



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

JUNE 2021

**MATHEMATICS P2
(EXEMPLAR)**

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages, including an information sheet.

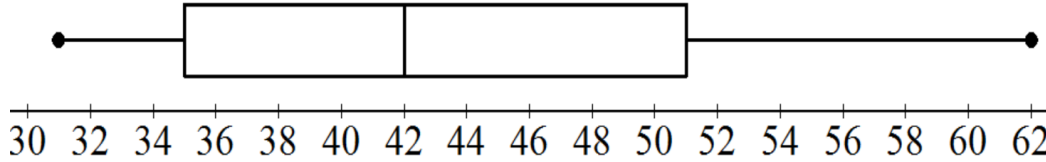
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The box and whisker diagram below represents soccer clubs' standings from position 1 to 14 after playing an equal number of games.



The following table is partly completed, from top (position 1) to bottom (position 14):

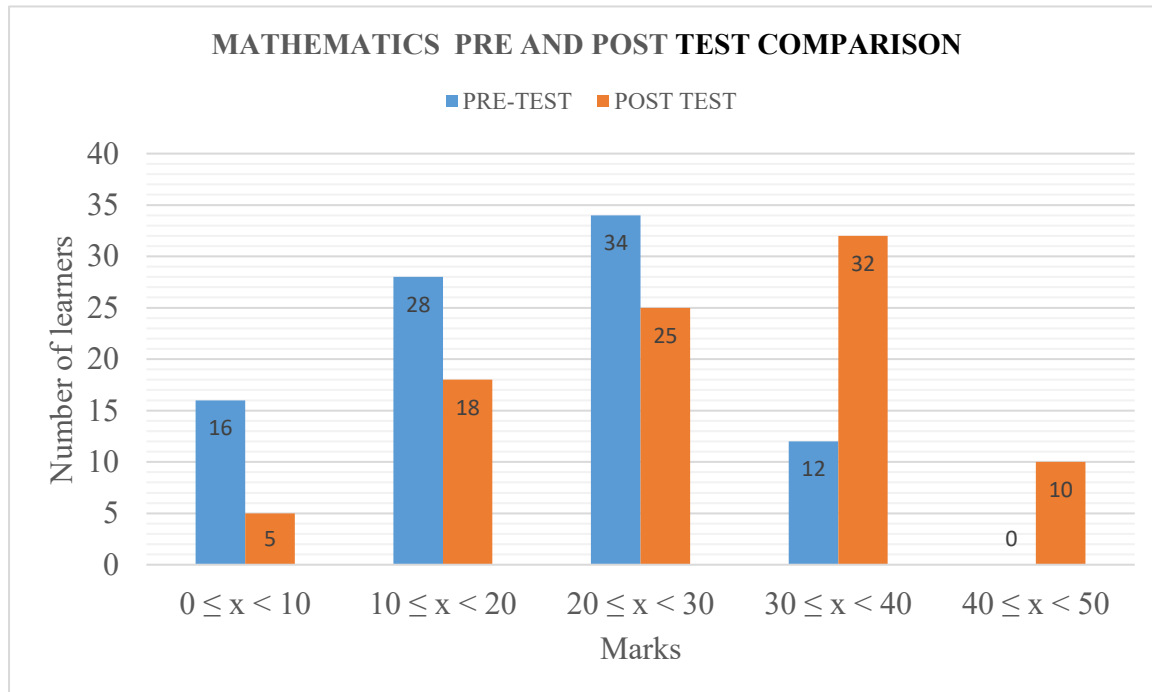
Position	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Points	<i>a</i>	59	58	<i>b</i>	49	45	<i>c</i>	42	37	36	<i>d</i>	32	32	<i>e</i>

- 1.1 Write down the values of *a*, *b*, *c*, *d* and *e*. (5)
- 1.2 Comment on the skewness of the data. (1)
- 1.3 One commentator says the top four teams each had at least 50 points. Do you agree with the commentator or not? Justify your answer. (2)

[8]

QUESTION 2

A school organised a weekend camp for the 90 grade 12 learners doing Mathematics. Learners wrote a pre-test (test before classes started) and a post-test (test after classes finished), out of 50 marks. Below is the graph representing the data.



- 2.1 Use the graph to conclude whether the camp had a positive impact (improved performance) or not. Give a reason for your answer. (2)
- 2.2 Write down the modal class of pre-test marks. (1)
- 2.3 Is the mean mark of the pre-test greater than, less than or the same as that of the post-test? (1)
- 2.4 Complete the frequency and cumulative frequency table in the ANSWER BOOK. (4)

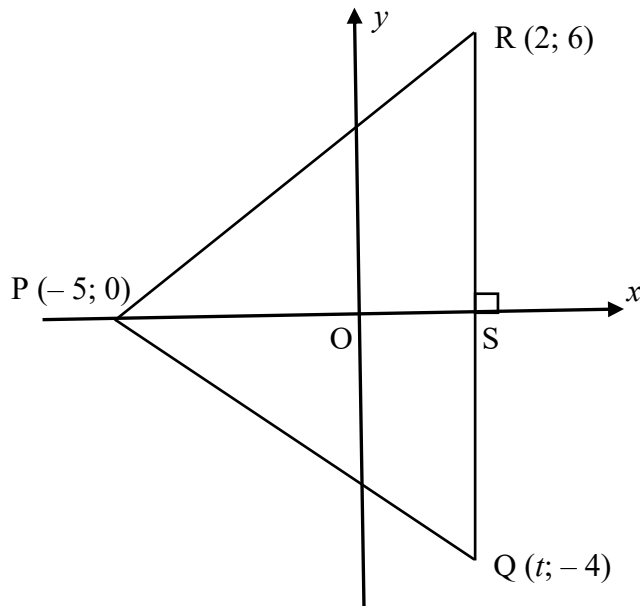
Marks	Frequency		Cumulative Frequency	
	Pre-test	Post-test	Pre-test	Post-test
$0 \leq x < 10$				
$10 \leq x < 20$				
$20 \leq x < 30$				
$30 \leq x < 40$				
$40 \leq x < 50$				

- 2.5 Draw the cumulative frequency graphs (ogives) using the grid provided in the ANSWER BOOK. (3)
- 2.6 The teacher targeted to have 50% more learners to get 60% or more in post-test compared to pre-test. Determine, with the necessary calculations or justification, whether the teacher achieved the target or not. (3)

[14]

QUESTION 3

$\triangle RPQ$ with vertices $R(2; 6)$, $P(-5; 0)$ and $Q(t; -4)$ is given below. RQ is perpendicular to the x -axis and cuts the x -axis at S . O is the origin.



