Province of the
EASTERN CAPE
EDUCATION

NASIONALE
SENIOR SERTIFIKAAT

GRAAD 12

JUNIE 2022

WISKUNDE V2

PUNTE: **150**

TYD: **3 uur**

Hierdie vraestel bestaan uit 12 bladsye en ’n antwoordeboek van 19 bladsye.

INSTRUKSIES EN INLIGTING

Lees die volgende instruksies noukeurig deur voordat die vrae beantwoord word.

1. Hierdie vraestel bestaan uit ELF vrae. Beantwoord AL die vrae in die SPESIALE ANTWOORDEBOEK wat verskaf word.

2. Dui ALLE berekeninge, diagramme, grafieke, ensovoorts wat jy in die bepaling van jou antwoorde gebruik het, duidelik aan.

3. Jy mag 'n goedgekeurde sakrekenaar (nieprogrammeerbaar en niegrafies) gebruik, tensy anders aangedui.

4. Volpunte sal nie noodwendig aan antwoorde alleen toegeken word nie.

5. Indien nodig, moet antwoorde tot TWEE desimale plekke afgerond word, tensy anders aangedui.

6. Diagramme is NIE noodwendig volgens skaal geteken NIE.

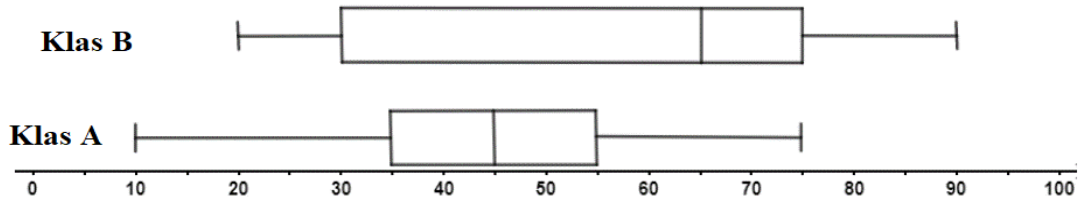
7. Nommer jou antwoorde korrek volgens die nommeringstelsel wat in hierdie vraestel gebruik is.

8. 'n Inligtingsblad met formules is aan die einde van die vraestel ingesluit.

9. Skryf netjies en leesbaar.

VRAAG 1

Die mond-en-snor diagramme hieronder toon die Wiskunde uitslae van klas A en klas B in die Junie eksamen. Dit word ook gegee dat klas B 'n mediaan van 65% het.



1.1 Watter klas het die top leerders? (1)

1.2 Bepaal watter klas die grootste Interkwartielvariasiewydte het? (1)

1.3 Watter persentasie van klas A het minder as 60% behaal? (1)

1.4 Indien al die leerders in klas A 'n ekstra 5% gegee word, wat sal met die standaardafwyking van die punte in klas A gebeur? (1)

1.5 Bepaal die semi-interkwartielvariasiewydte van klas B. (1)
[5]

VRAAG 2

'n Groep van 30 leerlinge was, tydens hulle graad 11-kamp, gevra om 'n hindernisbaan te voltooi. Die tye (in sekondes) wat dit leerlinge geneem het om die hindernisbaan te voltooi word in die tabel hieronder gegee.

Tyd geneem	$60 \leq t < 90$	$90 \leq t < 120$	$120 \leq t < 150$	$150 \leq t < 180$	$180 \leq t < 210$
Aantal leerlinge	3	6	7	8	6

2.1 Voltooi die kumulatiewe frekwensietabel vir bostaande data in die SPESIALE ANTWOORDEBOEK. (1)

2.2 Teken 'n kumulatiewe frekwensiekurwe vir die bostaande data op die rooster wat voorsien is. (4)

2.3 Dui op jou grafiek aan waar jy sal aflees:

2.3.1 Die aantal leerlinge wat 135 sekondes geneem het om die baan te voltooi. (Gebruik die letter A) (1)

2.3.2 Die waarde van t as 60% van die leerlinge minder as t sekondes geneem het om die hindernisbaan to voltooi. (Gebruik die letter B) (1)

2.3.3 Die 75ste persentiel. (Gebruik die letter C) (1)
[8]

VRAAG 3

Beskou die volgende stel van vier positiewe heelgetalle en hulle frekwensie.

Telling	$x+3$	$2x$	$x-1$	6
Frekwensie	4	3	2	2

3.1 Bepaal die mediaan-telling. (1)

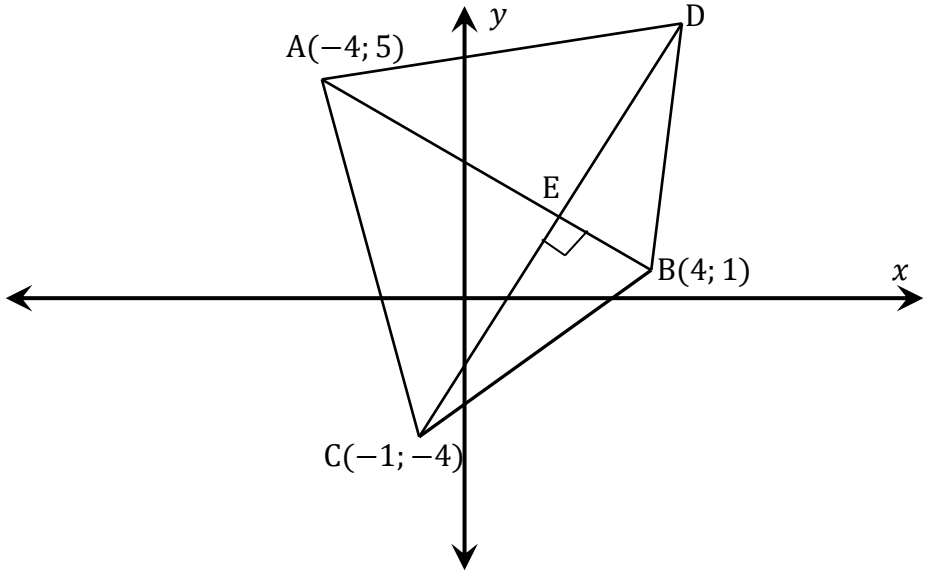
3.2 Bepaal die gemiddelde in terme van x. (3)

3.3 Indien slegs tellings in ag geneem word (sonder frekwensie), bepaal die standaardafwyking as dit gegee word dat $x = 5$. (2)

[6]

VRAAG 4

In die diagram hieronder is die koördinate van $\mathrm{A}(-4;5), \mathrm{C}(-1;-4)$ en $\mathrm{B}(4;1)$ die hoekpunte van 'n driehoek in die Cartesiese vlak. $\mathrm{CE} \perp \mathrm{AB}$ met E op AB. E is die middelpunt van reguitlyn CD.



4.1 Bepaal die gradiënt van AB. (2)

4.2 Bepaal die vergelyking van CD. (4)

4.3 Bepaal die koördinate van E. (6)

4.4 Bepaal die koördinate van D. (2)

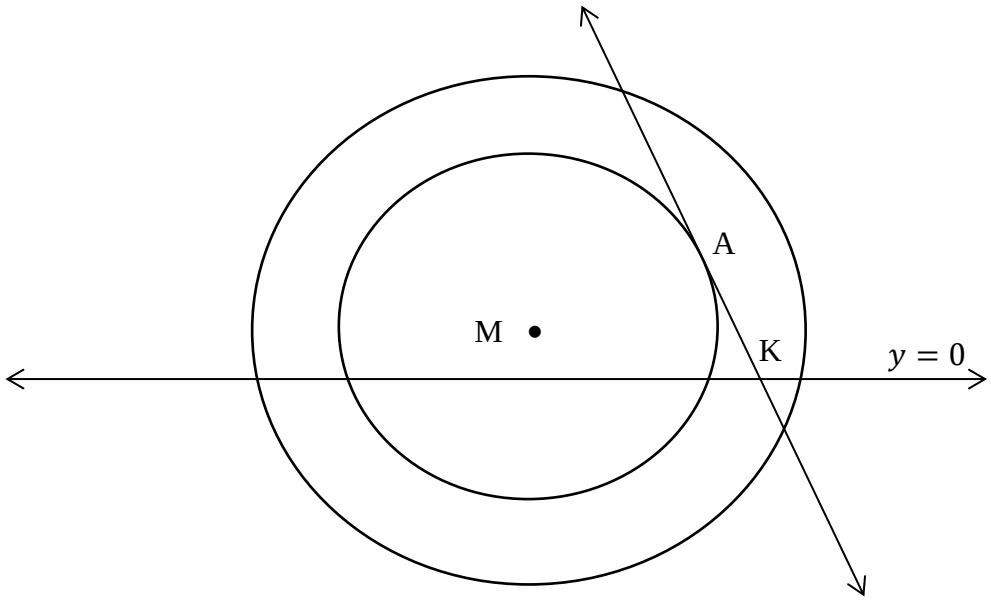
4.5 Bepaal die vergelyking van die reguitlyn, ewewydig aan AC en wat deur punt D gaan. (4)

4.6 Bepaal, deur ALLE berekeninge te toon, of die *x*-afsnit van die reguitlyn CD ook op die hoogtelyn van A na BC lê. (6)

[24]

VRAAG 5

In die figuur hieronder, is M die gemene middelpunt van twee sirkels. Die groter sirkel het vergelyking $x^2 + y^2 = 4y - 2x + 44$.
Die kleiner sirkel raak die reguitlyn $y = -x + 5$ by punt A. Die reguitlyn $y = 0$ sny beide sirkels.



