C(-1;-4)$ and $B(4;1)$ are the vertices of a triangle in a Cartesian plane. $CE \perp AB$ with E on AB. E is the midpoint of straight-line CD.



4.1 Determine the gradient of AB. (2)

4.2 Determine the equation of CD. (4)

4.3 Determine the coordinates of E. (6)

4.4 Determine the coordinates of D. (2)

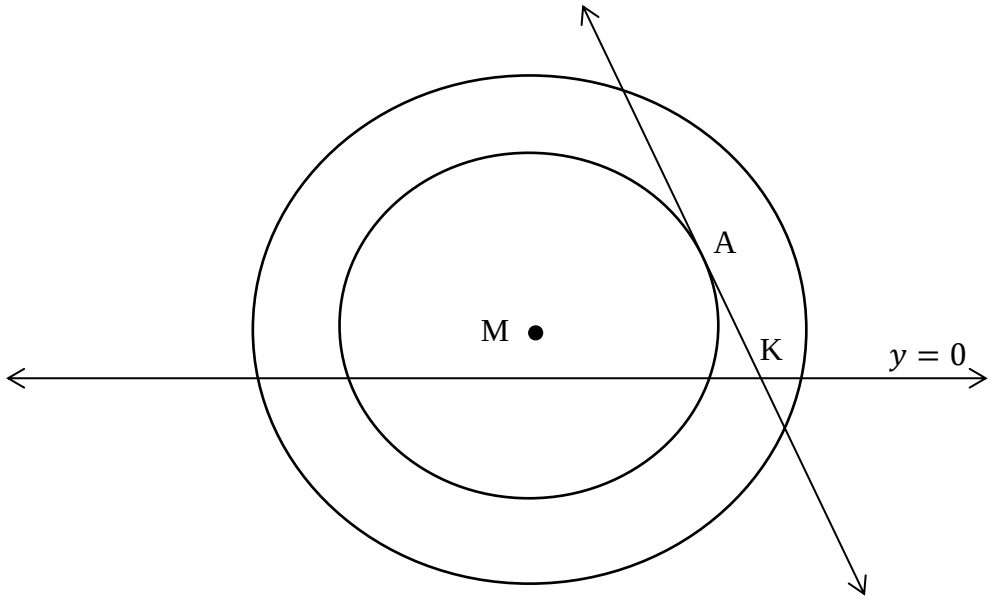
4.5 Determine the equation of the straight line passing through point D and parallel to AC. (4)

4.6 Determine, by showing ALL calculations, whether the *x*-intercept of the straight line CD also lies on the altitude (perpendicular height) from A to BC. (6)

[24]

QUESTION 5

In the figure below, M is the common centre of two circles. The larger circle has equation $x^2 + y^2 = 4y - 2x + 44$. The smaller circle touches the straight line $y = -x + 5$ at point A. The straight line $y = 0$ cuts both circles.



5.1 Determine the coordinates of M. (4)

5.2 Determine the coordinates of A. (5)

5.3 Determine the equation of the smaller circle. (3)

5.4 Write down the coordinates of K. (1)

5.5 The straight line $y = -x + 5$ meets the straight line $y = 0$ at point K. Determine the area of Δ AMK. (3)

[16]

QUESTION 6

6.1 If $\cos 26^\circ = \frac{1}{p}$. Determine the following in terms of p.

6.1.1 $\sin 26°$ (3)

6.1.2 $\cos 52°$ (3)

6.1.3 $\tan^2 64° \times (p + 1)$ (4)

6.2 Simplify: $\dfrac{\sin(-\beta) + \sin(360^\circ - \beta)}{\sin(180^\circ - \beta) + \sin 180^\circ}$ (5)

6.3 Determine the value of p, correct to two decimal places if $\theta = 82^\circ$ and $2p \tan\left(\frac{\theta}{2}\right) = \sin(2\theta)$ (3)

6.4 Prove the identity: $4 \sin\theta \cdot \cos^3\theta - 4 \cos\theta \cdot \sin^3\theta = \sin 4\theta$ (6)

[24]

QUESTION 7

Given: $f(x) = \sin(x - 30^\circ)$ and $g(x) = \cos 3x$.

7.1 Solve for x: $\cos 3x = \sin(x - 30^\circ)$ for $x \in [-60^\circ; 180^\circ]$. (7)

7.2 Draw the graphs of f and g for $x \in [-60^\circ; 180^\circ]$ on the grid provided. (6)

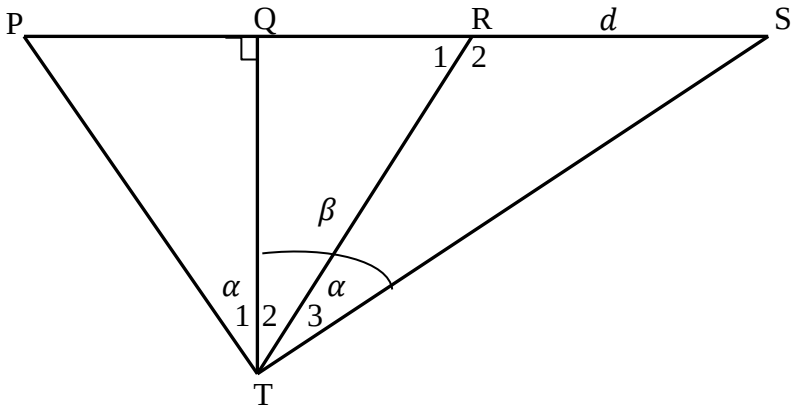
7.3 Use your graph and the answers to QUESTION 7.1 to answer the following question.

For which value(s) of x is $f(x) \times g(x) < 0$? (4)

[17]

QUESTION 8

Refer to the figure shown below. PQRS forms a straight road with TQ another road that is perpendicular to road PQRS. The distance, RS $= d$ kilometres.
$\hat{T}_1 = \hat{T}_3 = \alpha$ and $Q\hat{T}S = \beta$



8.1 Write down the size of $Q\hat{T}R$ in terms of α and β. (1)

8.2 In ΔSQT, write down the size of $\hat{S}$. (1)

8.3 In ΔPQT, write down the size of $\hat{P}$. (1)

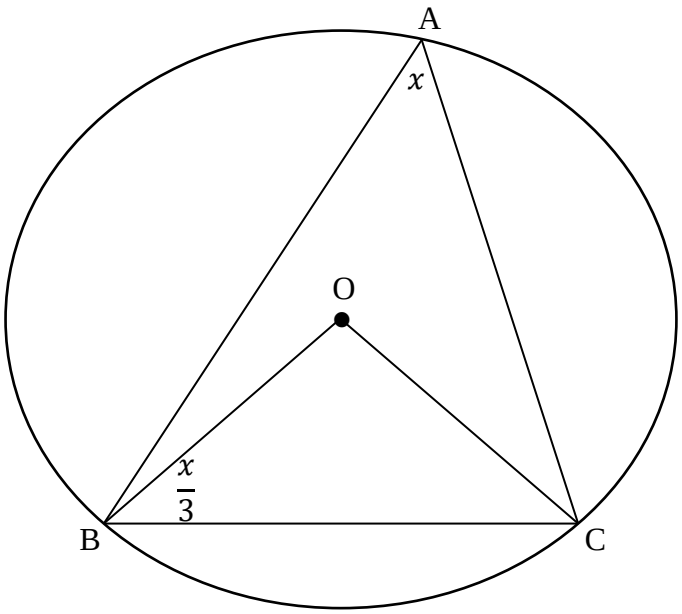
8.4 Determine the length of RT in terms of α and β. (3)

8.5 Hence, or otherwise, show that: $PR = \dfrac{d\cos\beta\sin\beta}{\sin\alpha.\cos\alpha}$ (3)
[9]

QUESTION 9

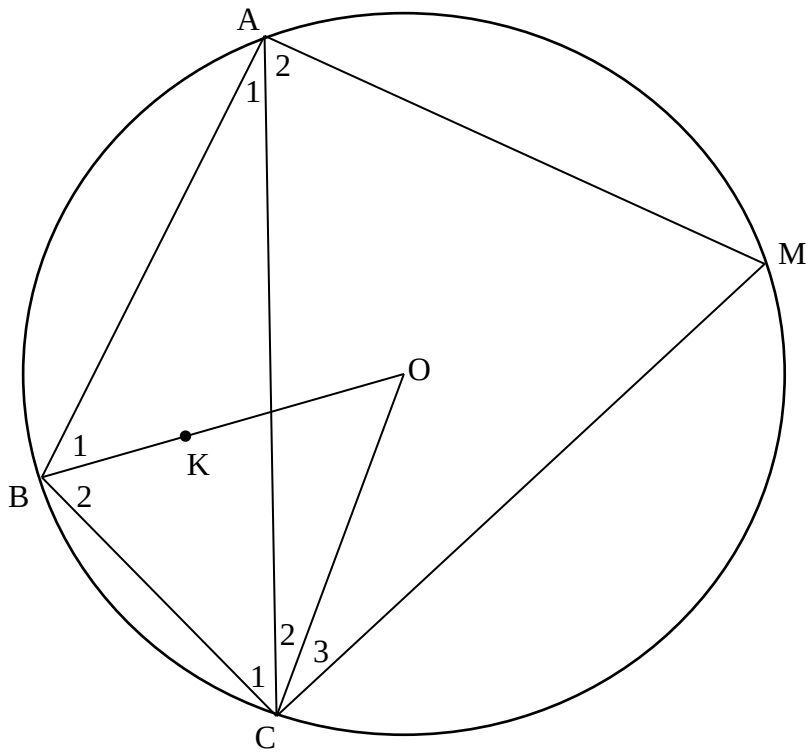
9.1 Complete the statement: The angle at the ... is equal to two times the angle at the circumference of the circle. (1)

9.2 See diagram below. O is the centre of the circle with points A, B and C on the circumference of the circle. $B\widehat{A}C = x$ and $O\widehat{B}C = \frac{x}{3}$.
Determine, with reasons, the value of x.