- 3.1 Write down the value of t . (1)
- 3.2 Determine:
- 3.2.1 the length of PR . Leave your answer in simplest surd form. (2)
- 3.2.2 the gradient of PR . (2)
- 3.3 Determine the size of \widehat{PRQ} . (5)
- 3.4 Determine whether $\triangle QPR$ is right angled at P or not. (4)
- 3.5 Determine the equation of the line parallel to PQ and passing through the origin. (3)
- 3.6 Determine the value of $\frac{\text{Area of } \triangle SPR}{\text{Area of } \triangle PRQ}$. (5)

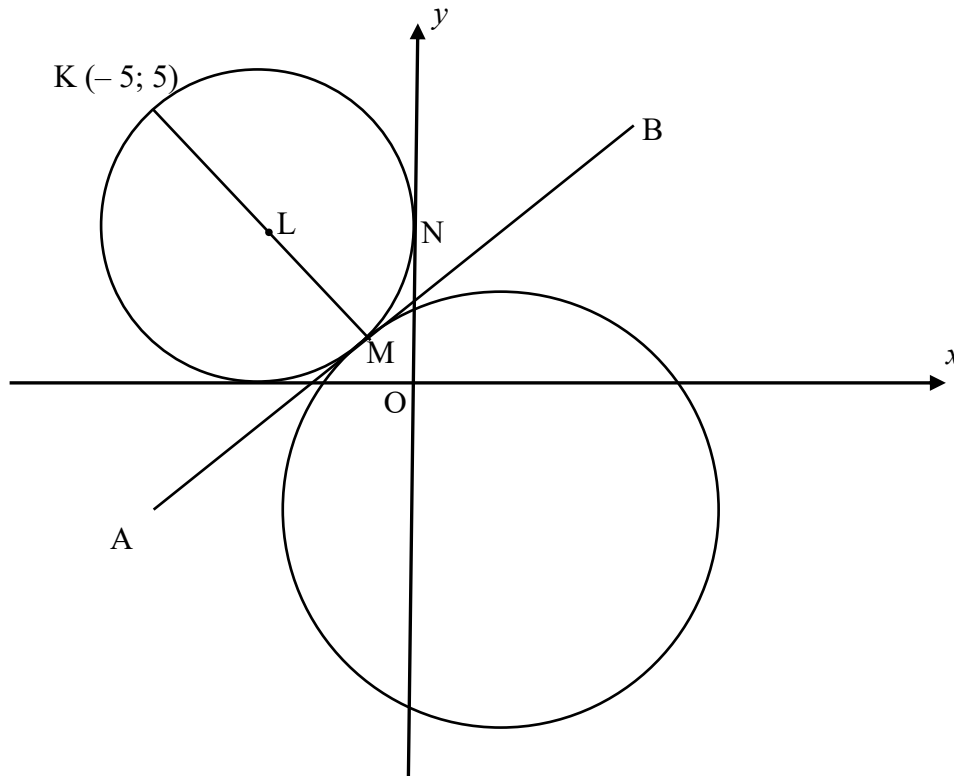
[22]

QUESTION 4

In the diagram below, the smaller circle with diameter KM passing through centre L has a tangent at M and y-intercept at N. The equation of the smaller circle is

$$x^2 + y^2 + 6x - 6y + 9 = 0$$

The bigger circle passes through M. The origin, O and K (-5 ; 5) is given.



4.1 Determine:

- 4.1.1 the coordinates of L and the length of the radius of the smaller circle (4)
- 4.1.2 the coordinates of M (3)
- 4.1.3 the equation of tangent AMB in the form $y = \dots$ (4)
- 4.1.4 the coordinates of N (2)

4.2 If the coordinates of the centre of the bigger circle is a result of shifting the coordinates of L, 5 units to the right and 7 units down.

- 4.2.1 Write down the coordinates of the centre of the new circle. (2)
- 4.2.2 Determine whether the diameter of the bigger circle from a common point of contact M passes through the origin or not. (4)

[19]

QUESTION 5

- 5.1 Given that $\sin \alpha = -\frac{5}{13}$ and $\tan \beta = -\frac{3}{4}$ where $\alpha, \beta \in [90^\circ; 270^\circ]$, calculate, **without the use of a calculator**, the value of:

5.1.1 $\sin (\alpha + \beta)$ (5)

5.1.2 $\cos 2\beta$ (3)

5.1.3 $\tan (-\alpha - 180^\circ)$ (2)

- 5.2 Consider the identity: $\frac{\sin \theta}{1 - \cos \theta} - \frac{\cos \theta}{\sin \theta} = \frac{1}{\sin \theta}$

5.2.1 For which value(s) of θ , for $\theta \in [0^\circ; 360^\circ]$ is the identity undefined? (2)

5.2.2 Prove the identity. (4)

- 5.3 If $\tan x = 3k$ and $\tan y = 2k$,

determine $\frac{\sin (x - y)}{\cos x \cdot \cos y}$ in terms of k (4)

[20]

QUESTION 6

Given the functions:

$$f(x) = \cos(x - 60^\circ) \text{ and } g(x) = \sin 3x \text{ for } x \in [-90^\circ; 180^\circ]$$

6.1 Write down:

6.1.1 the amplitude of f (1)

6.1.2 the period of g (1)

6.2 Determine the values of x for which $f(x) = g(x)$ for $x \in [-90^\circ; 180^\circ]$ (6)

6.3 On the same set of axes, sketch the graph of f and g for $x \in [-90^\circ; 180^\circ]$ in the SPECIAL ANSWER BOOK. Show ALL intercepts with the axes as well as turning and end points. (5)