5.1 Bepaal die koördinate van M. (4)

5.2 Bepaal die koördinate van A. (5)

5.3 Bepaal die vergelyking van die kleiner sirkel. (3)

5.4 Skryf die koördinate van K neer. (1)

5.5 Die reguitlyn $y = -x + 5$ ontmoet die reguitlyn $y = 0$ by punt K. Bepaal die oppervlakte van Δ AMK. (3)
[16]

VRAAG 6

6.1 As $\cos 26^{o} = \frac{1}{p}$. Bepaal die volgende in terme van p.

6.1.1 $\sin 26^{\circ}$ (3)

6.1.2 $\cos 52^{\circ}$ (3)

6.1.3 $\tan^2 64^{\circ} \times (p + 1)$ (4)

6.2 Vereenvoudig: $\frac{\sin(-\beta) + \sin(360^{\circ} - \beta)}{\sin(180^{\circ} - \beta) + \sin 180^{\circ}}$ (5)

6.3 Bepaal die waarde van p, korrek tot twee desimale plekke, as $\theta = 82^{\circ}$ en $2p \tan\left(\frac{\theta}{2}\right) = \sin(2\theta)$. (3)

6.4 Bewys die identiteit: $4 \sin\theta \cdot \cos^3\theta - 4 \cos\theta \cdot \sin^3\theta = \sin 4\theta$ (6)

[24]

VRAAG 7

Gegee: $f(x) = \sin(x - 30^{\circ})$ en $g(x) = \cos 3x$.

7.1 Los op vir x: $\cos 3x = \sin(x - 30^{o})$ vir $x \in [-60^{\circ}; 180^{\circ}]$. (7)

7.2 Teken die grafieke van f en g vir $x \in [-60^{\circ}; 180^{\circ}]$ op die voorsiende rooster. (6)

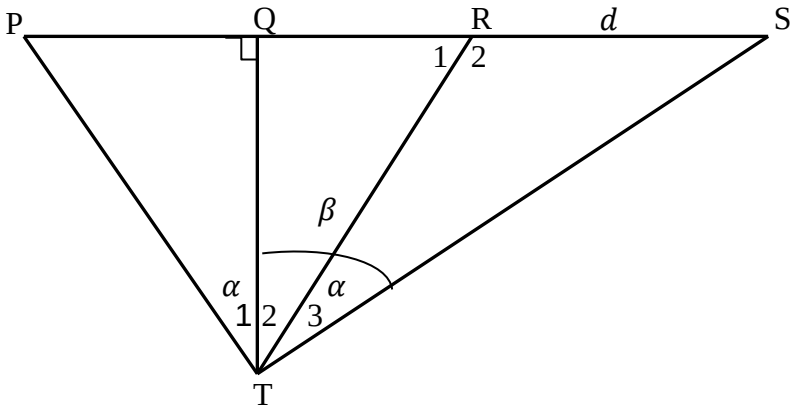
7.3 Gebruik jou grafiek en antwoorde in VRAAG 7.1 om die volgende vraag te beantwoord.

Vir watter waarde(s) van x is $f(x) \times g(x) < 0$? (4)

[17]

VRAAG 8

Verwys na die onderstaande figuur. PQRS stel 'n reguit pad voor met TQ 'n ander pad wat loodreg met pad PQRS is. Die afstand, RS $= d$ kilometer.
$\hat{T}_1 = \hat{T}_3 = \alpha$ en $Q\hat{T}S = \beta$



8.1 Skryf die grootte van $Q\hat{T}R$ in terme van α en β neer. (1)

8.2 In ΔSQT, skryf die grootte van $\hat{S}$ neer. (1)

8.3 In ΔPQT, skryf die grootte van $\hat{P}$ neer. (1)

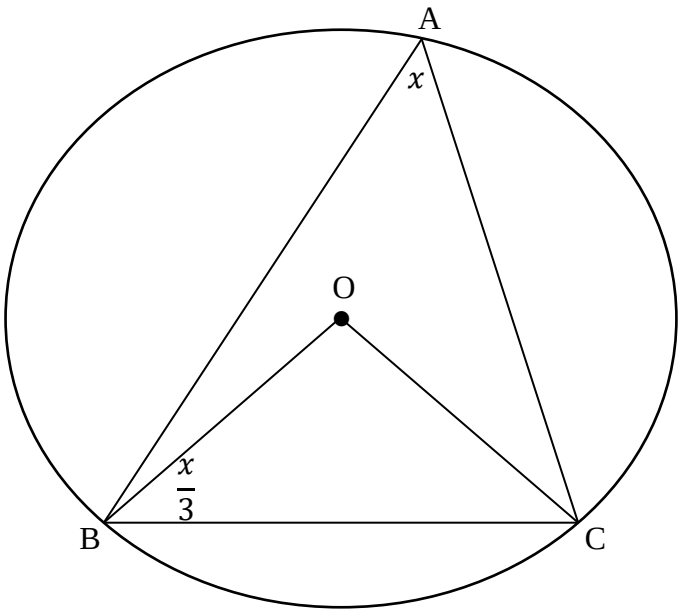
8.4 Bepaal die lengte van RT in terme van α en β. (3)

8.5 Vervolgens, of andersins, toon aan dat: $PR = \dfrac{d\cos\beta\sin\beta}{\sin\alpha.\cos\alpha}$ (3)
[9]

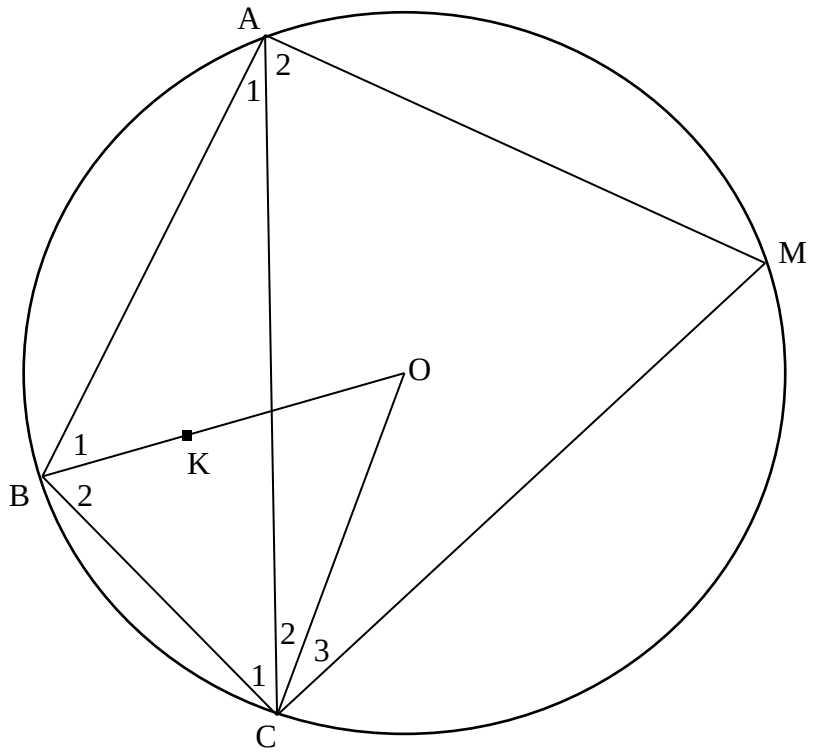
VRAAG 9

9.1 Voltooi die stelling: Die hoek by die ... is gelyk aan tweemaal die omtrekshoek van die sirkel. (1)

9.2 Sien diagram hieronder. O is die middelpunt van die sirkel met punte A, B en C op die omtrek van die sirkel. $B\widehat{A}C = x$ en $O\widehat{B}C = \frac{x}{3}$.
Bepaal, met redes, die waarde van x. (6)



9.3 In die diagram hieronder, is O die middelpunt van die sirkel wat deur A, B, C en M gaan. K is die middelpunt van 'n sirkel (nie geteken nie) wat deur punte A, B en C van Δ ABC gaan sodat K op radius BO lê. $\widehat{A}_1 = 30^\circ$. BO halveer $A\widehat{B}C$.



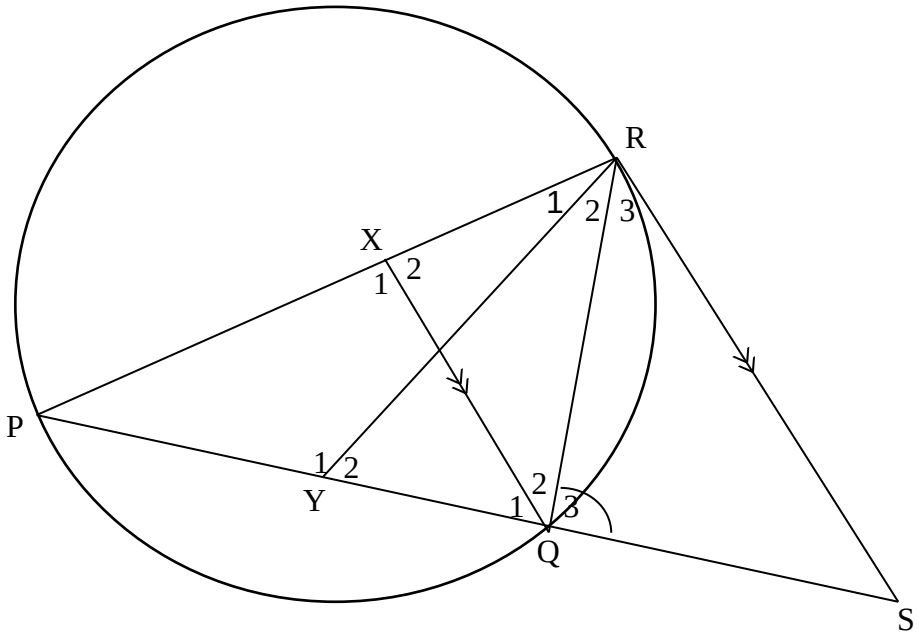
9.3.1 Bepaal die grootte van $\hat{B}_1$. (Verskaf redes vir jou antwoord.) (5)

9.3.2 Bewys dat $\widehat{M} = 2\hat{A}_1$ (3)

[15]

VRAAG 10

In die diagram hieronder is, P, Q en R punte op 'n sirkel. YR halveer $P\hat{R}Q$ met Y op PQ. PQ is verleng en ontmoet RS by S sodat SR = SY. QX ∥ SR.