(6)

9.3 In the diagram below, O is the centre of the circle passing through A, B, C and M. K is the centre of the circle (not drawn) passing through points A, B and C of Δ ABC such that K lies on radius BO. $\widehat{A}_1 = 30^\circ$. BO bisects $A\widehat{B}C$.



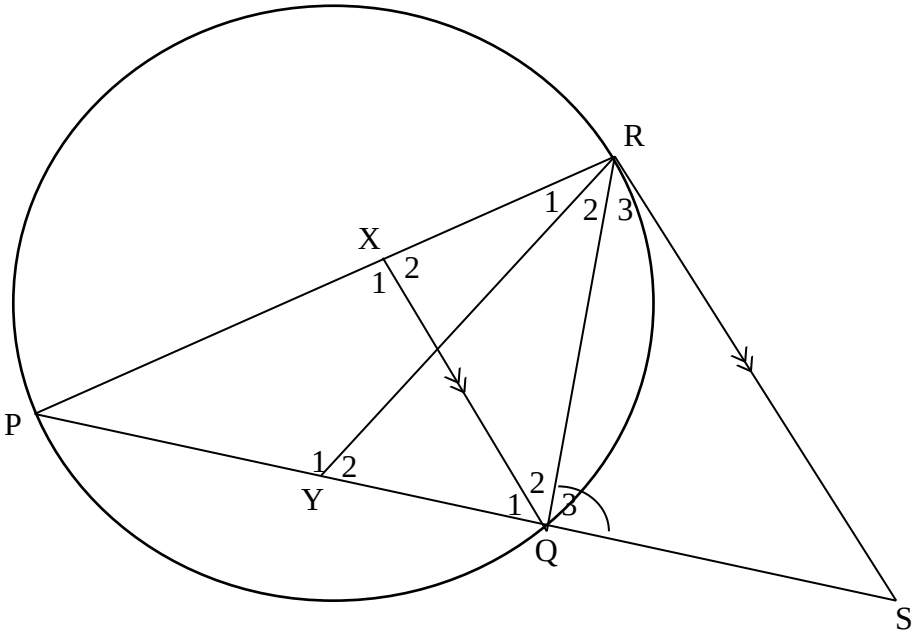
9.3.1 Determine the size of $\hat{B}_1$. (Supply reasons for your answer.) (5)

9.3.2 Prove that $\widehat{M} = 2\hat{A}_1$ (3)

[15]

QUESTION 10

In the diagram below, P, Q and R are points on a circle. YR bisects $P\hat{R}Q$ with Y on PQ. PQ is produced to meet RS at S such that SR = SY. QX ∥ SR.

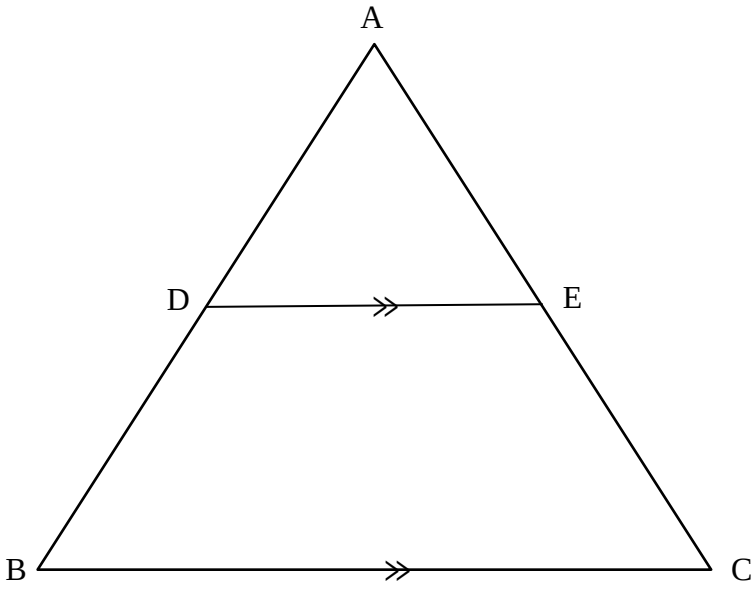


10.1 Prove that SR is a tangent to the circle at R. (6)

10.2 Prove that QR is a tangent to the circle passing through Q, X and P. (3)
[9]

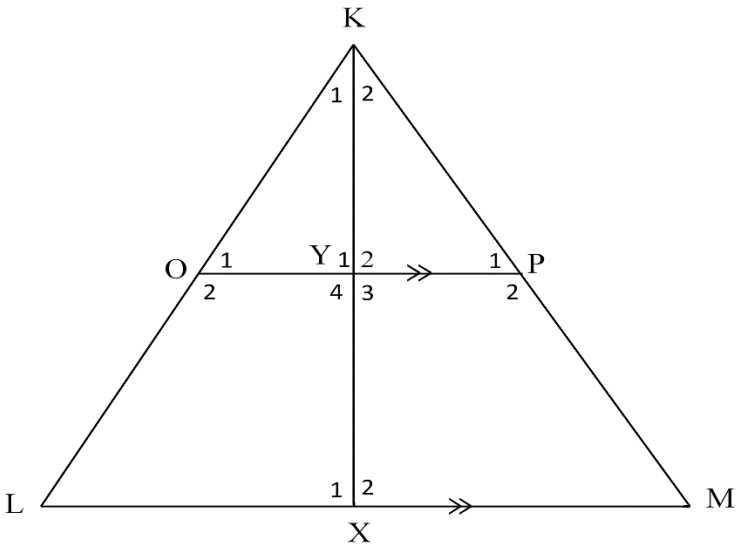
QUESTION 11

11.1 In the diagram below D and E are points on sides AB and AC of ΔABC such that DE ∥ BC. Use the diagram to prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$.



(6)

11.2 In the diagram below, OP ∥ LM such that the area of ΔKOP = area of quadrilateral OLMP. KYX is perpendicular to OP and LM at Y and X respectively.



Prove that:

11.2.1 $\Delta KOP ||| \Delta KLM$ (3)

11.2.2 $\frac{KY}{KX} = \frac{OP}{LM}$ (2)

11.2.3 $\frac{KO}{KL} = \frac{1}{\sqrt{2}}$ (6)

[17]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;; -1 < r < 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad \text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n} \qquad P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

Province of the
EASTERN CAPE
EDUCATION

LEARNER'S NAME: *LEERDER SE NAAM:*	

GRADE 12: *GRAAD 12:*	

NATIONAL/*NASIONALE* SENIOR CERTIFICATE/*SERTIFIKAAT*

GRADE/*GRAAD* 12

JUNE/*JUNIE* 2022

MATHEMATICS P2/*WISKUNDE V2*
SPECIAL ANSWER BOOK/*SPESIALE ANTWOORDEBOEK*

Marker/*Merker*					**Moderator's Initials / *Moderator se paraaf***																
Question *Vraag*	**Mark** *Punt*			**Initial** **Parafeer**	**Marks** *Punte*			**S M**	**Marks** *Punte*			**D M**	**Marks** *Punte*			**P M**	**Marks** *Punte*			**NM**
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11																				
TOTAL *TOTAAL*																				

This special answer book consists of 19 pages.
Hierdie spesiale antwoordeboek bestaan uit 19 bladsye.