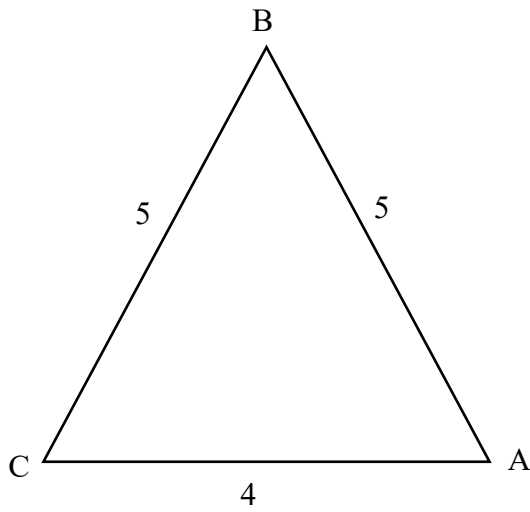
6.4 For which value(s) of x is $\frac{g(x)}{f(x)}$ undefined for $x \in [-90^\circ; 180^\circ]$? (1)

6.5 Write down the equation of $h(x)$ if $h(x)$ is a result of shifting $f(x)$, 15° to the left. (1)

[15]

QUESTION 7

The diagram below shows $\triangle ABC$ with lengths 5, 5 and 4 units.



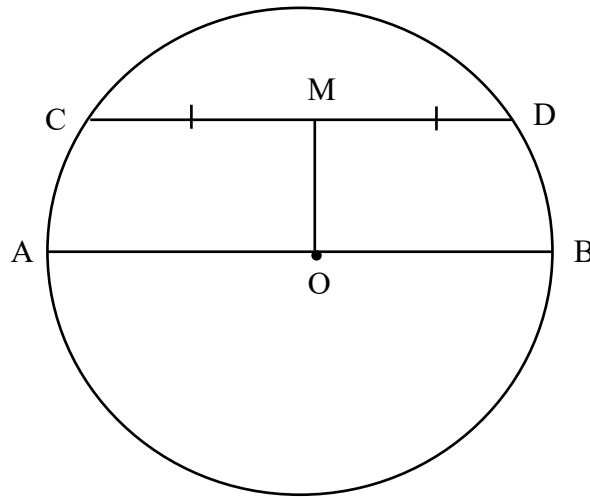
Determine the numerical value of $\cos A - \cos B$

(5)
[5]

Give reasons for your statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

In the diagram below, O is the centre of circle A, B, C and D. M is the midpoint of chord CD. Line OM is drawn. AB is the diameter. $AB = 22$ cm and $OM = 7$ cm.



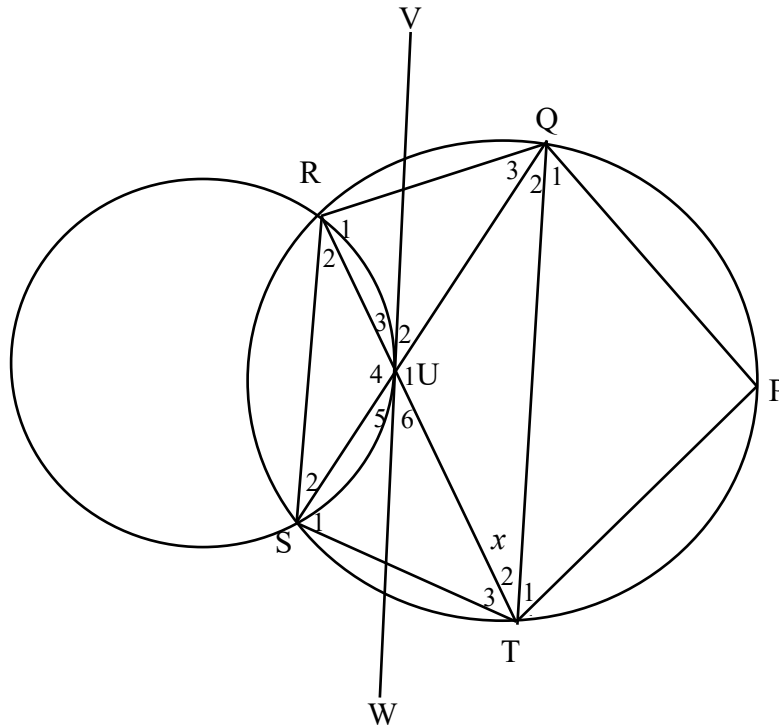
Determine, with reasons, the length of CD.

(5)
[5]

QUESTION 9

In the diagram below, a bigger circle PQRST intersects a smaller circle at R and S. VW is a tangent of the smaller circle at U. SUQ and TUR are straight lines.

Chords RQ, QP, PT, QT, TS and SR are also drawn. $\widehat{R\hat{T}Q} = x$.

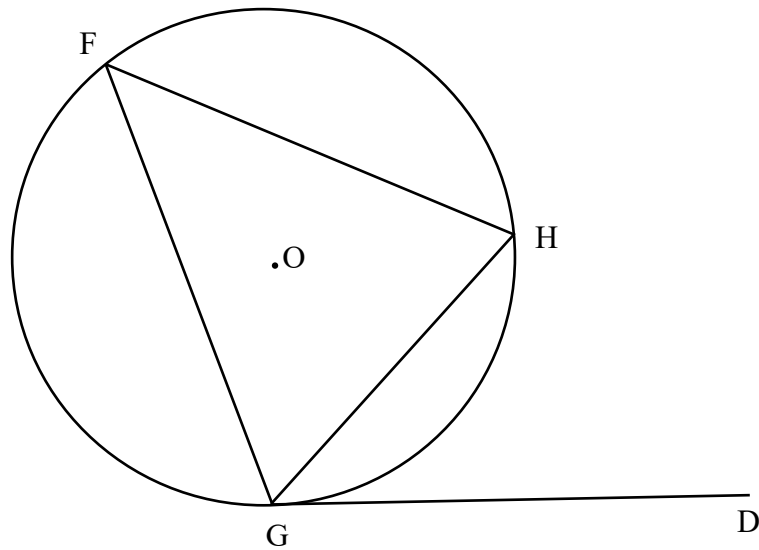


- 9.1 Prove, with reasons, that $\Delta RUS \parallel \Delta QUT$. (3)
- 9.2 Determine, with reasons, THREE other angles each equal to x . (4)
- 9.3 If $\widehat{RQT} = 90^\circ - x$, determine :
 - 9.3.1 whether QT is a diameter or not. (4)
 - 9.3.2 \widehat{P} (2)
- 9.4 If it is further given that $UQ = UT$, show that:
 - 9.4.1 $RS \parallel QT$ (2)
 - 9.4.2 VW is also a tangent to the circle passing through QUT at U. (2)