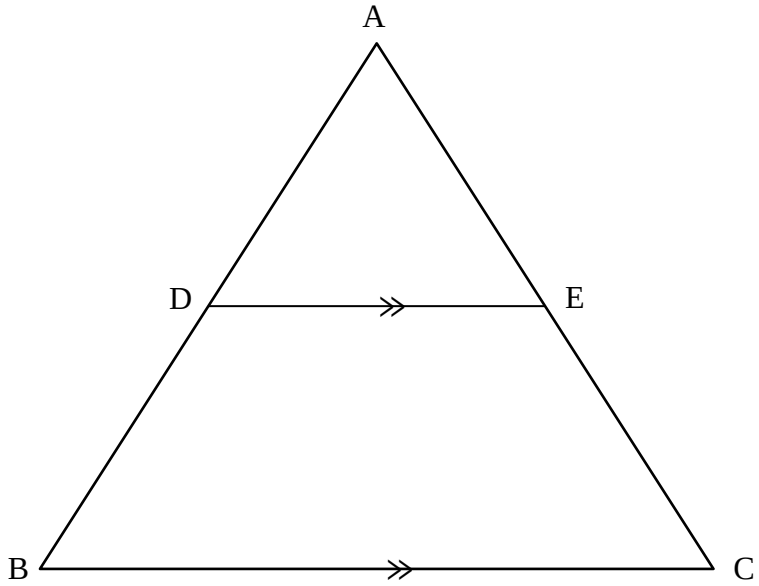
10.1 Bewys dat SR 'n raaklyn aan die sirkel by R is. (6)

10.2 Bewys dat QR 'n raaklyn aan die sirkel, wat deur Q, X en P gaan, is. (3)
[9]

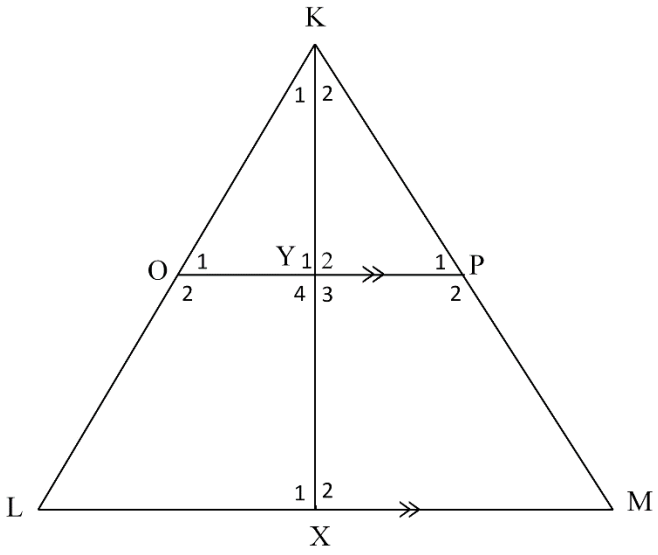
VRAAG 11

11.1 In die diagram hieronder, is D en E punte op sye AB en AC van ΔABC sodat DE ∥ BC.

Gebruik die diagram om die stelling te bewys wat meld dat: $\frac{AD}{DB} = \frac{AE}{EC}$ (6)



11.2 In die diagram hieronder, is OP ∥ LM sodat die oppervlakte van Δ KOP = oppervlakte van vierhoek OLMP. KYX is loodreg op OP en LM by Y en X onderskeidelik.



Bewys dat:

11.2.1 Δ KOP ||| Δ KLM (3)

11.2.2 $\frac{KY}{KX} = \frac{OP}{LM}$ (2)

11.2.3 $\frac{KO}{KL} = \frac{1}{\sqrt{2}}$ (6)

[17]

TOTAAL: 150

INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \Delta ABC: \; \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad \text{area } \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n} \qquad P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

Province of the
EASTERN CAPE
EDUCATION

LEARNER'S NAME: *LEERDER SE NAAM:*	

GRADE 12: *GRAAD 12:*	

NATIONAL/*NASIONALE* SENIOR CERTIFICATE/*SERTIFIKAAT*

GRADE/*GRAAD* 12

JUNE/*JUNIE* 2022

MATHEMATICS P2/*WISKUNDE V2*
SPECIAL ANSWER BOOK/*SPESIALE ANTWOORDEBOEK*

Marker/*Merker*					**Moderator's Initials / *Moderator se paraaf***																
Question *Vraag*	**Mark** *Punt*			**Initial** **Parafeer**	**Marks** *Punte*			**S M**	**Marks** *Punte*			**D M**	**Marks** *Punte*			**P M**	**Marks** *Punte*			**NM**
1																				
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8																				
9																				
10																				
11																				
TOTAL *TOTAAL*																				

This special answer book consists of 19 pages.
Hierdie spesiale antwoordeboek bestaan uit 19 bladsye.

QUESTION/*VRAAG* 1

1.1		(1)
1.2		(1)
1.3		(1)
1.4		(1)
1.5		(1)
		[5]

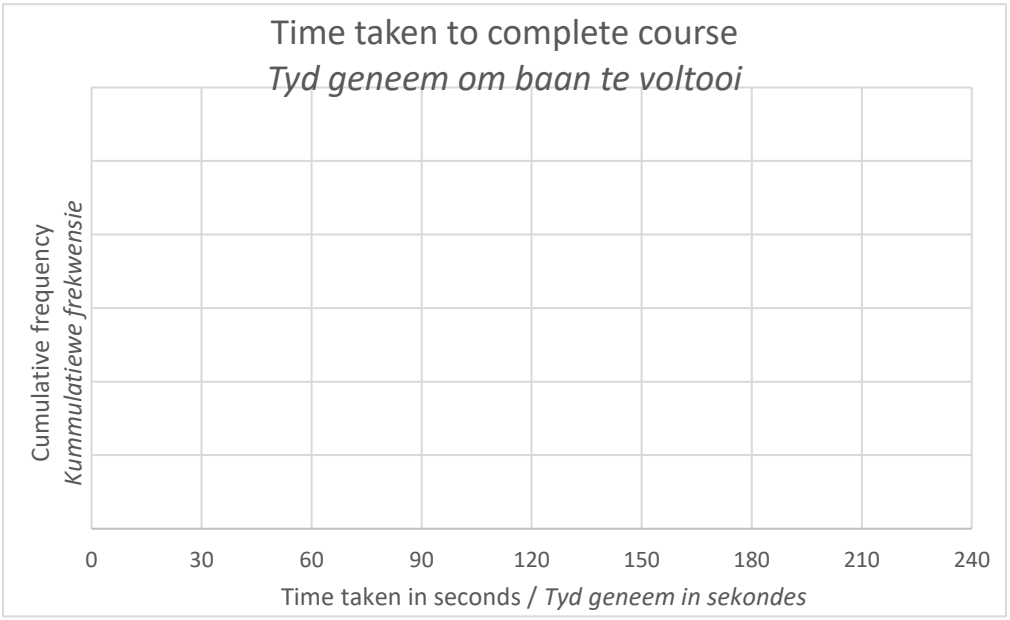
QUESTION/*VRAAG* 2

2.1

Time taken / *Tyd geneem*	No. of pupils *Aantal leerlinge*	Cumulative frequency *Kummulatiewe frekwensie*
$60 \leq t \leq 90$	3	
$90 \leq t \leq 120$	6	
$120 \leq t \leq 150$	7	
$150 \leq t \leq 180$	8	
$180 \leq t \leq 210$	6	

(1)

2.2



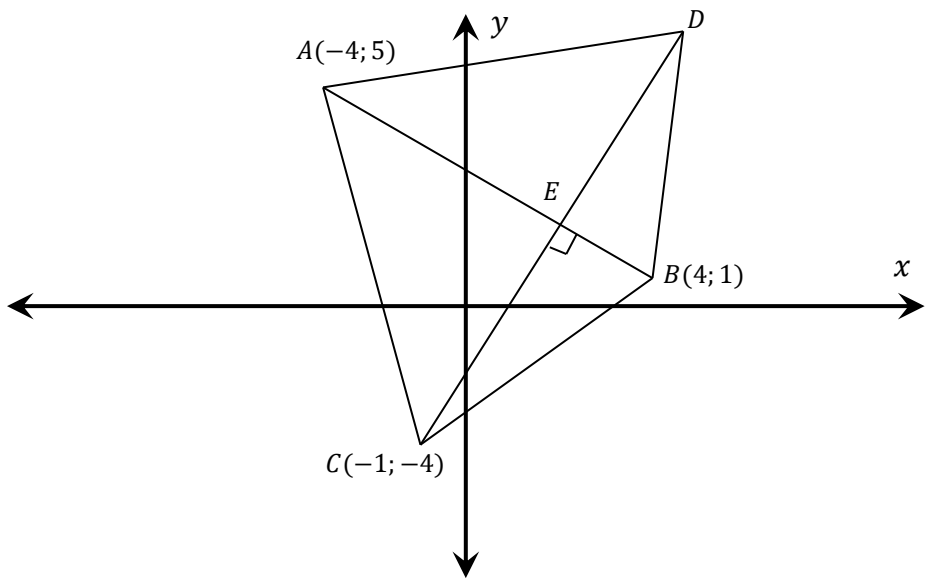
(4)

2.3.1		(1)
2.3.2		(1)
2.3.3		(1)
		[8]

QUESTION/*VRAAG* 3

3.1		(1)
3.2		(3)
3.3		(2)
		[6]