QUESTION/*VRAAG* 1

1.1		(1)
1.2		(1)
1.3		(1)
1.4		(1)
1.5		(1)
		[5]

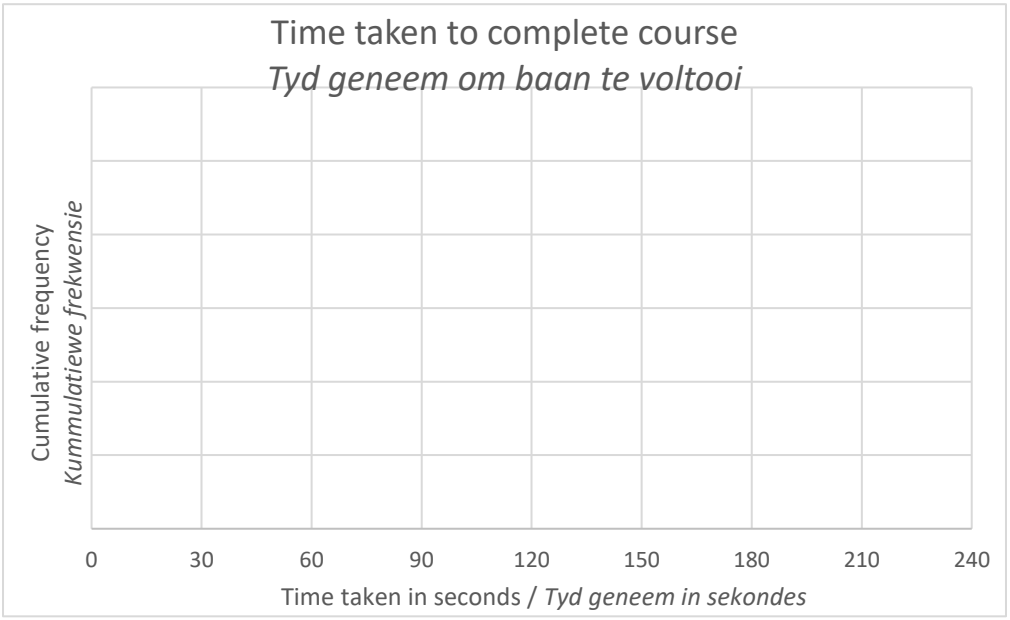
QUESTION/*VRAAG* 2

2.1

Time taken / *Tyd geneem*	No. of pupils *Aantal leerlinge*	Cumulative frequency *Kummulatiewe frekwensie*
$60 \leq t \leq 90$	3	
$90 \leq t \leq 120$	6	
$120 \leq t \leq 150$	7	
$150 \leq t \leq 180$	8	
$180 \leq t \leq 210$	6	

(1)

2.2



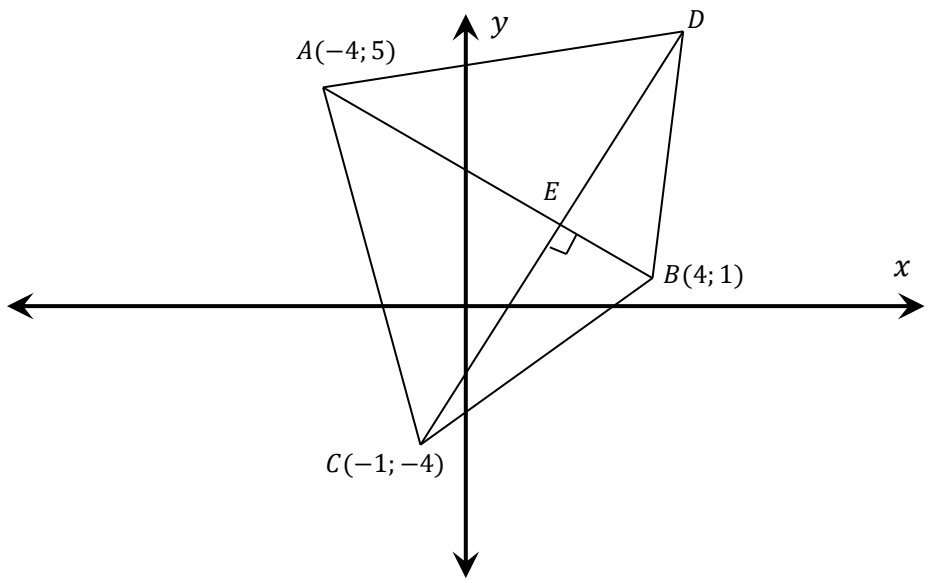
(4)

2.3.1		(1)
2.3.2		(1)
2.3.3		(1)
		[8]

QUESTION/*VRAAG* 3

3.1		(1)
3.2		(3)
3.3		(2)
		[6]

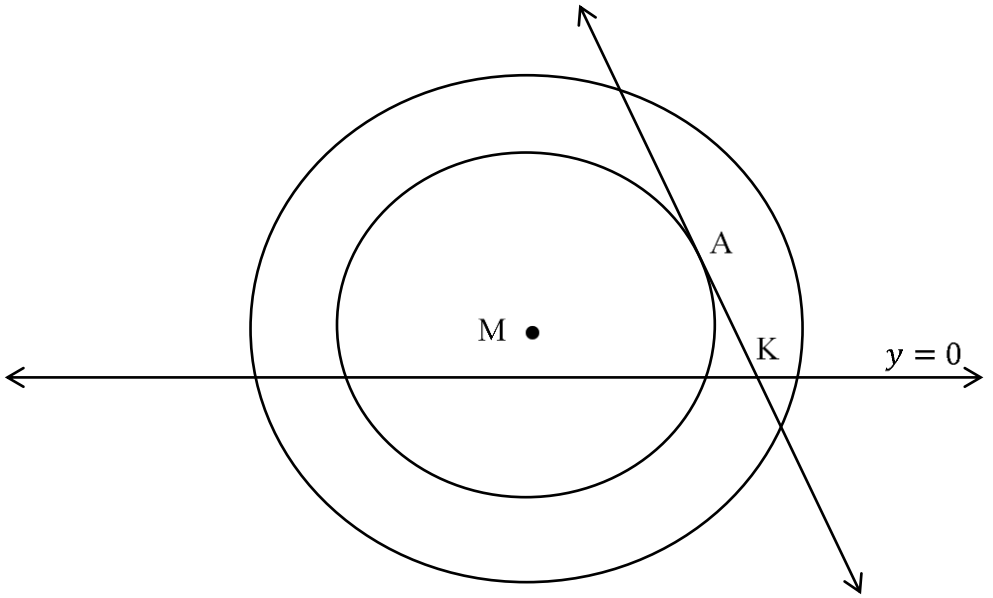
QUESTION 4/*VRAAG 4*



4.1		(2)
4.2		(4)
4.3		(6)

4.4		(2)
4.5		(4)
4.6		(6)
		[24]

QUESTION 5/*VRAAG 5*



5.1		(4)
5.2		(5)

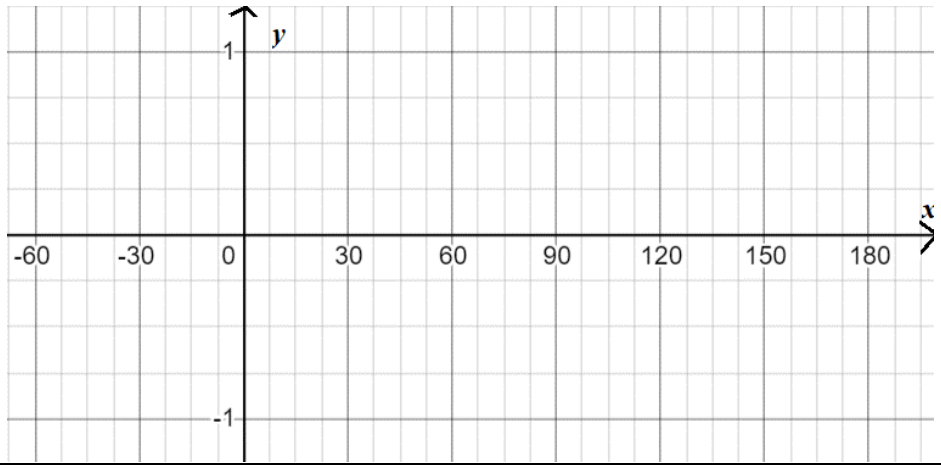
5.3		(3)
5.4		(1)
5.5		(3)
		[16]

QUESTION 6/*VRAAG 6*

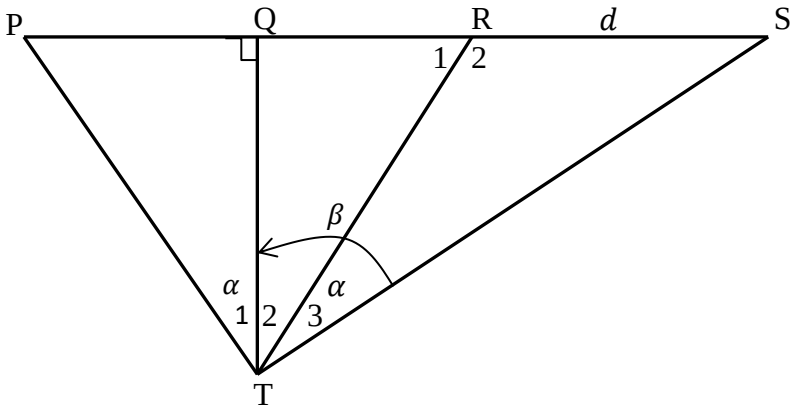
6.1	Draw your sketch here / *Teken jou skets hier.*	Do your calculations here / *Doen jou berekeninge hier.*	
6.1.1			(3)
6.1.2			(3)

6.1.3		(4)
6.2		(5)
6.3		(3)
6.4		(6)
		[24]

QUESTION 7/*VRAAG 7*

7.1		(7)
7.2		(6)
7.3		(4)
		[17]