[17]

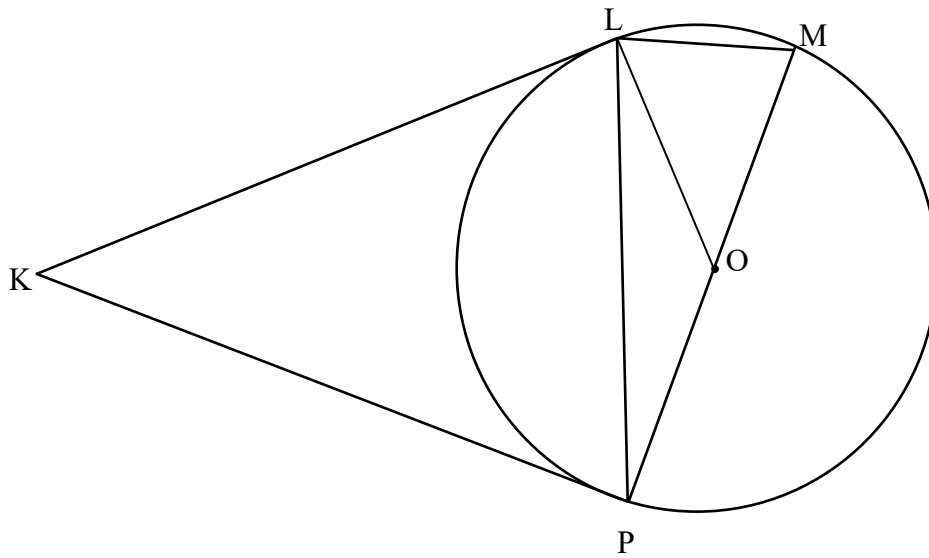
QUESTION 10

10.1 In the diagram below, O is the centre of circle FGH with DG a tangent at G.



Prove the theorem which states that $\widehat{DGH} = \widehat{F}$. (5)

10.2 In the diagram below, O is the centre of circle LMP with tangents KL and KP at L and P respectively. $\widehat{OLM} = 67^\circ$



10.2.1 What type of quadrilateral is KLOP? (1)

10.2.2 Give, with reasons, 3 angles each equal to 90° . (5)

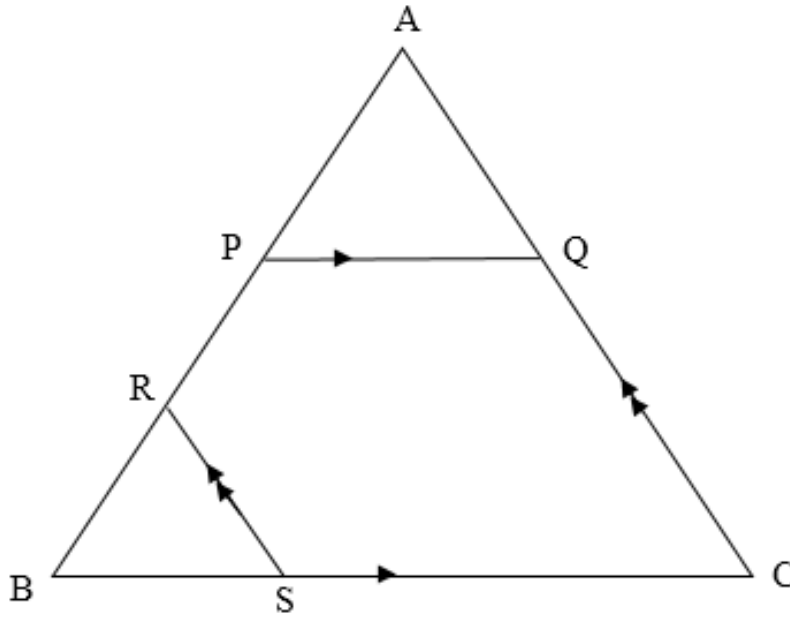
10.2.3 Prove, stating reasons, that KLOP is a cyclic quadrilateral. (2)

10.2.4 Hence, determine \widehat{K} . (5)

[18]

QUESTION 11

In the diagram below, $\triangle ABC$ is drawn with $PQ \parallel BC$ and $RS \parallel AC$.
 $AQ : QC = 3 : 5$ and $BR : RA = 1 : 3$



Prove that $AP = PR$.

(7)
[7]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



Province of the
EASTERN CAPE
EDUCATION

LEARNER'S NAME: <i>LEERDERNAAM:</i>	
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GRADE 12 <i>GRAAD 12</i>	
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**NATIONAL/NASIONALE
SENIOR
CERTIFICATE/SERTIFIKAAT**