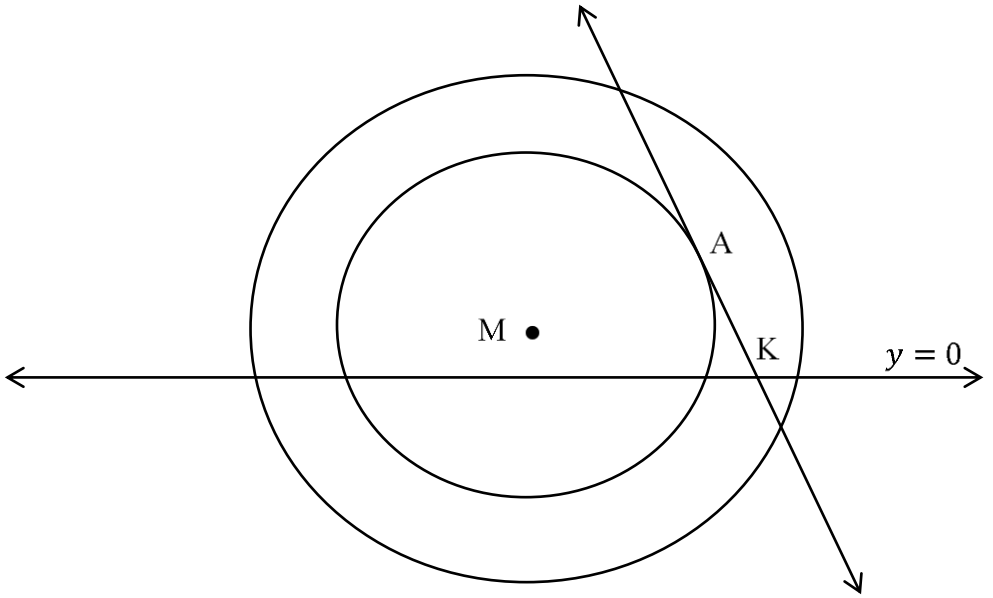
QUESTION 4/*VRAAG 4*



4.1		(2)
4.2		(4)
4.3		(6)

4.4		(2)
4.5		(4)
4.6		(6)
		[24]

QUESTION 5/*VRAAG 5*



5.1		(4)
5.2		(5)

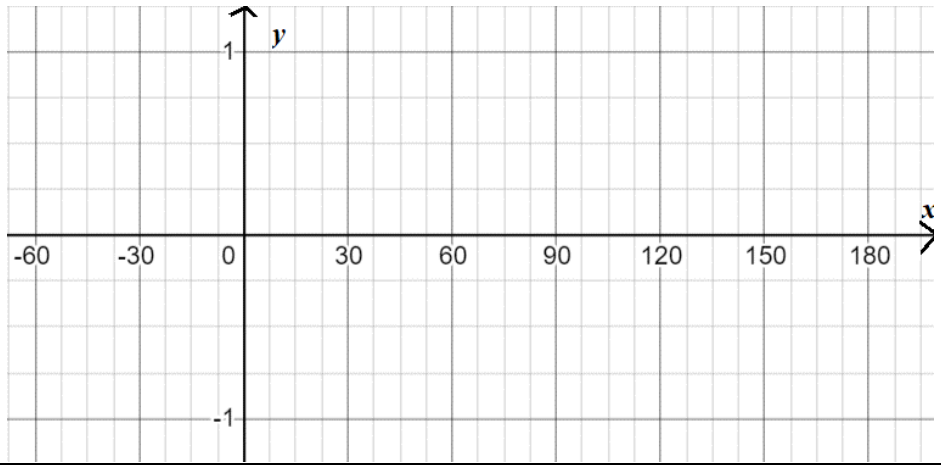
5.3		(3)
5.4		(1)
5.5		(3)
		[16]

QUESTION 6/*VRAAG 6*

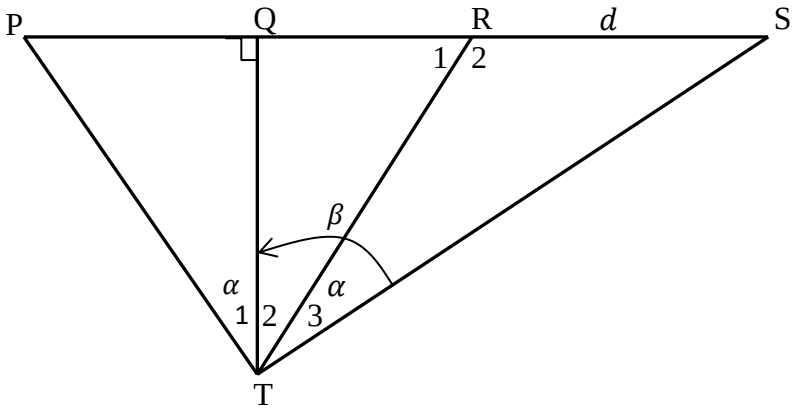
6.1	Draw your sketch here / *Teken jou skets hier.*	Do your calculations here / *Doen jou berekeninge hier.*	
6.1.1			(3)
6.1.2			(3)

6.1.3		(4)
6.2		(5)
6.3		(3)
6.4		(6)
		[24]

QUESTION 7/*VRAAG 7*

7.1		(7)
7.2		(6)
7.3		(4)
		[17]

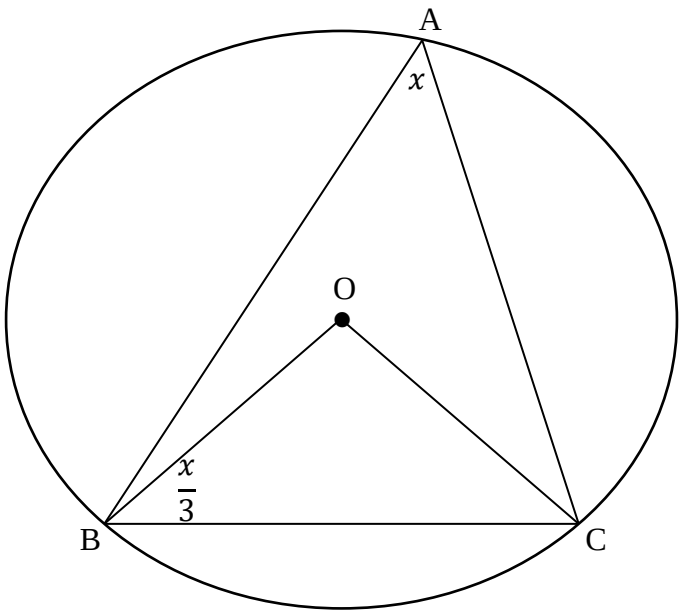
QUESTION 8/*VRAAG 8*



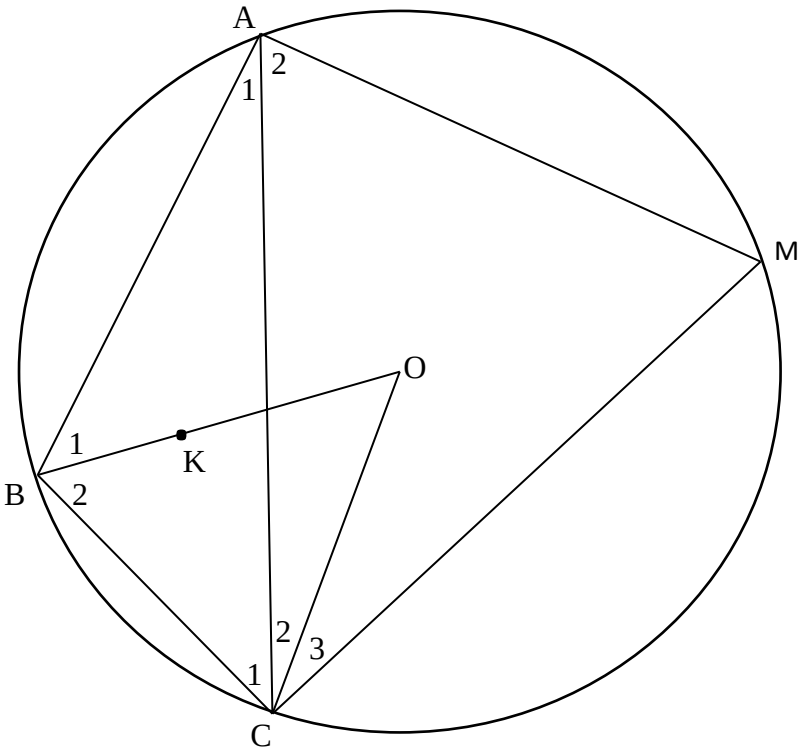
8.1		(1)
8.2		(1)
8.3		(1)
8.4		(3)
8.5		(3)
		[9]

QUESTION 9/*VRAAG 9*

9.1 (1)



9.2 (6)



9.3.1

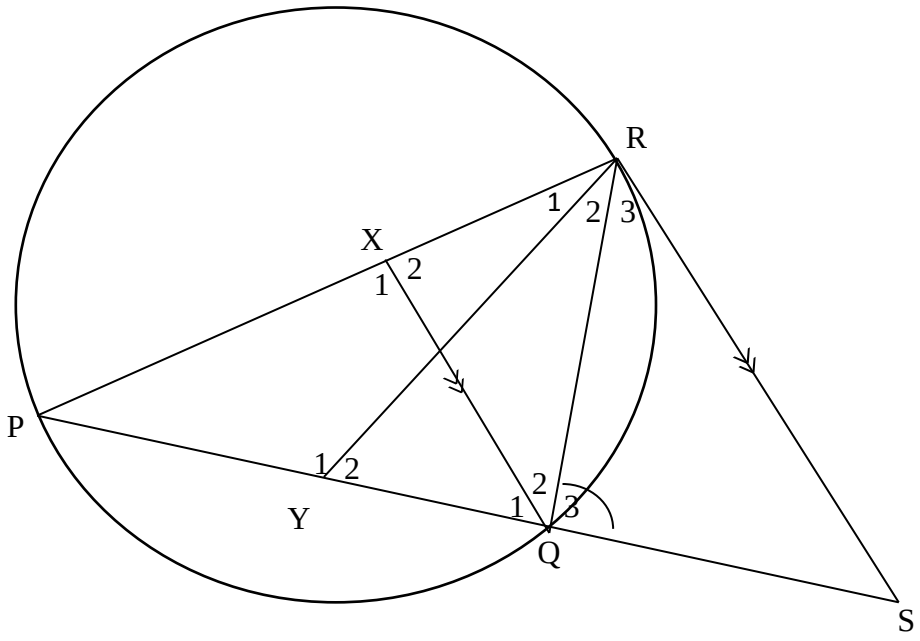
(5)

9.3.2

(3)

[15]