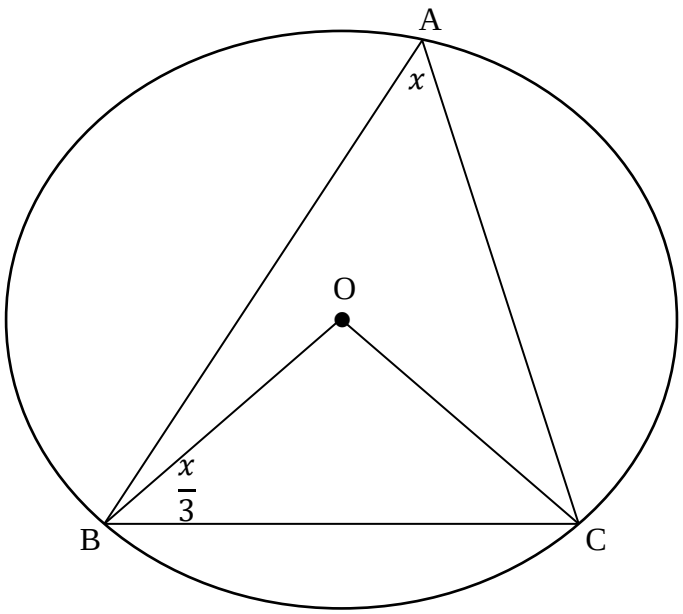
QUESTION 8/*VRAAG 8*



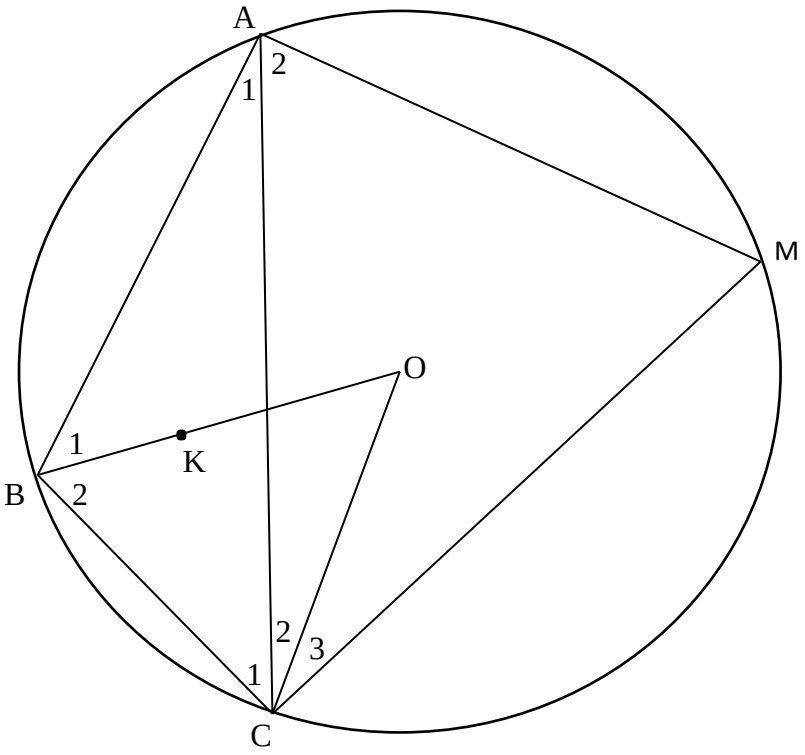
8.1		(1)
8.2		(1)
8.3		(1)
8.4		(3)
8.5		(3)
		[9]

QUESTION 9/*VRAAG 9*

9.1 (1)



9.2 (6)



9.3.1

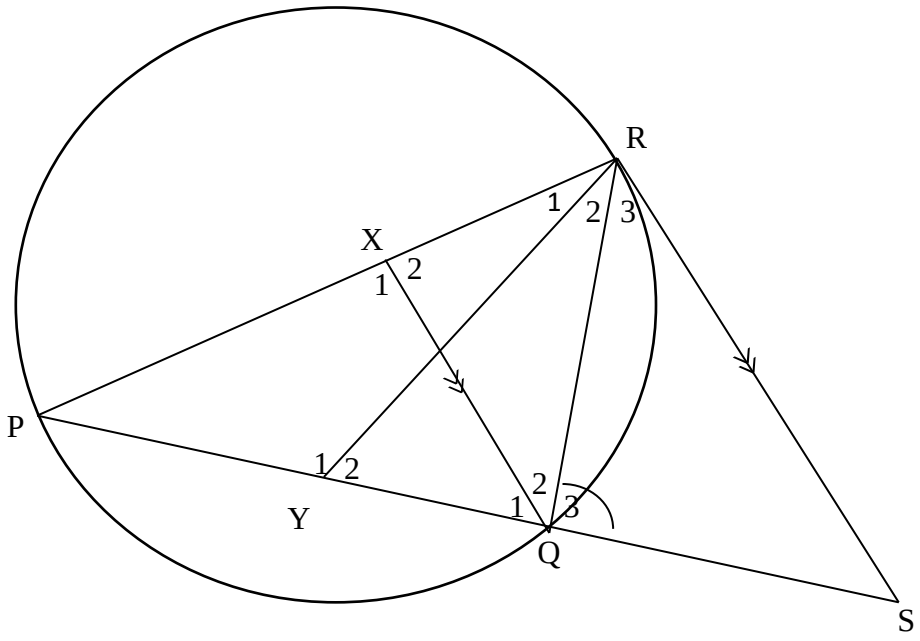
(5)

9.3.2

(3)

[15]

QUESTION 10/*VRAAG 10*

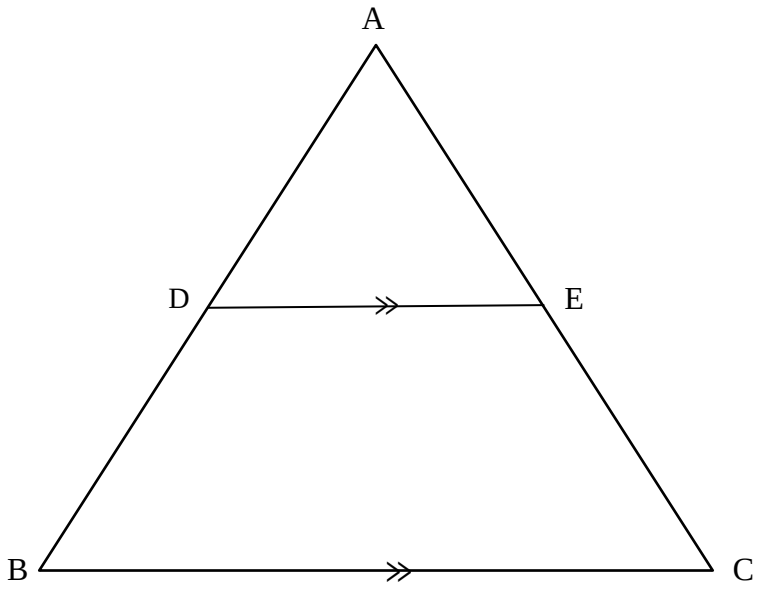


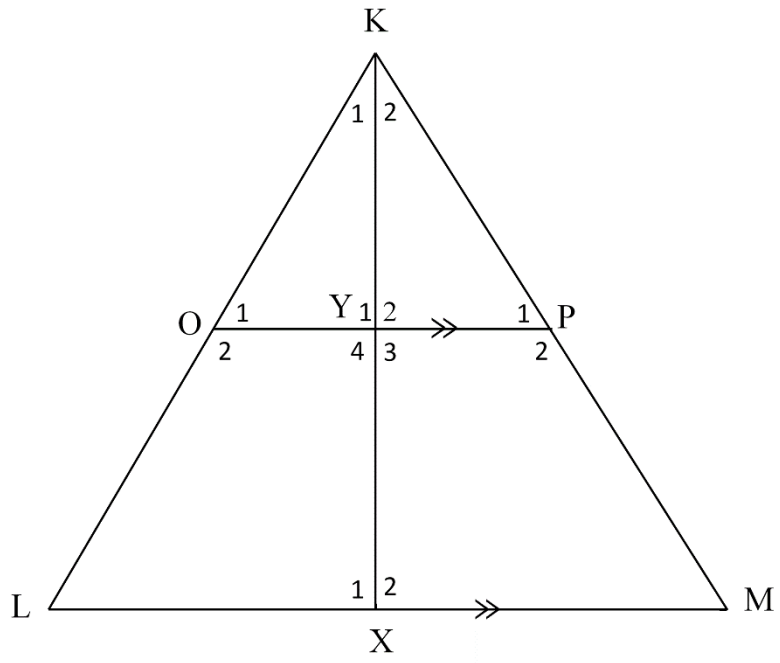
10.1 (6)

10.2 (3)

[9]

QUESTION 11/*VRAAG 11*

		
11.1		(6)

11.2		
11.2.1		(3)
11.2.2		(2)
11.2.3		(6)
		[17]

TOTAL/*TOTAAL*: 150

Additional Space/*Addisionele ruimte*

Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE/*GRAAD 12*

JUNE/*JUNIE* 2022

MATHEMATICS P2/*WISKUNDE V2* MARKING GUIDELINE/*NASIENRIGLYN*

MARKS/*PUNTE*: **150**

This marking guideline consists of 15 pages.
Hierdie nasienriglyn bestaan uit 15 bladsye.