GRADE 12/GRAAD 12

**MATHEMATICS P2/WISKUNDE V2
SPECIAL ANSWER BOOK/SPEZIALE ANTWOORDEBOEK**

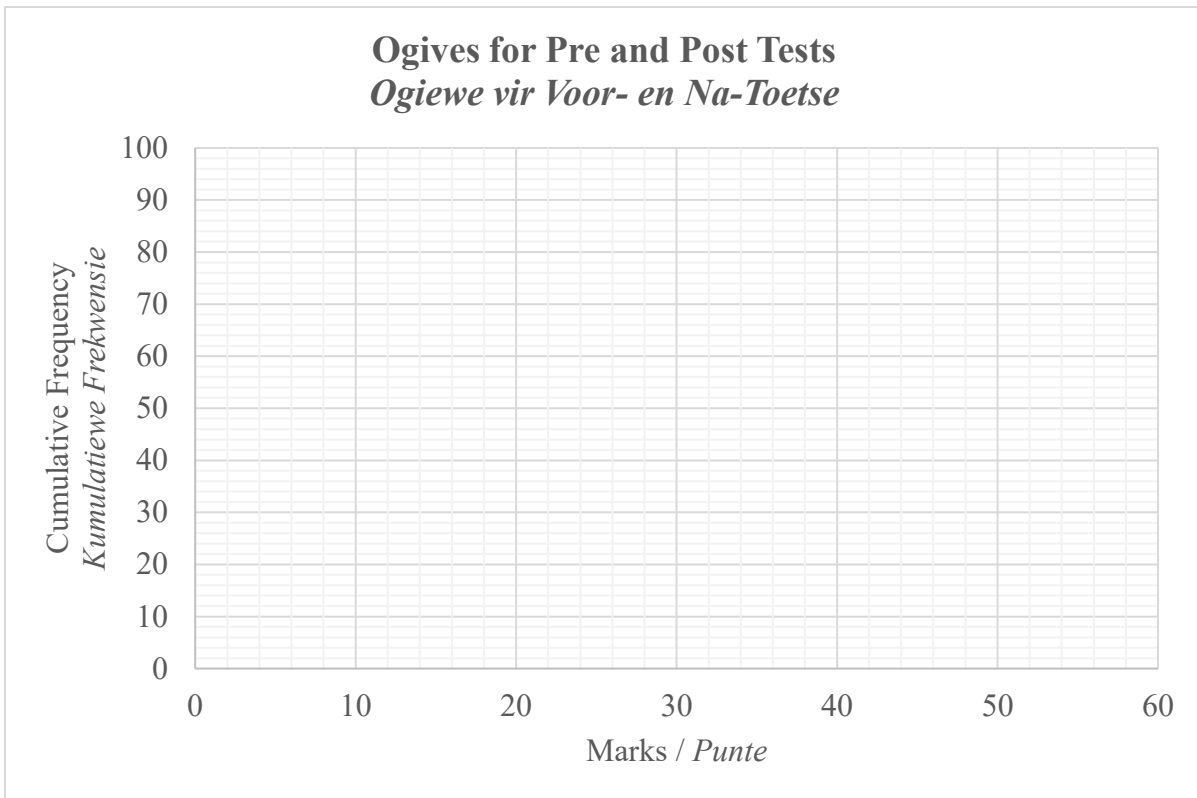
Marker/Merker			Moderator's Initials / Moderator se paraaf							
Question <i>Vraag</i>	Mark <i>Punt</i>	Initial <i>Parafeer</i>	Marks <i>Punte</i>	S M	Marks <i>Punte</i>	D M	Marks <i>Punte</i>	P M	Marks <i>Punte</i>	NM
1										
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This special answer book consists of 21 pages.
Hierdie spesiale antwoordeboek bestaan uit 21 bladsye.

QUESTION 1/VRAAG 1		
1.1		(5)
1.2		(1)
1.3		(2)
		[8]

QUESTION 2/VRAAG 2																																							
	<p>MATHEMATICS PRE AND POST TEST COMPARISON WISKUNDE VOOR EN NA TOETS VERGELYKING</p> <p style="text-align: center;"> ■ PRE-TEST VOOR TOETS ■ POST TEST NA TOETS </p> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <caption>Chart Data</caption> <thead> <tr> <th>Marks / Punte</th> <th>Pre-Test / Voor Toets</th> <th>Post Test / Na Toets</th> </tr> </thead> <tbody> <tr> <td>$0 \leq x < 10$</td> <td>16</td> <td>5</td> </tr> <tr> <td>$10 \leq x < 20$</td> <td>28</td> <td>18</td> </tr> <tr> <td>$20 \leq x < 30$</td> <td>34</td> <td>25</td> </tr> <tr> <td>$30 \leq x < 40$</td> <td>12</td> <td>32</td> </tr> <tr> <td>$40 \leq x < 50$</td> <td>0</td> <td>10</td> </tr> </tbody> </table>			Marks / Punte	Pre-Test / Voor Toets	Post Test / Na Toets	$0 \leq x < 10$	16	5	$10 \leq x < 20$	28	18	$20 \leq x < 30$	34	25	$30 \leq x < 40$	12	32	$40 \leq x < 50$	0	10																		
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2.3				(1)																																			
2.4	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th rowspan="2" style="padding: 5px;">Marks Punte</th> <th colspan="2" style="padding: 5px;">Frequency / Frekwensie</th> <th colspan="2" style="padding: 5px;">Cumulative Frequency Kumulatiewe Frekwensie</th> </tr> <tr> <th style="padding: 5px;">Pre-Test / Voor Toets</th> <th style="padding: 5px;">Post Test Na Toets</th> <th style="padding: 5px;">Pre-Test Voor Toets</th> <th style="padding: 5px;">Post Test Na Toets</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">$0 \leq x < 10$</td> <td style="width: 20%;"></td> <td style="width: 20%;"></td> <td style="width: 20%;"></td> <td style="width: 20%;"></td> </tr> <tr> <td style="padding: 5px;">$10 \leq x < 20$</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td style="padding: 5px;">$20 \leq x < 30$</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td style="padding: 5px;">$30 \leq x < 40$</td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td style="padding: 5px;">$40 \leq x < 50$</td> <td></td> <td></td> <td></td> <td></td> </tr> </tbody> </table>				Marks Punte	Frequency / Frekwensie		Cumulative Frequency Kumulatiewe Frekwensie		Pre-Test / Voor Toets	Post Test Na Toets	Pre-Test Voor Toets	Post Test Na Toets	$0 \leq x < 10$					$10 \leq x < 20$					$20 \leq x < 30$					$30 \leq x < 40$					$40 \leq x < 50$					(4)
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2.5



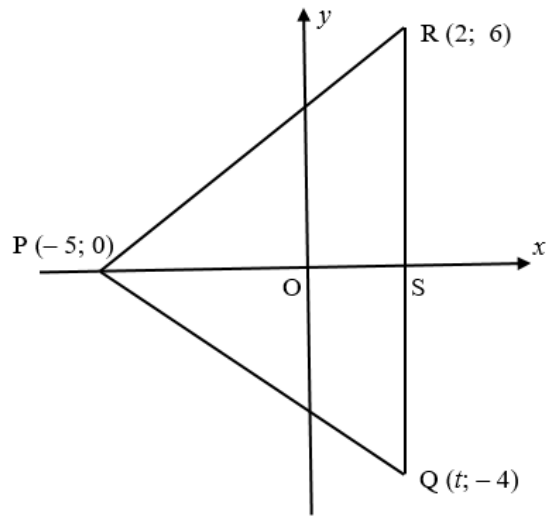
(3)

2.6

(3)

[14]