QUESTION 10/*VRAAG 10*

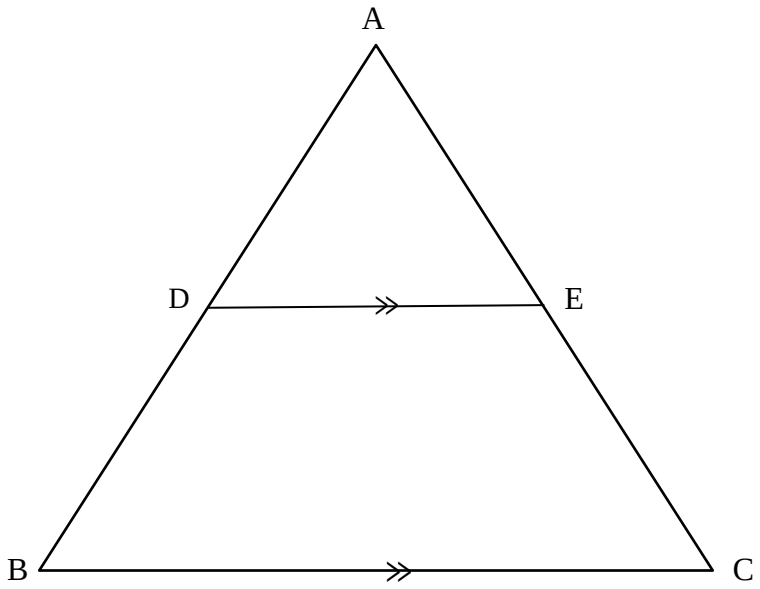


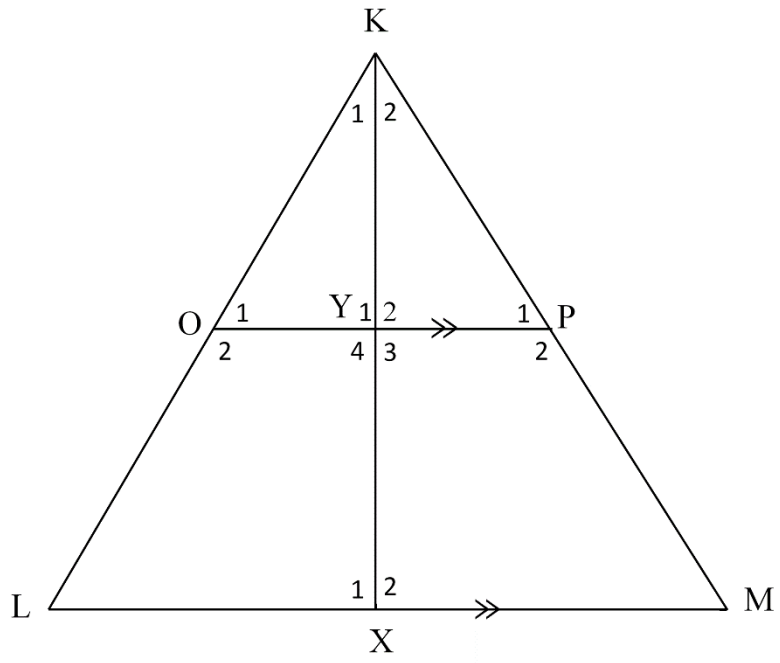
10.1 (6)

10.2 (3)

[9]

QUESTION 11/*VRAAG 11*

		
11.1		(6)

11.2		
11.2.1		(3)
11.2.2		(2)
11.2.3		(6)
		[17]

TOTAL/*TOTAAL*: 150

Additional Space/*Addisionele ruimte*

Province of the
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NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE/*GRAAD 12*

JUNE/*JUNIE* 2022

MATHEMATICS P2/*WISKUNDE V2* MARKING GUIDELINE/*NASIENRIGLYN*

MARKS/*PUNTE*: **150**

This marking guideline consists of 15 pages.
Hierdie nasienriglyn bestaan uit 15 bladsye.

QUESTION/*VRAAG* 1

1.1	B	✓ answer/*antwoord*	(1)
1.2	B	✓ answer/*antwoord*	(1)
1.3	75%	✓ answer/*antwoord*	(1)
1.4	Nothing. It remains the same. No change in standard deviation. *Niks. Dit bly dieselfde. Geen verandering in standaardafwyking.*	✓reason/*rede*	(1)
1.5	$\text{Semi} - \text{IQR: IKV} = \frac{75 - 30}{2}$ $\text{Semi} - \text{IQR: IKV} = 22{,}5$	✓ answer/*antwoord*	(1)
			[5]

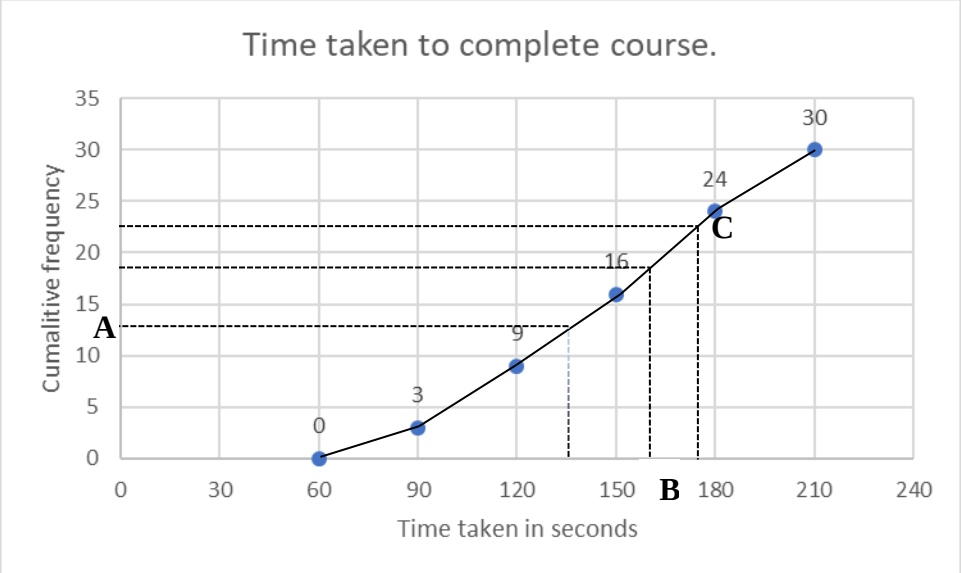
QUESTION/*VRAAG* 2

2.1

Time taken *Tyd geneem*	No. of pupils *Aantal leerlinge*	Cumulative frequency *Kummulatiewe frekwensie*
$60 \leq t \leq 90$	3	3
$90 \leq t \leq 120$	6	9
$120 \leq t \leq 150$	7	16
$150 \leq t \leq 180$	8	24
$180 \leq t \leq 210$	6	30

✓ for values *vir waardes* (1)

2.2



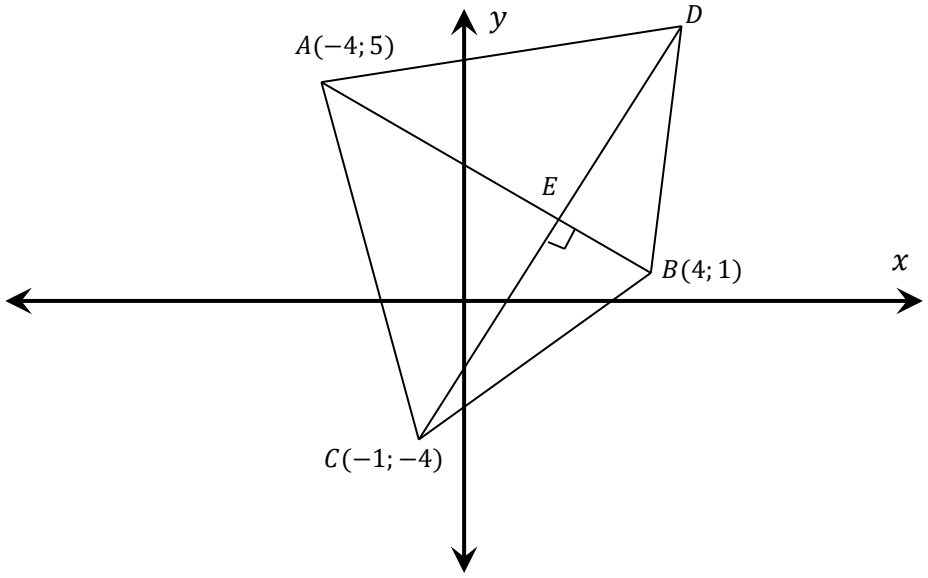
✓ anchor point/ *ankerpunt* (60;0)
✓ (120; 9)
✓ (150; 16)
✓ (210; 30) (4)

2.3	2.3.1	See diagram above / *Sien diagram hierbo*	✓ A	(1)
	2.3.2	See diagram above / *Sien diagram hierbo*	✓ B	(1)
	2.3.3	See diagram above / *Sien diagram hierbo*	✓ C	(1)
				[8]

QUESTION/*VRAAG* 3

3.1	Median score / *Mediaan telling* = $2x$	✓ answer/*antwoord*	(1)
3.2	Mean/*Gemiddelde* $= \frac{\sum x}{n}$ $= \frac{4(x+3)+3(2x)+2(x-1)+2(6)}{11}$ $= \frac{12x+22}{11}$	✓ substitution/*vervanging* ✓ simplification/*vereenvoudiging* ✓ answer/*antwoord*	(3)
3.3	Use of a calculator where the four values are as follows: *Gebruik van 'n sakrekenaar waar die vier waardes soos volg is:* 8 ; 10 ; 4 and/*en* 6 $sd(\sigma) = \sqrt{5}$	✓ four values/*vier waardes* ✓ answer/*antwoord*	(2)
			[6]