QUESTION/*VRAAG* 1

1.1	B	✓ answer/*antwoord*	(1)
1.2	B	✓ answer/*antwoord*	(1)
1.3	75%	✓ answer/*antwoord*	(1)
1.4	Nothing. It remains the same. No change in standard deviation. *Niks. Dit bly dieselfde. Geen verandering in standaardafwyking.*	✓reason/*rede*	(1)
1.5	$\text{Semi} - \text{IQR: IKV} = \frac{75 - 30}{2}$ $\text{Semi} - \text{IQR: IKV} = 22{,}5$	✓ answer/*antwoord*	(1)
			[5]

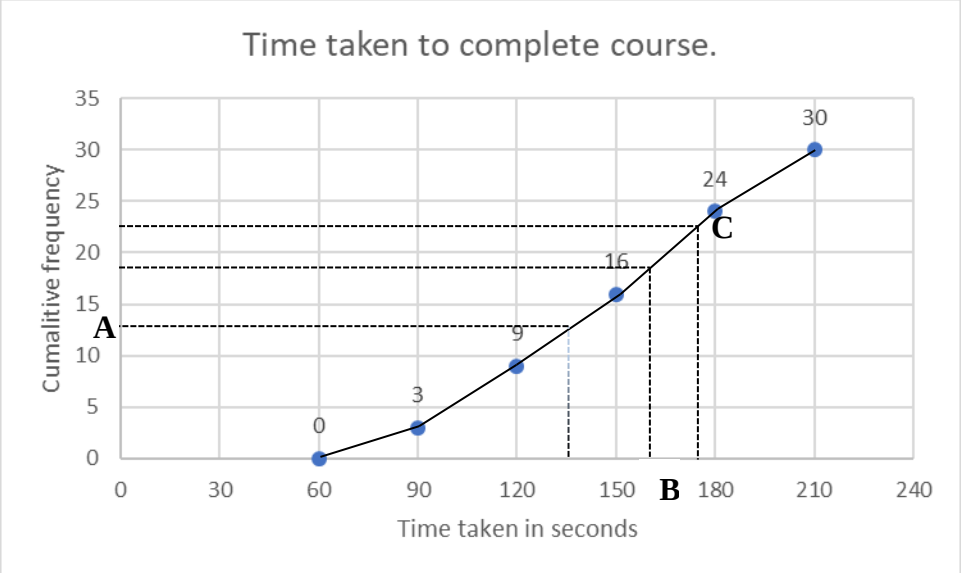
QUESTION/*VRAAG* 2

2.1

Time taken *Tyd geneem*	No. of pupils *Aantal leerlinge*	Cumulative frequency *Kummulatiewe frekwensie*
$60 \leq t \leq 90$	3	3
$90 \leq t \leq 120$	6	9
$120 \leq t \leq 150$	7	16
$150 \leq t \leq 180$	8	24
$180 \leq t \leq 210$	6	30

✓ for values *vir waardes* (1)

2.2



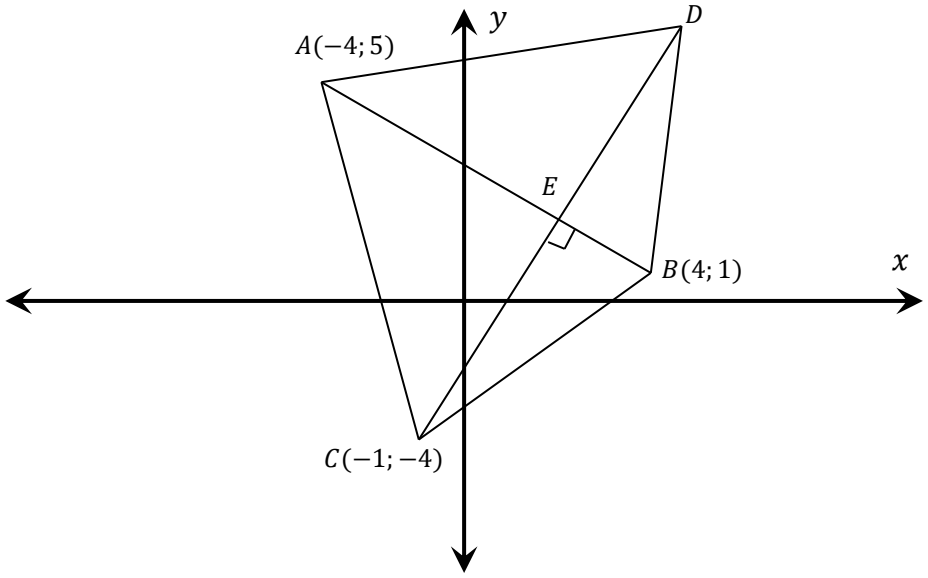
✓ anchor point/ *ankerpunt* (60;0)
✓ (120; 9)
✓ (150; 16)
✓ (210; 30) (4)

2.3	2.3.1	See diagram above / *Sien diagram hierbo*	✓ A	(1)
	2.3.2	See diagram above / *Sien diagram hierbo*	✓ B	(1)
	2.3.3	See diagram above / *Sien diagram hierbo*	✓ C	(1)
				[8]

QUESTION/*VRAAG* 3

3.1	Median score / *Mediaan telling* = $2x$	✓ answer/*antwoord*	(1)
3.2	Mean/*Gemiddelde* $= \frac{\sum x}{n}$ $= \frac{4(x+3)+3(2x)+2(x-1)+2(6)}{11}$ $= \frac{12x+22}{11}$	✓ substitution/*vervanging* ✓ simplification/*vereenvoudiging* ✓ answer/*antwoord*	(3)
3.3	Use of a calculator where the four values are as follows: *Gebruik van 'n sakrekenaar waar die vier waardes soos volg is:* 8 ; 10 ; 4 and/*en* 6 $sd(\sigma) = \sqrt{5}$	✓ four values/*vier waardes* ✓ answer/*antwoord*	(2)
			[6]

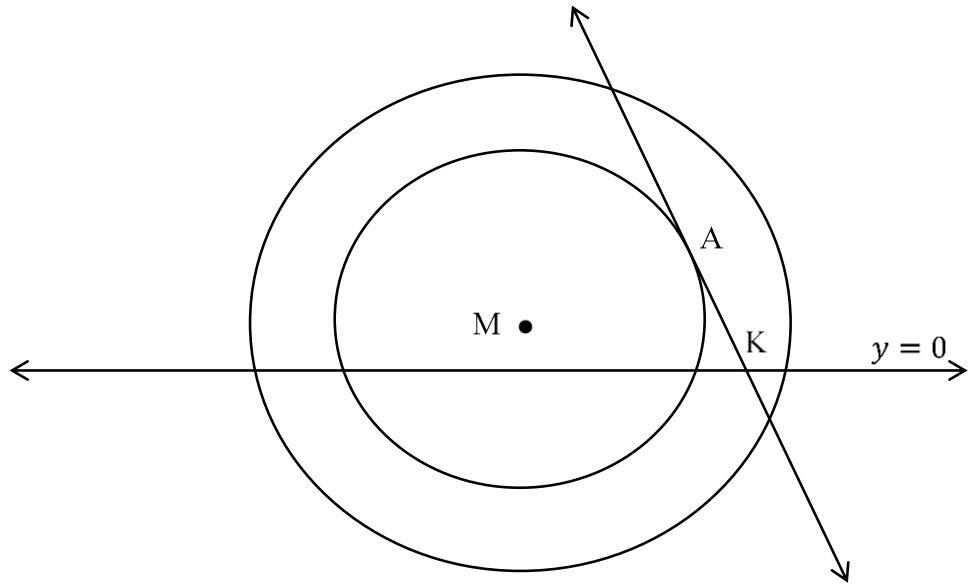
QUESTION/*VRAAG* 4



4.1	$m_{AB} = \frac{5-1}{-4-4}$ $= \frac{4}{-8}$ $m_{AB} = -\frac{1}{2}$ $\therefore\ m_{CD} = 2$	✓ subst. into gradient formula/ *verv. in gradiënt formule* ✓ $m_{AB} = -\frac{1}{2}$	(2)
4.2	$\therefore\ m_{CD} = 2$ $\overline{CD}:\quad y = 2x + c$ $-4 = 2(-1) + c$ $-2 = c$ $\overline{CD}:\quad y = 2x - 2$	✓ $m_{CD} = 2$ ✓sub of point/*verv. van punt* $(-1; -4)$ ✓ $-2 = c$ ✓ equation of CD / *vergelyking van* CD	(4)
4.3	$\overline{AB}:\quad y = -\frac{1}{2}x + c$ $1 = -\frac{1}{2}(4) + c$ $3 = c$ $y = -\frac{1}{2}x + 3$ $-\frac{1}{2}x + 3 = 2x - 2$ $-x + 6 = 4x - 4$ $5x = 10$ $x = 2$ $y = 2(2) - 2$ $y = 2$ $E(2; 2)$	✓ substitution of point (4; 1) *vervanging van punt* (4;1) ✓ equation of $\overline{AB}$ *vergelyking van* $\overline{AB}$ ✓ equating of AB and CD *gelykstel van* AB en CD ✓ *x* –value/*waarde* ✓ substitution of/*vervanging van* $x-$ value/*waarde* ✓ $y-$ value/*waarde*	(6)