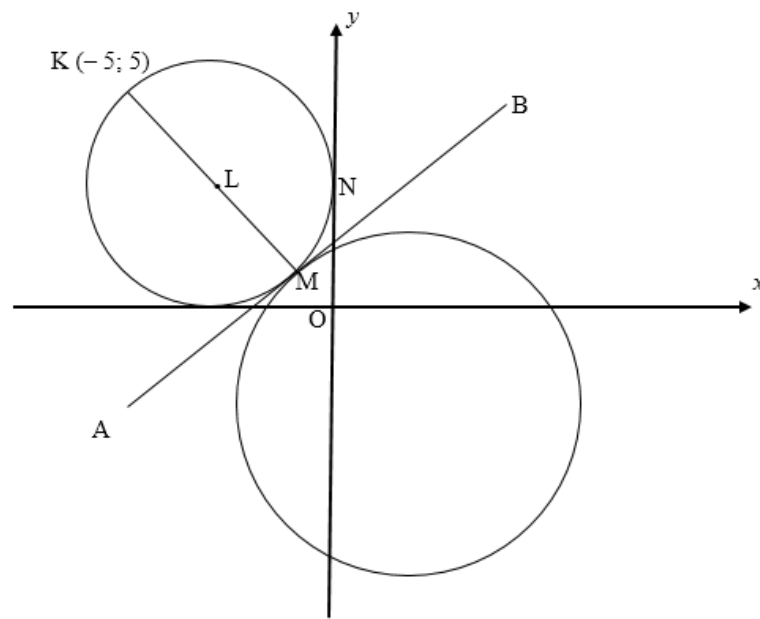
QUESTION 3/VRAAG 3



3.1		(1)
3.2.1		(2)
3.2.2		(2)
3.3		(5)

3.4		
		(4)
3.5		
		(3)
3.6		
		(5)
	[22]	

QUESTION 4/VRAAG 4

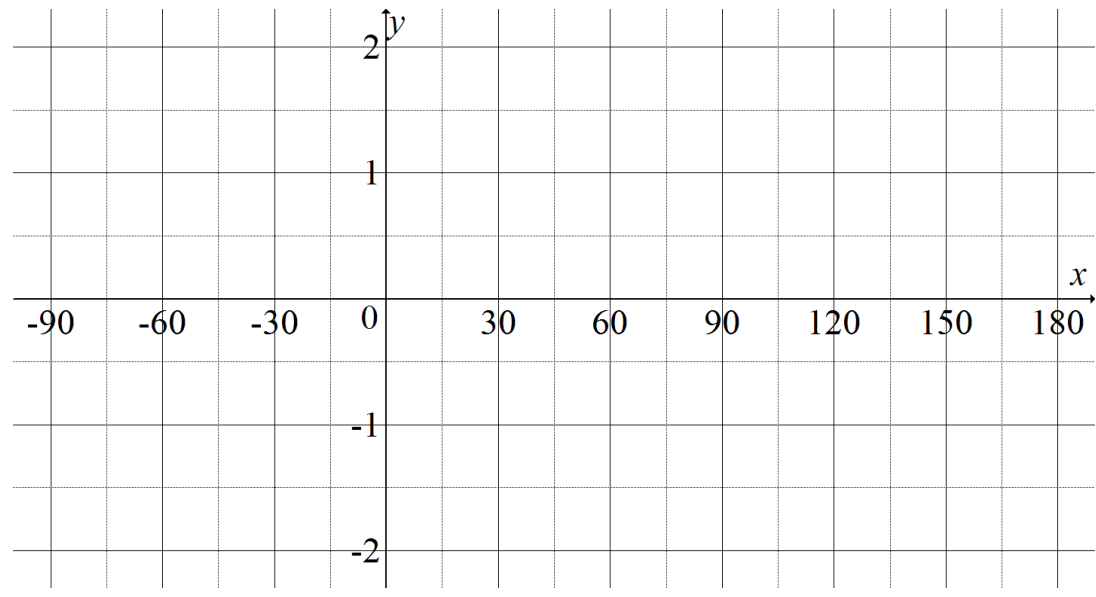


4.1.1		(4)
4.1.2		(3)
4.1.3		(4)

4.1.4		(2)
4.2.1		(2)
4.2.2		(4)
		[19]

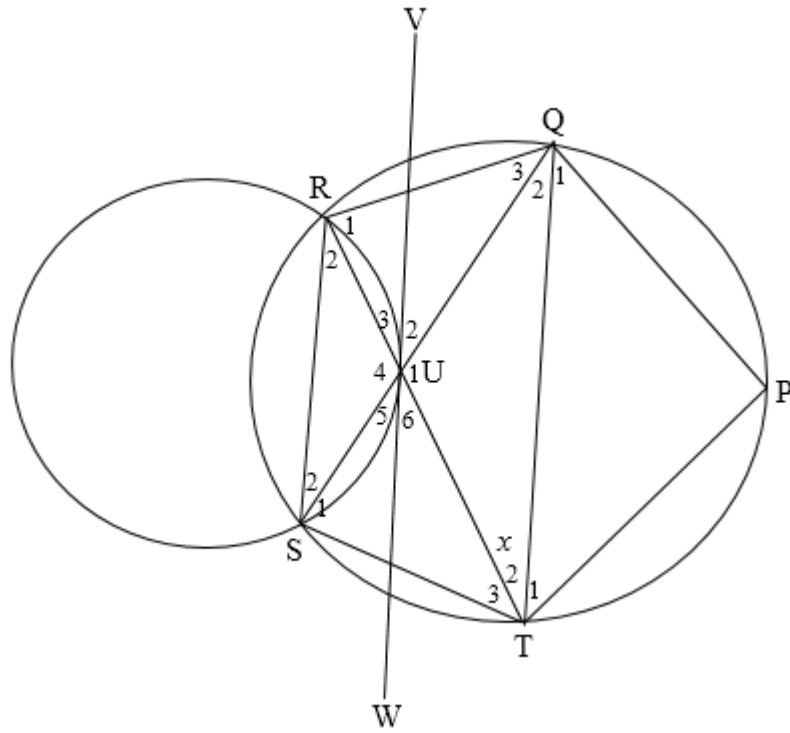
QUESTION 5/VRAAG 5		
5.1.1		(5)
5.1.2		(3)
5.1.3		(2)
5.2.1		(2)

5.2.2		
		(4)
5.3		
		(4)
		[20]