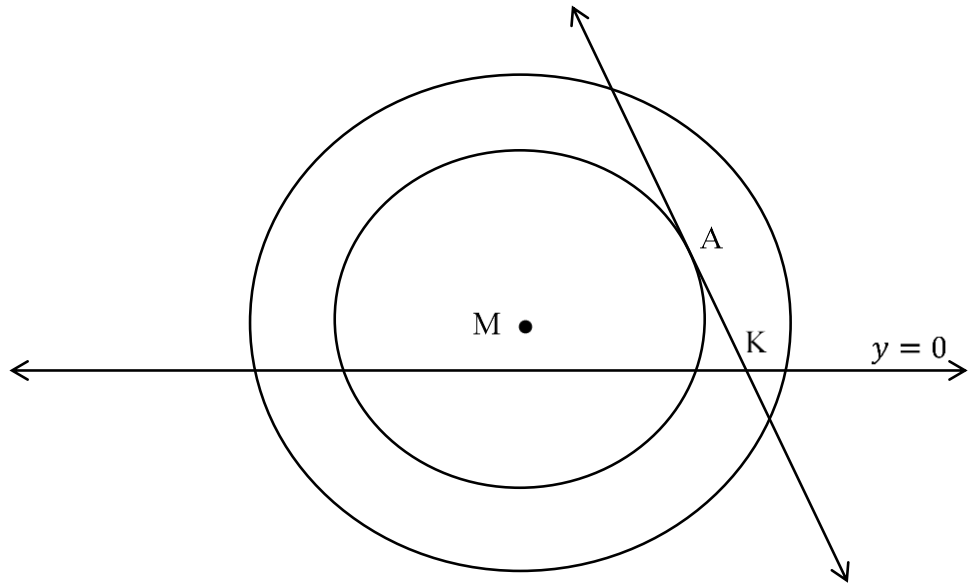
QUESTION/*VRAAG* 4



4.1	$m_{AB} = \frac{5-1}{-4-4}$ $= \frac{4}{-8}$ $m_{AB} = -\frac{1}{2}$ $\therefore\ m_{CD} = 2$	✓ subst. into gradient formula/ *verv. in gradiënt formule* ✓ $m_{AB} = -\frac{1}{2}$	(2)
4.2	$\therefore\ m_{CD} = 2$ $\overline{CD}:\quad y = 2x + c$ $-4 = 2(-1) + c$ $-2 = c$ $\overline{CD}:\quad y = 2x - 2$	✓ $m_{CD} = 2$ ✓sub of point/*verv. van punt* $(-1; -4)$ ✓ $-2 = c$ ✓ equation of CD / *vergelyking van* CD	(4)
4.3	$\overline{AB}:\quad y = -\frac{1}{2}x + c$ $1 = -\frac{1}{2}(4) + c$ $3 = c$ $y = -\frac{1}{2}x + 3$ $-\frac{1}{2}x + 3 = 2x - 2$ $-x + 6 = 4x - 4$ $5x = 10$ $x = 2$ $y = 2(2) - 2$ $y = 2$ $E(2; 2)$	✓ substitution of point (4; 1) *vervanging van punt* (4;1) ✓ equation of $\overline{AB}$ *vergelyking van* $\overline{AB}$ ✓ equating of AB and CD *gelykstel van* AB en CD ✓ *x* –value/*waarde* ✓ substitution of/*vervanging van* $x-$ value/*waarde* ✓ $y-$ value/*waarde*	(6)

4.4	$D(x;y)$ $\frac{x-1}{2}=2$ $\qquad$ $\frac{y-4}{2}=2$ $x-1=4$ $\qquad$ $y-4=4$ $x=5$ $\qquad$ $y=8$ $D(5;8)$	✓ $x=5$ ✓ $y=8$	(2)
4.5	$m_{AC}=\frac{5+4}{-4+1}$ $m_{AC}=-3$ Line parallel to AC has same gradient. *Lyn ewewydig aan AC het dieselfde gradient.* $y=-3x+c$ $8=-3(5)+c$ $c=23$ $y=-3x+23$	✓ substitution into gradient formula/*vervanging in gradiënt formule* ✓ $m_{AC}=-3$ ✓ $c=23$ ✓ equation of line/*vergelyking van lyn*	(4)
4.6	x intercept of CD : *x afsnit van* CD $2x-2=0$ $x=1$ $m_{BC}=1$ Equation of Altitude/*Vergelyking van hoogtelyn* $y=-x+c$ $5=-(-4)+c$ $c=1$ $\therefore y=-x+1$ x intercept of Altitude / *x-afsnit van hoogtelyn* $x=1$ x intercept of CD $=$ x intercept of altitude *x-afsnit van* CD = *x-afsnit van hoogtelyn*	✓ $x=1$ ✓ $m_{BC}=1$ ✓ gradient of altitude -1 *gradiënt van hoogtelyn* -1 ✓ substitution of point *vervanging van punt* ✓equation of altitude *vergelyking van hoogtelyn* ✓ $x=1$	(6)
			[24]

QUESTION/*VRAAG* 5

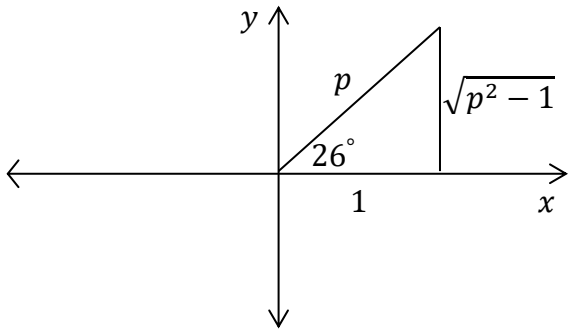


5.1	$x^2 + y^2 = 4y - 2x + 44$ $x^2 + 2x + 1 + y^2 - 4y + 4 = 44 + 1 + 4$ $(x + 1)^2 + (y - 2)^2 = 49$ $\therefore M(-1; 2)$	✓✓ completing the square *voltooiing van die vierkant* ✓ factorizing / *faktorisering* ✓ $M(-1; 2)$	(4)
5.2	$m_{MA} = \frac{y-2}{x+1}$ $\frac{y-2}{x+1} = 1$ $y - 2 = x + 1$ $y = x + 3$ $x + 3 = -x + 5$ $2x = 2$ $x = 1$ $\therefore \ y = 4$ $A(1; 4)$	✓ gradient of MA/*gradiënt van* MA ✓ equating it to 1/*stel dit gelyk aan* 1 ✓ making y or x the subject. *maak y* of *x die onderwerp* ✓ equating the two linear functions. *gelykstel van twee lineêre funksies* ✓ coordinates of $A(1; 4)$ *koördinate van* $A(1; 4)$	(5)
5.3	$(x + 1)^2 + (y - 2)^2 = r^2$ $(1 + 1)^2 + (4 - 2)^2 = r^2$ $8 = r^2$ $(x + 1)^2 + (y - 2)^2 = 8$	✓ substitution of A/*vervanging van* A ✓ $8 = r^2$ ✓ equation of the circle/*vergelyking van* . *die sirkel*	(3)

5.4	$K(5;0)$	✓ $K(5;0)$	(1)
5.5	$AK = \sqrt{32}$ Area of/*van* $\Delta AMK = \frac{1}{2} AK \times AM$ Area of/*van* $\Delta AMK = \frac{1}{2}\sqrt{32} \times \sqrt{8}$ Area of/*van* $\Delta AMK = 8$ units^2/*eenhede*^{2}	✓ $AK = \sqrt{32}$ ✓ $AM = \sqrt{8}$ ✓ 8 units^2/*eenhede*^{2}	(3)
			[16]

QUESTION/*VRAAG* 6

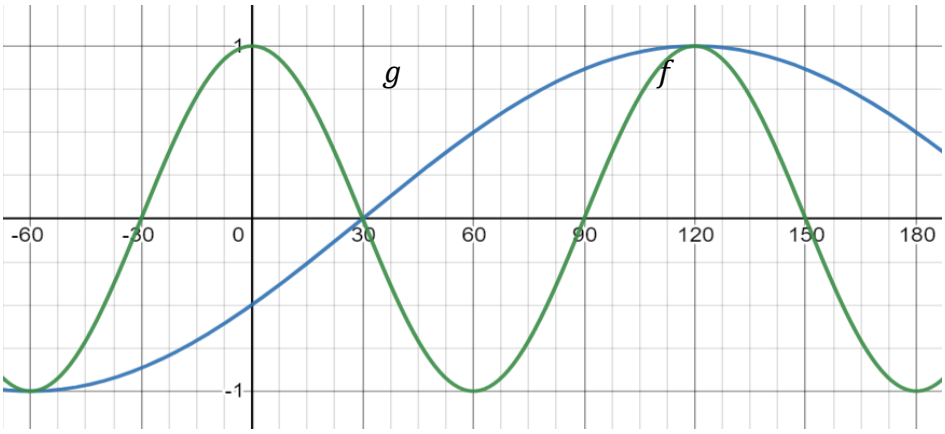
6.1



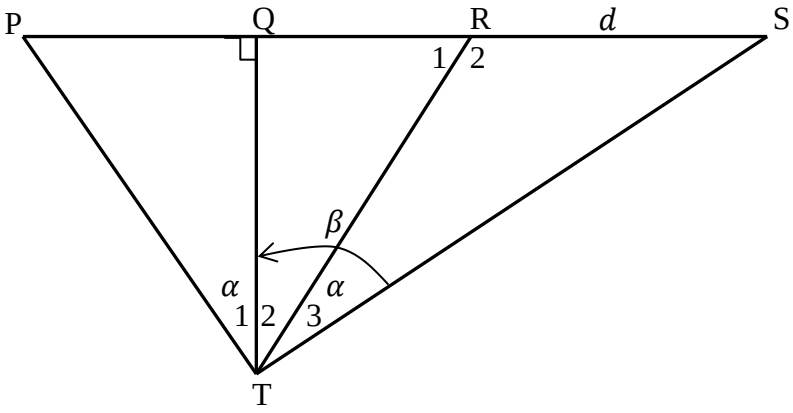
	6.1.1	$\sin 26^\circ = \frac{\sqrt{p^2-1}}{p}$	✓ sketch in Quadrant 1 *skets in kwadrant* 1 ✓ $\sqrt{p^2-1}$ ✓ answer/*antwoord*	(3)
	6.1.2	$\cos 52^\circ = \cos 2(26^\circ)$ $= 2\,cos^2 26^\circ - 1$ $= 2\left(\frac{1}{p}\right)^2 - 1$ $= \frac{2}{p^2} - 1$	✓$\cos 2(26^\circ)$ ✓$2\,cos^2 26^\circ - 1$ ✓ answer/*antwoord*	(3)
	6.1.3	$tan^2 64^\circ \times (p+1)$ $= \left(\frac{1}{\sqrt{p^2-1}}\right)^2 \times (p+1)$ $= \frac{1}{p^2-1} \times (p+1)$ $= \frac{1}{(p-1)(p+1)} \times (p+1)$ $= \frac{1}{p-1}$	✓ $\left(\frac{1}{\sqrt{p^2-1}}\right)^2$ ✓ $\frac{1}{p^2-1}$ ✓ $(p-1)(p+1)$ ✓ answer/*antwoord*	(4)