4.4	$D(x;y)$ $\frac{x-1}{2}=2$ $\qquad$ $\frac{y-4}{2}=2$ $x-1=4$ $\qquad$ $y-4=4$ $x=5$ $\qquad$ $y=8$ $D(5;8)$	✓ $x=5$ ✓ $y=8$	(2)
4.5	$m_{AC}=\frac{5+4}{-4+1}$ $m_{AC}=-3$ Line parallel to AC has same gradient. *Lyn ewewydig aan AC het dieselfde gradient.* $y=-3x+c$ $8=-3(5)+c$ $c=23$ $y=-3x+23$	✓ substitution into gradient formula/*vervanging in gradiënt formule* ✓ $m_{AC}=-3$ ✓ $c=23$ ✓ equation of line/*vergelyking van lyn*	(4)
4.6	x intercept of CD : *x afsnit van* CD $2x-2=0$ $x=1$ $m_{BC}=1$ Equation of Altitude/*Vergelyking van hoogtelyn* $y=-x+c$ $5=-(-4)+c$ $c=1$ $\therefore y=-x+1$ x intercept of Altitude / *x-afsnit van hoogtelyn* $x=1$ x intercept of CD $=$ x intercept of altitude *x-afsnit van* CD = *x-afsnit van hoogtelyn*	✓ $x=1$ ✓ $m_{BC}=1$ ✓ gradient of altitude -1 *gradiënt van hoogtelyn* -1 ✓ substitution of point *vervanging van punt* ✓equation of altitude *vergelyking van hoogtelyn* ✓ $x=1$	(6)
			[24]

QUESTION/*VRAAG* 5

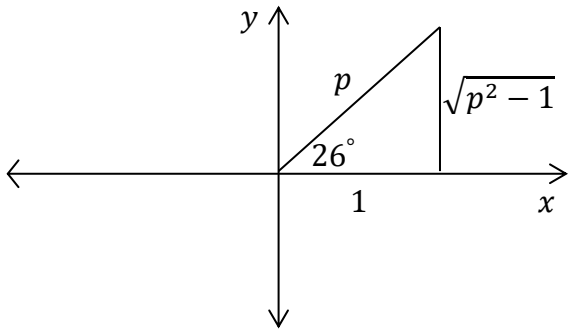


5.1	$x^2 + y^2 = 4y - 2x + 44$ $x^2 + 2x + 1 + y^2 - 4y + 4 = 44 + 1 + 4$ $(x + 1)^2 + (y - 2)^2 = 49$ $\therefore M(-1; 2)$	✓✓ completing the square *voltooiing van die vierkant* ✓ factorizing / *faktorisering* ✓ $M(-1; 2)$	(4)
5.2	$m_{MA} = \frac{y-2}{x+1}$ $\frac{y-2}{x+1} = 1$ $y - 2 = x + 1$ $y = x + 3$ $x + 3 = -x + 5$ $2x = 2$ $x = 1$ $\therefore \ y = 4$ $A(1; 4)$	✓ gradient of MA/*gradiënt van* MA ✓ equating it to 1/*stel dit gelyk aan* 1 ✓ making y or x the subject. *maak y* of *x die onderwerp* ✓ equating the two linear functions. *gelykstel van twee lineêre funksies* ✓ coordinates of $A(1; 4)$ *koördinate van* $A(1; 4)$	(5)
5.3	$(x + 1)^2 + (y - 2)^2 = r^2$ $(1 + 1)^2 + (4 - 2)^2 = r^2$ $8 = r^2$ $(x + 1)^2 + (y - 2)^2 = 8$	✓ substitution of A/*vervanging van* A ✓ $8 = r^2$ ✓ equation of the circle/*vergelyking van* . *die sirkel*	(3)

5.4	$K(5;0)$	✓ $K(5;0)$	(1)
5.5	$AK = \sqrt{32}$ Area of/*van* $\Delta AMK = \frac{1}{2} AK \times AM$ Area of/*van* $\Delta AMK = \frac{1}{2}\sqrt{32} \times \sqrt{8}$ Area of/*van* $\Delta AMK = 8$ units^2/*eenhede*^{2}	✓ $AK = \sqrt{32}$ ✓ $AM = \sqrt{8}$ ✓ 8 units^2/*eenhede*^{2}	(3)
			[16]

QUESTION/*VRAAG* 6

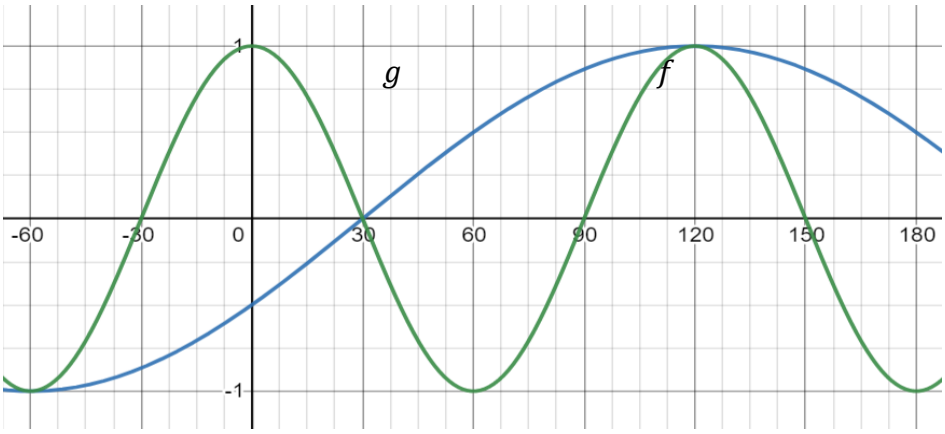
6.1



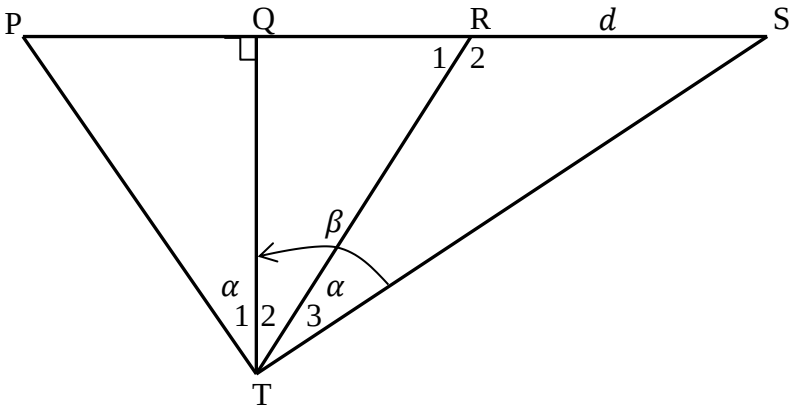
	6.1.1	$\sin 26^\circ = \frac{\sqrt{p^2-1}}{p}$	✓ sketch in Quadrant 1 *skets in kwadrant* 1 ✓ $\sqrt{p^2-1}$ ✓ answer/*antwoord*	(3)
	6.1.2	$\cos 52^\circ = \cos 2(26^\circ)$ $= 2\,cos^2 26^\circ - 1$ $= 2\left(\frac{1}{p}\right)^2 - 1$ $= \frac{2}{p^2} - 1$	✓$\cos 2(26^\circ)$ ✓$2\,cos^2 26^\circ - 1$ ✓ answer/*antwoord*	(3)
	6.1.3	$tan^2 64^\circ \times (p+1)$ $= \left(\frac{1}{\sqrt{p^2-1}}\right)^2 \times (p+1)$ $= \frac{1}{p^2-1} \times (p+1)$ $= \frac{1}{(p-1)(p+1)} \times (p+1)$ $= \frac{1}{p-1}$	✓ $\left(\frac{1}{\sqrt{p^2-1}}\right)^2$ ✓ $\frac{1}{p^2-1}$ ✓ $(p-1)(p+1)$ ✓ answer/*antwoord*	(4)

6.2	$\frac{\sin(-\beta)+\sin(360^\circ-\beta)}{\sin(180^o-\beta)+\sin 180^\circ}$ $=\frac{-\sin\beta+(-\sin\beta)}{\sin\beta+0}$ $=\frac{-2\sin\beta}{\sin\beta}$ $=-2$	✓ $-\sin\beta$ ✓ $-\sin\beta$ ✓ $\sin\beta$ ✓ simplification *vereenvoudiging* ✓ answer/*antwoord*	(5)
6.3	$2p\tan\left(\frac{\theta}{2}\right)=\sin(2\theta)$ $2p\tan\left(\frac{82^\circ}{2}\right)=\sin(2\times 82^\circ)$ $p=\frac{\sin 162^\circ}{2\tan 41^\circ}$ $p=0{,}16$	✓ substitution/*vervanging* ✓ simplification/*vereenvoudiging* ✓ answer/*antwoord*	(3)
6.4	$4\sin\theta.\cos^3\theta-4\cos\theta.\sin^3\theta$ $=\sin 4\theta$ $LHS/LK=4\sin\theta.\cos^3\theta$ $-4\cos\theta.\sin^3\theta$ $=4\sin\theta.\cos\theta(\cos^2\theta-\sin^2\theta)$ $=2\times 2\sin\theta\cos\theta(\cos 2\theta)$ $=2.\sin 2\theta.\cos 2\theta$ $=\sin 4\theta$ $=RHS/RK$	✓ common factor/*gemene faktor* ✓ $2\times 2\sin\theta\cos\theta$ ✓ $(\cos 2\theta)$ ✓$\sin 2\theta.$ ✓ $2.\sin 2\theta.\cos 2\theta$ ✓ answer/*antwoord*	(6)
			[24]