QUESTION 6/VRAAG 6		
6.1.1		(1)
6.1.2		(1)
6.2		(6)
6.3		(5)

6.4		(1)
6.5		(1)
		[15]

QUESTION 9/VRAAG 9



9.1

(3)

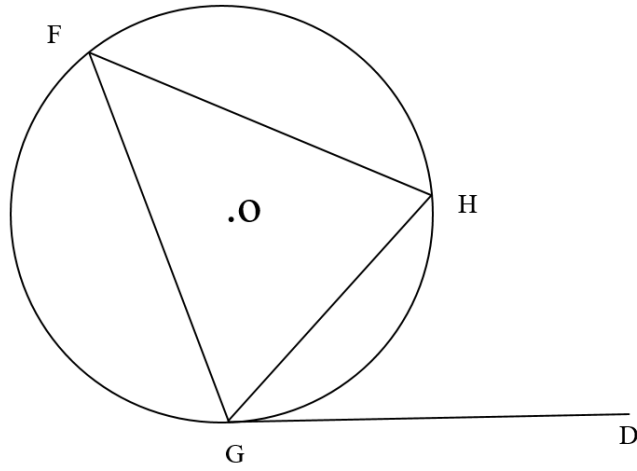
9.2

(4)

9.3.1		
		(4)
9.3.2		
		(2)
9.4.1		
		(2)
9.4.2		
		(2)
		[17]

QUESTION 10/VRAAG 10

10.1



(5)

10.2		
10.2.1		
		(1)
10.2.2		
		(5)
10.2.3		
		(2)
10.2.4		
		(5)
		[18]



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GRADE/*GRAAD* 12

JUNE/*JUNIE* 2021

**MATHEMATICS P2/*WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN*
(*EXEMPLAR/EKSEMPLAAR*)**

MARKS/*PUNTE*: 150

This marking guideline consists of 17 pages.
Hierdie nasienriglyn bestaan uit 17 bladsye.

QUESTION 1/VRAAG 1

1.1	$a = 62$ $b = 51$ $c = 42$ $d = 35$ $e = 31$	✓ value of a / waarde van a ✓ value of b / waarde van b ✓ value of c / waarde van c ✓ value of d / waarde van d ✓ value of e / waarde van e (5)
1.2	Skewed to the right / <i>Skeef na regs</i> OR/OF Positively skewed / <i>Positief skeef</i>	✓ answer / <i>antwoord</i> (1)
1.3	Yes / <i>Ja</i> $Q_3 = 51$ and the upper 25% is from Q_3 above. $Q_3 = 51$ en die boonste 25% is vanaf Q_3 en op.	✓ Yes / <i>Ja</i> ✓ Reason / <i>Rede</i> (2)
		[8]

QUESTION 2/VRAAG 2

2.1	<p>Positive impact / <i>Positiewe impak</i></p> <p>The number of learners obtaining lower marks decreased while those obtaining higher marks increased in the Post Test. <i>Die aantal leerders wat laer punte behaal het, het verminder terwyl die wat hoër punte behaal het in die na-toets vermeerder het.</i></p>	<p>✓ Positive impact <i>Positiewe impak</i></p> <p>✓ Reason / <i>Rede</i></p>	(2)
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2.2	$20 < x \leq 30$	✓ answer / <i>antwoord</i>	(1)
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2.3	Less / <i>Minder</i>	✓ answer / <i>antwoord</i>	(1)
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2.4			
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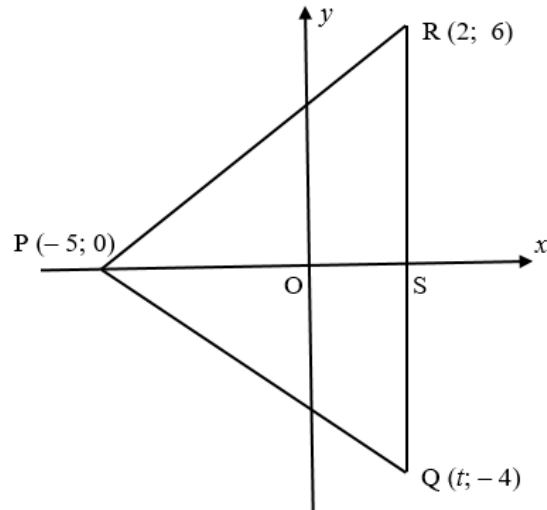
	<table border="1"> <thead> <tr> <th rowspan="2">Marks <i>Punte</i></th> <th colspan="2">Frequency <i>Frekwensie</i></th> <th colspan="2">Cumulative Frequency <i>Kumulatiewe Frekwensie</i></th> </tr> <tr> <th>Pre-Test <i>Voor Toets</i></th> <th>Post Test <i>Na Toets</i></th> <th>Pre-Test <i>Voor Toets</i></th> <th>Post Test <i>Na Toets</i></th> </tr> </thead> <tbody> <tr> <td>$0 \leq x < 10$</td> <td>16</td> <td>5</td> <td>16</td> <td>5</td> </tr> <tr> <td>$10 \leq x < 20$</td> <td>28</td> <td>18</td> <td>44</td> <td>23</td> </tr> <tr> <td>$20 \leq x < 30$</td> <td>34</td> <td>25</td> <td>78</td> <td>48</td> </tr> <tr> <td>$30 \leq x < 40$</td> <td>12</td> <td>32</td> <td>90</td> <td>80</td> </tr> <tr> <td>$40 \leq x < 50$</td> <td>0</td> <td>10</td> <td>90</td> <td>90</td> </tr> </tbody> </table>	Marks <i>Punte</i>	Frequency <i>Frekwensie</i>		Cumulative Frequency <i>Kumulatiewe Frekwensie</i>		Pre-Test <i>Voor Toets</i>	Post Test <i>Na Toets</i>	Pre-Test <i>Voor Toets</i>	Post Test <i>Na Toets</i>	$0 \leq x < 10$	16	5	16	5	$10 \leq x < 20$	28	18	44	23	$20 \leq x < 30$	34	25	78	48	$30 \leq x < 40$	12	32	90	80	$40 \leq x < 50$	0	10	90	90	<p>Frequency / <i>Frekwensie</i></p> <p>✓ Pre-Test / <i>Voor Toets</i></p> <p>✓ Post Test / <i>Na Toets</i></p> <p>Cumulative Frequency <i>Kumulatiewe Frekwensie</i></p> <p>✓ Pre-Test / <i>Voor Toets</i></p> <p>✓ Post Test / <i>Na Toets</i></p>	(4)
Marks <i>Punte</i>	Frequency <i>Frekwensie</i>		Cumulative Frequency <i>Kumulatiewe Frekwensie</i>																																		
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2.5	<p style="text-align: center;">Ogives for Pre and Post Tests <i>Ogiewe vir Voor en Na Toetse</i></p>	<p>✓ grounding / <i>anker</i></p> <p>✓ upper limits used <i>boonste limiete</i></p> <p>✓ shape / <i>vorm</i></p>	(3)
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2.6	<p>Pre: $90 - 78 = 12$ learners obtained 60% and more Post: $90 - 48 = 42$ learners obtained 60% and more Therefore, the teacher achieved the target. <i>Voor - toets: $90 - 78 = 12$ leerders het 60% en meer behaal</i> <i>Na - toets: $90 - 48 = 42$ leerders het 60% en meer behaal</i> <i>Daarom het die onderwyser die doelwit behaal</i></p>	<p>✓ 12</p> <p>✓ 42</p> <p>✓ conclusion <i>gevolgtrekking</i></p>	(3)
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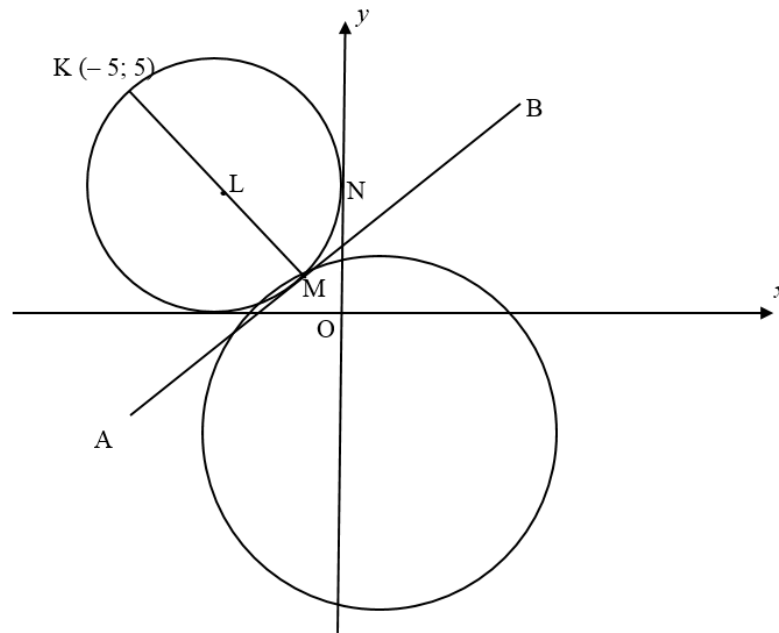
QUESTION 3/VRAAG 3