6.2	$\frac{\sin(-\beta)+\sin(360^\circ-\beta)}{\sin(180^o-\beta)+\sin 180^\circ}$ $=\frac{-\sin\beta+(-\sin\beta)}{\sin\beta+0}$ $=\frac{-2\sin\beta}{\sin\beta}$ $=-2$	✓ $-\sin\beta$ ✓ $-\sin\beta$ ✓ $\sin\beta$ ✓ simplification *vereenvoudiging* ✓ answer/*antwoord*	(5)
6.3	$2p\tan\left(\frac{\theta}{2}\right)=\sin(2\theta)$ $2p\tan\left(\frac{82^\circ}{2}\right)=\sin(2\times 82^\circ)$ $p=\frac{\sin 162^\circ}{2\tan 41^\circ}$ $p=0{,}16$	✓ substitution/*vervanging* ✓ simplification/*vereenvoudiging* ✓ answer/*antwoord*	(3)
6.4	$4\sin\theta.\cos^3\theta-4\cos\theta.\sin^3\theta$ $=\sin 4\theta$ $LHS/LK=4\sin\theta.\cos^3\theta$ $-4\cos\theta.\sin^3\theta$ $=4\sin\theta.\cos\theta(\cos^2\theta-\sin^2\theta)$ $=2\times 2\sin\theta\cos\theta(\cos 2\theta)$ $=2.\sin 2\theta.\cos 2\theta$ $=\sin 4\theta$ $=RHS/RK$	✓ common factor/*gemene faktor* ✓ $2\times 2\sin\theta\cos\theta$ ✓ $(\cos 2\theta)$ ✓$\sin 2\theta.$ ✓ $2.\sin 2\theta.\cos 2\theta$ ✓ answer/*antwoord*	(6)
			[24]

QUESTION/*VRAAG 7*

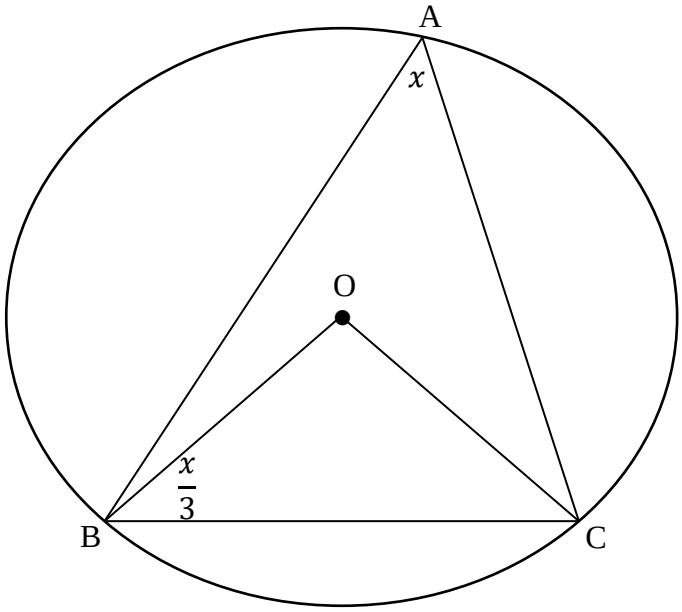
7.1	$\cos 3x = \sin(x - 30^\circ)$ $\cos 3x = \cos[90^\circ - (x - 30^\circ)]$ $\cos 3x = \cos[120 - x]$ $3x = 120 - x$ (ref angle) $3x = 120^\circ - x + k.360^\circ$ **OR/*OF*** $3x = 360^\circ - [120^\circ - x]k.360^\circ$ $4x = 120^\circ + k.360^\circ$ $\quad 2x = 240^\circ + k.360^\circ$ $x = 30^\circ + k.90^\circ$ $\quad x = 120^\circ + k.180^\circ$ $x = 30^\circ\ ;\ -60^\circ\ ;\ 120^\circ$	✓ Co- ratio *Ko-verhoud.* ✓ ref angle *verwys.* $\angle$ ✓ quadrant 1 *kwadrant* 1 ✓ quadrant 4 *kwadrant* 4 ✓ 30° ✓ -60° ✓ 120°	(7)
7.2	 ✓ Shape of f. ✓ x intercepts ✓ start and end points *Vorm van* f *x afsnitte* *begin en eindpunte* ✓ Shape of g. ✓ x intercepts ✓ start and end points *Vorm van* g *x afsnitte* *begin en eindpunte*		(6)
7.3	$-30^\circ < x < 30^\circ$ ***OR/OF*** $30^\circ < x < 90^\circ$ ***OR/OF*** $150^\circ < x < 180^\circ$	✓✓ $-30^\circ < x < 30^\circ$ ✓ $30^\circ < x < 90^\circ$ ✓ $150^\circ < x < 180^\circ$	(4)
			[17]

QUESTION/*VRAAG* 8

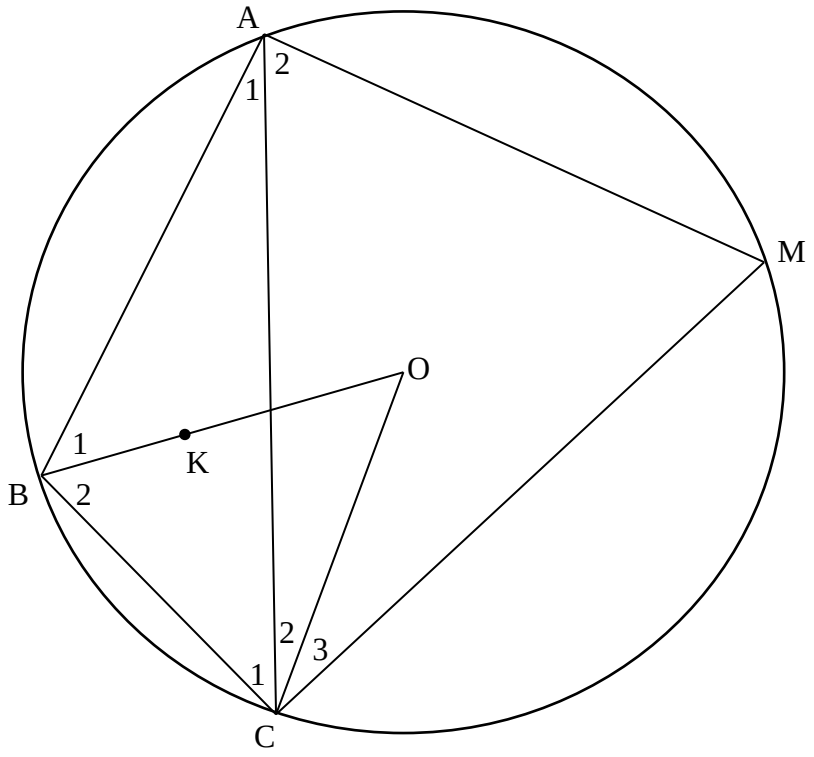


8.1	$Q\hat{T}R = \beta - \alpha$	✓ answer/*antwoord*	(1)
8.2	$\hat{S} = 90^\circ - \beta$	✓ answer/*antwoord*	(1)
8.3	$\hat{S} = 90^\circ - \alpha$	✓ answer/*antwoord*	(1)
8.4	In $\Delta\, RST$ $\frac{d}{\sin\alpha} = \frac{RT}{\sin(90^\circ - \beta)}$ $RT = \frac{d\cos\beta}{\sin\alpha}$	✓ use of sine rule *gebruik van sinusreël* ✓ use of co-function *gebruik van ko-funksie* ✓ answer/*antwoord*	(3)
8.5	$\frac{PR}{\sin\beta} = \frac{RT}{\sin(90^\circ - \alpha)}$ $PR = \frac{RT\sin\beta}{\cos\alpha}$ $PR = \frac{d\cos\beta\sin\beta}{\sin\alpha\cos\alpha}$	✓ use of sine rule *gebruik van sinusreël* ✓ sub of RT *vervanging van* RT ✓ answer/*antwoord*	(3)
			[9]

QUESTION/*VRAAG* 9

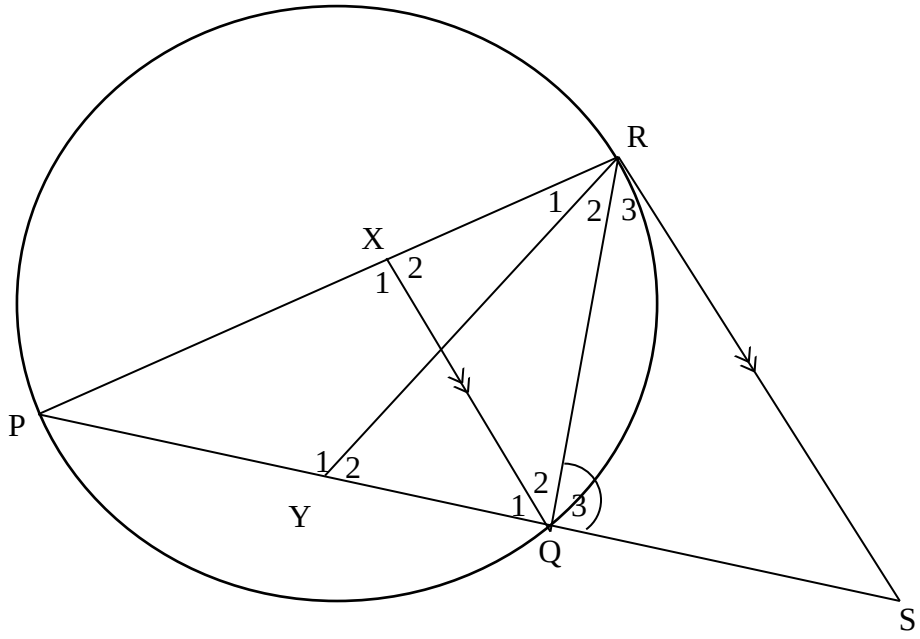
9.1	centre / *middelpuntshoek*	✓ answer/*antwoord*	(1)
			