QUESTION/*VRAAG 7*

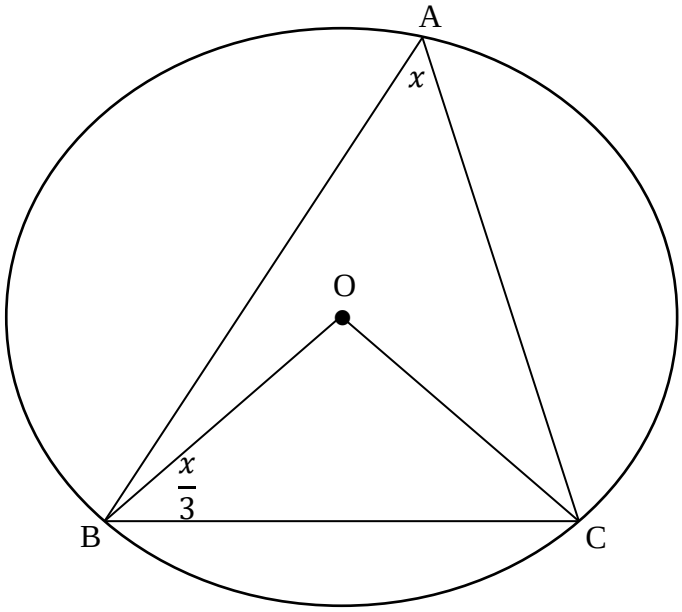
7.1	$\cos 3x = \sin(x - 30^\circ)$ $\cos 3x = \cos[90^\circ - (x - 30^\circ)]$ $\cos 3x = \cos[120 - x]$ $3x = 120 - x$ (ref angle) $3x = 120^\circ - x + k.360^\circ$ **OR/*OF*** $3x = 360^\circ - [120^\circ - x]k.360^\circ$ $4x = 120^\circ + k.360^\circ$ $\quad 2x = 240^\circ + k.360^\circ$ $x = 30^\circ + k.90^\circ$ $\quad x = 120^\circ + k.180^\circ$ $x = 30^\circ\ ;\ -60^\circ\ ;\ 120^\circ$	✓ Co- ratio *Ko-verhoud.* ✓ ref angle *verwys.* $\angle$ ✓ quadrant 1 *kwadrant* 1 ✓ quadrant 4 *kwadrant* 4 ✓ 30° ✓ -60° ✓ 120°	(7)
7.2	 ✓ Shape of f. ✓ x intercepts ✓ start and end points *Vorm van* f *x afsnitte* *begin en eindpunte* ✓ Shape of g. ✓ x intercepts ✓ start and end points *Vorm van* g *x afsnitte* *begin en eindpunte*		(6)
7.3	$-30^\circ < x < 30^\circ$ ***OR/OF*** $30^\circ < x < 90^\circ$ ***OR/OF*** $150^\circ < x < 180^\circ$	✓✓ $-30^\circ < x < 30^\circ$ ✓ $30^\circ < x < 90^\circ$ ✓ $150^\circ < x < 180^\circ$	(4)
			[17]

QUESTION/*VRAAG* 8

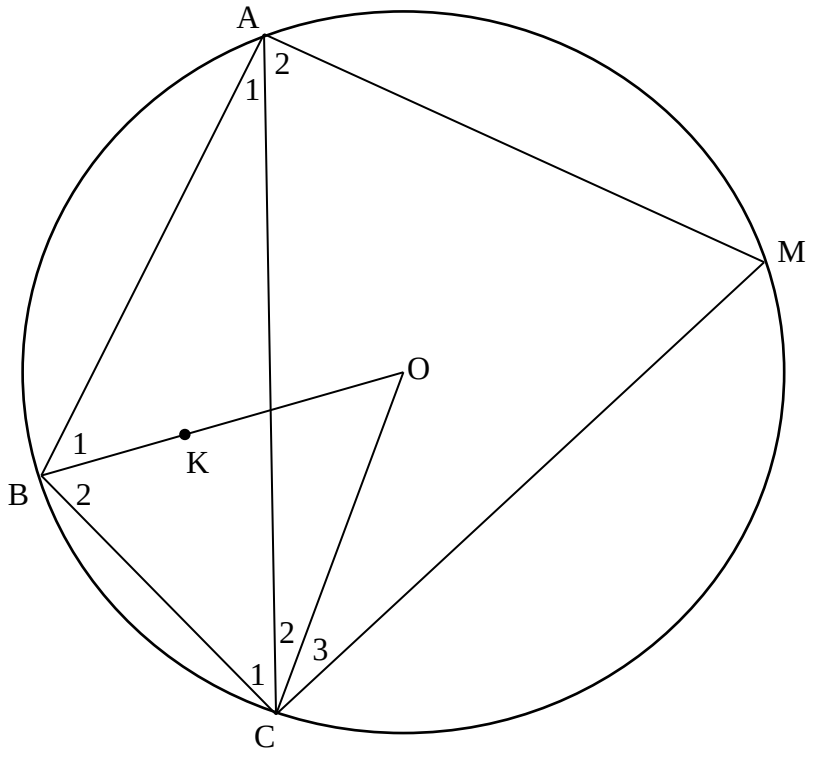


8.1	$Q\hat{T}R = \beta - \alpha$	✓ answer/*antwoord*	(1)
8.2	$\hat{S} = 90^\circ - \beta$	✓ answer/*antwoord*	(1)
8.3	$\hat{S} = 90^\circ - \alpha$	✓ answer/*antwoord*	(1)
8.4	In $\Delta\, RST$ $\frac{d}{\sin\alpha} = \frac{RT}{\sin(90^\circ - \beta)}$ $RT = \frac{d\cos\beta}{\sin\alpha}$	✓ use of sine rule *gebruik van sinusreël* ✓ use of co-function *gebruik van ko-funksie* ✓ answer/*antwoord*	(3)
8.5	$\frac{PR}{\sin\beta} = \frac{RT}{\sin(90^\circ - \alpha)}$ $PR = \frac{RT\sin\beta}{\cos\alpha}$ $PR = \frac{d\cos\beta\sin\beta}{\sin\alpha\cos\alpha}$	✓ use of sine rule *gebruik van sinusreël* ✓ sub of RT *vervanging van* RT ✓ answer/*antwoord*	(3)
			[9]

QUESTION/*VRAAG* 9

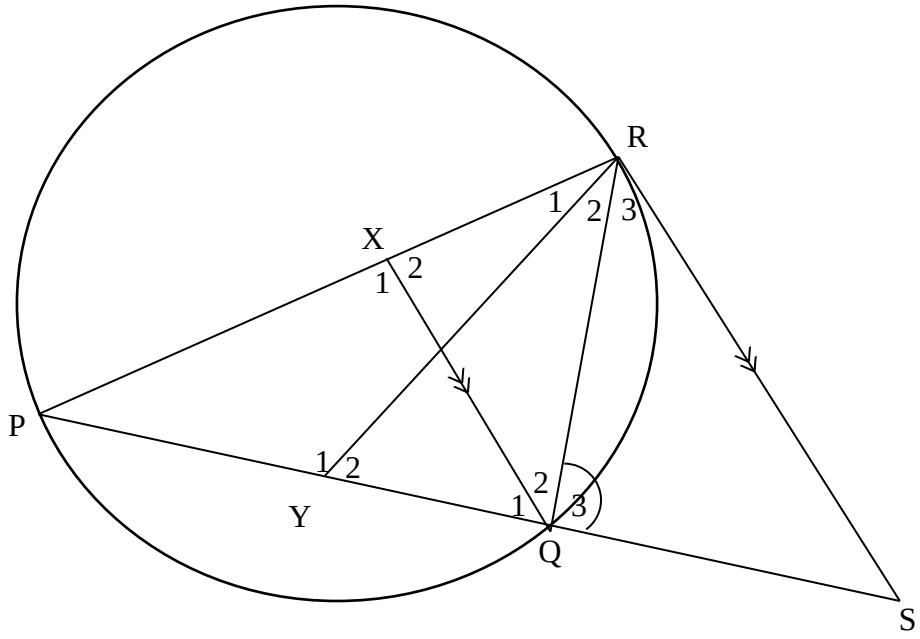
9.1	centre / *middelpuntshoek*	✓ answer/*antwoord*	(1)
			
9.2	$B\widehat{O}C = 2x$ [angle at centre $= 2 \times$ angle at circum] [*Middelpuntshoek* = 2 $\times$ *Omtrekshoek*] $B\widehat{C}O = \frac{x}{3}$ [angles opp $=$ sides ; OB $=$ OC] [*hoeke teenoor = sye* ; OB = OC] $\therefore \frac{x}{3} + \frac{x}{3} + 2x = 180^\circ$ [sum of angles of Δ] [*som van hoeke van* Δ] $8x = 540^\circ$ $x = 67{,}5^\circ$	✓ statement / *stelling* (**S**) ✓ reason/*rede* (**R**) ✓statement / *stelling* (**S**) ✓reason/*rede* (**R**) ✓ **S/R** ✓ answer/*antwoord*	(6)

9.3

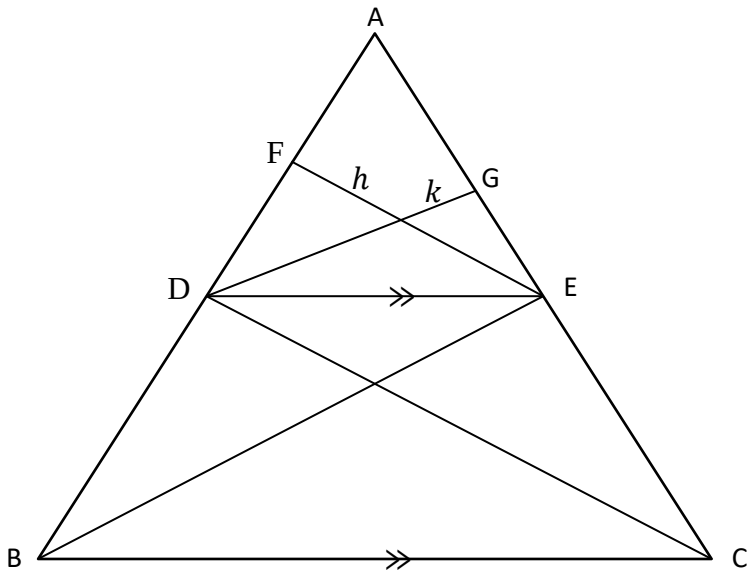


	9.3.1	$\hat{A}_1 = 30^\circ$ $B\hat{O}C = 60^\circ$ [angle at centre = 2 × angle at circum] [*middelpuntshoek* = 2 x *omtrekshoek*] $\hat{B}_2 = \hat{C}_1 + \hat{C}_2$ [angles opp = sides ; OB = OC] [*hoeke teenoor = sye* ; OB = OC] $\therefore 2\hat{B}_2 = 180^\circ - 60^\circ$ [sum of angles of Δ] [*som van hoeke van* Δ] $\hat{B}_2 = 60^\circ$ $\hat{B}_1 = 60^\circ$ [BO bisects A$\hat{B}$C] / [BO *halveer* A$\hat{B}$C]	✓ S/R ✓ S/R ✓ S/R ✓ S ✓ S/R	(5)
	9.3.2	$\hat{A}_1 = 30^\circ$ $\hat{B}_1 + \hat{B}_2 + \hat{M} = 180^\circ$ [opp angles of cyclic quad] [*teenoorst. hoeke van koordevierhoek*] $\hat{M} = 180^\circ - 120^\circ$ $\hat{M} = 60^\circ$ $\therefore \hat{M} = 2 \times \hat{A}_1$	 ✓ S/R ✓ S ✓ answer/ *antwoord*	(3)
				[15]

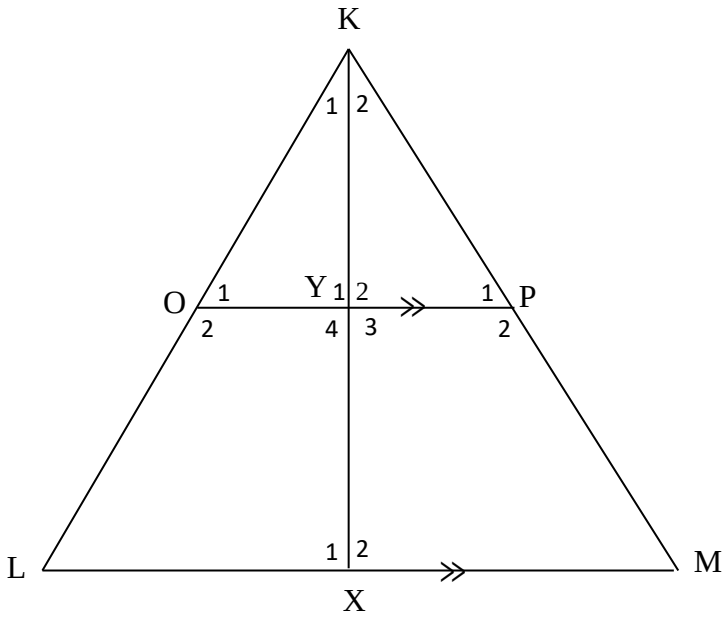
QUESTION/*VRAAG* 10

			
10.1	$\widehat{R}_1 = \widehat{R}_2$ [YR bisects $P\widehat{R}Q$] / [YR *halveer* $P\widehat{R}Q$] $\widehat{R}_2 + \widehat{R}_3 = \widehat{Y}_2$ [angles opp = sides; RS = YS] [*hoeke teenoor = sye* ; RS = YS] $\widehat{R}_1 + \widehat{P} = \widehat{Y}_2$ [ext angle of Δ]/[*buitehoek van* Δ] $\therefore\ \widehat{R}_3 = \widehat{P}$ ∴ SR = tangent/*raaklyn* [converse tan − chord theorem] [*omgekeerde raaklyn-koord stelling*]	✓ S/R ✓ S ✓ R ✓ S ✓ R ✓ S	(6)
10.2	$\widehat{Q}_2 = \widehat{R}_3$ [alt angles = ; SR ∥ QX] [*verw. hoeke* = ; SR || QX] $\widehat{P} = \widehat{R}_3$ [proven] / [*bewys*] $\widehat{Q}_2 = \widehat{P}$ ∴ QR = tangent [converse tan − chord theorem] QR = *raaklyn* [*omgekeerde raaklyn-koord stelling*]	✓ S/R ✓ S ✓ S	(3)
			[9]

QUESTION/*VRAAG* 11



11.1	Proof / *Bewys*: Construct Perpendicular heights DG (k) and EF (h) in $\Delta\ ADE$. Join BE and DC. *Teken loodregte hoogtes* DG (k) *en* EF (h) *in* $\Delta\ ADE$ *Verbind* BE *en* DC $\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\frac{1}{2}\text{AD. h}}{\frac{1}{2}\text{BD. h}} = \frac{\text{AD}}{\text{BD}}$ $\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CED} = \frac{\frac{1}{2}\text{AE. k}}{\frac{1}{2}\text{CE. k}} = \frac{\text{AE}}{\text{CE}}$ But/*Maar* Area ΔBDE = Area ΔCED (same base DE, same height) (*dieselfde basis* DE, *dieselfde hoogte*) DE ∥ BC $\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CED}$ $\therefore\ \frac{\text{AD}}{\text{BD}} = \frac{\text{AE}}{\text{CE}}$	✓ constr. *konstruk.* ✓ S/R ✓S/R ✓S ✓ R ✓ S	(6)

11.2				
	11.2.1	In ΔKOP and/*en* ΔKLM $\hat{K} = \hat{K}$ [common] / [*gemeen*] $\hat{O}_1 = \hat{L}$ [corresponding angles = ; OP ‖ LM] [*ooreenkomstige hoeke* = ; OP ‖ LM] $\hat{P}_1 = \hat{M}$ [corresponding angles =, OP ‖ LM] [*ooreenkomstige hoeke* = ; OP ‖ LM] $\therefore$ Δ KOP ||| ΔKLM [A; A; A]	✓ S/R ✓S/R ✓ R	(3)
	11.2.2	$\frac{KO}{KL} = \frac{OP}{LM}$ [similarity : *gelykvormig*] $\frac{KO}{KL} = \frac{KY}{KX}$ [line parallel to one side of Δ] [*lyn ewewydig aan een sy van* Δ] $\therefore$ $\frac{KY}{KX} = \frac{OP}{LM}$	✓ S/R ✓ S/R	(2)
	11.2.3	Area of Δ KOP = Area of Quad/*Vierhoek* OLMP $\therefore$ Area of Δ KLM $= 2 \times$ Area of Δ KOP $\frac{1}{2} \times LM \times KX = 2 \times \frac{1}{2} \times OP \times KY$ $\frac{1}{2} = \frac{OP.KY}{LM.KX}$ but/*maar* $\frac{OP}{LM} = \frac{KY}{KX}$ $\frac{OP^2}{LM^2} = \frac{1}{2}$ $\frac{OP}{LM} = \frac{1}{\sqrt{2}}$ $\frac{OP}{LM} = \frac{KO}{KL}$ [Δ KOP |||ΔKLM] $\therefore \frac{KO}{KL} = \frac{1}{\sqrt{2}}$	✓ S ✓ S ✓S ✓S ✓S ✓ S/R	(6)
				[17]
			TOTAL/*TOTAAL:*	**150**