9.2	$B\widehat{O}C = 2x$ [angle at centre $= 2 \times$ angle at circum] [*Middelpuntshoek* = 2 $\times$ *Omtrekshoek*] $B\widehat{C}O = \frac{x}{3}$ [angles opp $=$ sides ; OB $=$ OC] [*hoeke teenoor = sye* ; OB = OC] $\therefore \frac{x}{3} + \frac{x}{3} + 2x = 180^\circ$ [sum of angles of Δ] [*som van hoeke van* Δ] $8x = 540^\circ$ $x = 67{,}5^\circ$	✓ statement / *stelling* (**S**) ✓ reason/*rede* (**R**) ✓statement / *stelling* (**S**) ✓reason/*rede* (**R**) ✓ **S/R** ✓ answer/*antwoord*	(6)

9.3

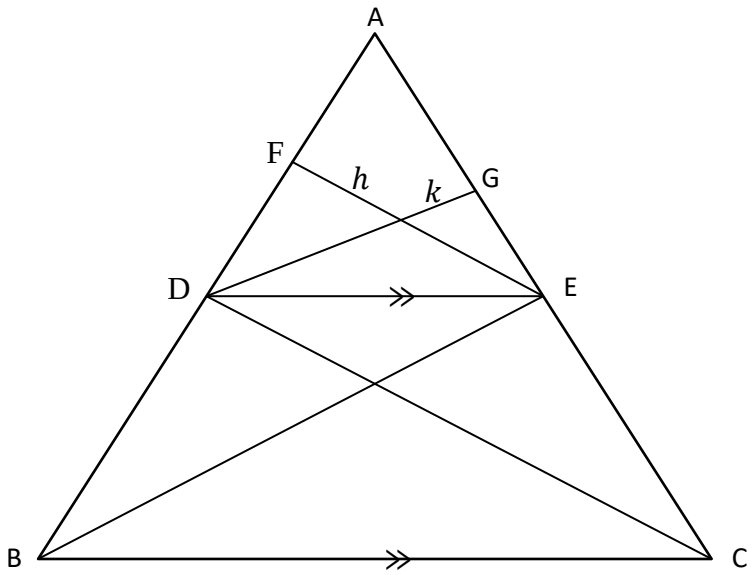


	9.3.1	$\hat{A}_1 = 30^\circ$ $B\hat{O}C = 60^\circ$ [angle at centre = 2 × angle at circum] [*middelpuntshoek* = 2 x *omtrekshoek*] $\hat{B}_2 = \hat{C}_1 + \hat{C}_2$ [angles opp = sides ; OB = OC] [*hoeke teenoor = sye* ; OB = OC] $\therefore 2\hat{B}_2 = 180^\circ - 60^\circ$ [sum of angles of Δ] [*som van hoeke van* Δ] $\hat{B}_2 = 60^\circ$ $\hat{B}_1 = 60^\circ$ [BO bisects A$\hat{B}$C] / [BO *halveer* A$\hat{B}$C]	✓ S/R ✓ S/R ✓ S/R ✓ S ✓ S/R	(5)
	9.3.2	$\hat{A}_1 = 30^\circ$ $\hat{B}_1 + \hat{B}_2 + \hat{M} = 180^\circ$ [opp angles of cyclic quad] [*teenoorst. hoeke van koordevierhoek*] $\hat{M} = 180^\circ - 120^\circ$ $\hat{M} = 60^\circ$ $\therefore \hat{M} = 2 \times \hat{A}_1$	 ✓ S/R ✓ S ✓ answer/ *antwoord*	(3)
				[15]

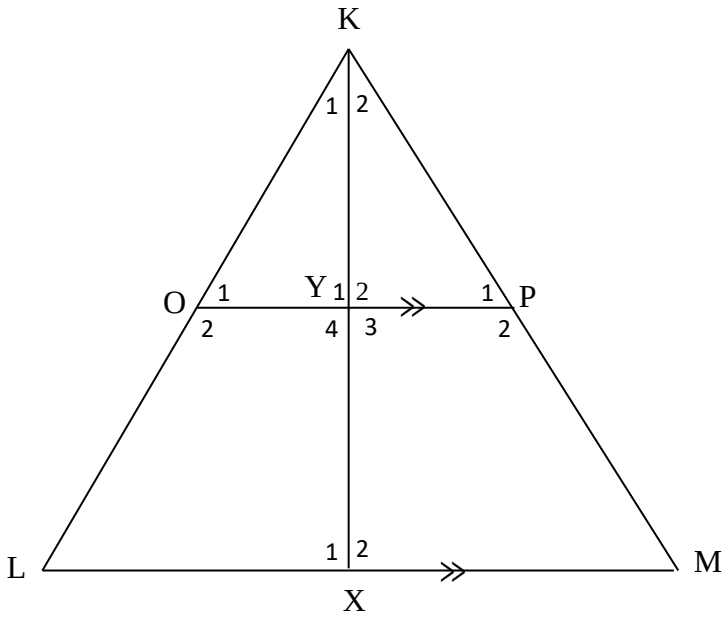
QUESTION/*VRAAG* 10

			
10.1	$\widehat{R}_1 = \widehat{R}_2$ [YR bisects $P\widehat{R}Q$] / [YR *halveer* $P\widehat{R}Q$] $\widehat{R}_2 + \widehat{R}_3 = \widehat{Y}_2$ [angles opp = sides; RS = YS] [*hoeke teenoor = sye* ; RS = YS] $\widehat{R}_1 + \widehat{P} = \widehat{Y}_2$ [ext angle of Δ]/[*buitehoek van* Δ] $\therefore\ \widehat{R}_3 = \widehat{P}$ ∴ SR = tangent/*raaklyn* [converse tan − chord theorem] [*omgekeerde raaklyn-koord stelling*]	✓ S/R ✓ S ✓ R ✓ S ✓ R ✓ S	(6)
10.2	$\widehat{Q}_2 = \widehat{R}_3$ [alt angles = ; SR ∥ QX] [*verw. hoeke* = ; SR || QX] $\widehat{P} = \widehat{R}_3$ [proven] / [*bewys*] $\widehat{Q}_2 = \widehat{P}$ ∴ QR = tangent [converse tan − chord theorem] QR = *raaklyn* [*omgekeerde raaklyn-koord stelling*]	✓ S/R ✓ S ✓ S	(3)
			[9]

QUESTION/*VRAAG* 11



11.1	Proof / *Bewys*: Construct Perpendicular heights DG (k) and EF (h) in $\Delta\ ADE$. Join BE and DC. *Teken loodregte hoogtes* DG (k) *en* EF (h) *in* $\Delta\ ADE$ *Verbind* BE *en* DC $\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\frac{1}{2}\text{AD. h}}{\frac{1}{2}\text{BD. h}} = \frac{\text{AD}}{\text{BD}}$ $\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CED} = \frac{\frac{1}{2}\text{AE. k}}{\frac{1}{2}\text{CE. k}} = \frac{\text{AE}}{\text{CE}}$ But/*Maar* Area ΔBDE = Area ΔCED (same base DE, same height) (*dieselfde basis* DE, *dieselfde hoogte*) DE ∥ BC $\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CED}$ $\therefore\ \frac{\text{AD}}{\text{BD}} = \frac{\text{AE}}{\text{CE}}$	✓ constr. *konstruk.* ✓ S/R ✓S/R ✓S ✓ R ✓ S	(6)

11.2				
	11.2.1	In ΔKOP and/*en* ΔKLM $\hat{K} = \hat{K}$ [common] / [*gemeen*] $\hat{O}_1 = \hat{L}$ [corresponding angles = ; OP ‖ LM] [*ooreenkomstige hoeke* = ; OP ‖ LM] $\hat{P}_1 = \hat{M}$ [corresponding angles =, OP ‖ LM] [*ooreenkomstige hoeke* = ; OP ‖ LM] $\therefore$ Δ KOP ||| ΔKLM [A; A; A]	✓ S/R ✓S/R ✓ R	(3)
	11.2.2	$\frac{KO}{KL} = \frac{OP}{LM}$ [similarity : *gelykvormig*] $\frac{KO}{KL} = \frac{KY}{KX}$ [line parallel to one side of Δ] [*lyn ewewydig aan een sy van* Δ] $\therefore$ $\frac{KY}{KX} = \frac{OP}{LM}$	✓ S/R ✓ S/R	(2)
	11.2.3	Area of Δ KOP = Area of Quad/*Vierhoek* OLMP $\therefore$ Area of Δ KLM $= 2 \times$ Area of Δ KOP $\frac{1}{2} \times LM \times KX = 2 \times \frac{1}{2} \times OP \times KY$ $\frac{1}{2} = \frac{OP.KY}{LM.KX}$ but/*maar* $\frac{OP}{LM} = \frac{KY}{KX}$ $\frac{OP^2}{LM^2} = \frac{1}{2}$ $\frac{OP}{LM} = \frac{1}{\sqrt{2}}$ $\frac{OP}{LM} = \frac{KO}{KL}$ [Δ KOP |||ΔKLM] $\therefore \frac{KO}{KL} = \frac{1}{\sqrt{2}}$	✓ S ✓ S ✓S ✓S ✓S ✓ S/R	(6)
				[17]
			TOTAL/*TOTAAL:*	**150**