3.1	$t = 2$	✓ value of t / waarde van t (1)
3.2.1	$PR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(-5 - 2)^2 + (0 - 6)^2}$ $= \sqrt{85}$	✓ substitution / vervanging ✓ answer / antwoord (2)
3.2.2	$m_{PR} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 6}{-5 - 2}$ $= \frac{6}{7}$	✓ substitution / vervanging ✓ gradient of PR gradiënt van PR (2)
3.3	$\tan \hat{RPS} = \frac{6}{7}$ $\hat{RPS} = 40,6^\circ$ $40,6^\circ + \hat{PRQ} = 90^\circ \text{ (ext. } \angle \text{ of } \Delta / \text{buite } \angle \text{ van } \Delta)$ $\therefore \hat{PRQ} = 49,4^\circ$	✓ $\tan \theta = \frac{6}{7}$ ✓ value of / waarde van \hat{PRQ} ✓ method / metode ✓ reason / rede ✓ value of / waarde van \hat{RPS} (5)
3.4	$m_{PR} = \frac{6}{7}$ $m_{PQ} = \frac{-4 - 0}{2 - (-5)} = \frac{-4}{7}$ $m_{PR} \times m_{PQ} = \frac{6}{7} \times \left(-\frac{4}{7}\right) = -\frac{24}{49} \neq -1$ $\therefore \Delta PRQ \text{ is not right angled at R}$ $\Delta PRQ \text{ is nie reghoekig by R nie}$ <p style="text-align: center;">OR / OF</p>	✓ substitution / vervanging ✓ gradient of PR / gradiënt van PR ✓ method / metode ✓ conclusion / gevolgtrekking <p style="text-align: center;">OR / OF</p>

	$PQ = \sqrt{(-5-2)^2 + (0-(-4))^2}$ $= \sqrt{65}$ $RQ^2 = 100$ $PR^2 = 85$ $PQ^2 = 65$ $\therefore RQ^2 \neq PR^2 + PQ^2$ $\therefore \Delta PQR \text{ is not right angled at P}$ $\Delta PRQ \text{ is nie reghoekig by R nie}$	<ul style="list-style-type: none"> ✓ substitution / <i>vervanging</i> ✓ squares / <i>vierkante</i> ✓ $RQ^2 \neq RP^2 + PQ^2$ ✓ conclusion / <i>gevolgtrekking</i> 	(4)
3.5	$m_{\text{newline}} = m_{PQ} = -\frac{4}{7}$ $y - 0 = -\frac{4}{7}(x - 0)$ $\therefore y = -\frac{4}{7}x$	<ul style="list-style-type: none"> ✓ gradient of new line <i>gradiënt van nuwe lyn</i> ✓ substitution / <i>vervanging</i> ✓ equation / <i>vergelyking</i> 	(3)
3.6	<p>SP = 7 units / <i>eenhede</i> SR = 6 units / <i>eenhede</i> RQ = 10 units / <i>eenhede</i></p> <p>Area of ΔSPR / <i>Oppervlakte van ΔSPR</i> $= \frac{1}{2} \times 7 \times 6$ $= 21 \text{ units}^2 / \text{eenhede}^2$</p> <p>Area of ΔRPQ / <i>Oppervlakte van ΔRPQ</i> $= \frac{1}{2} \times 7 \times 10$ $= 35 \text{ units}^2 / \text{eenhede}^2$</p> <p>$\frac{\text{Area of } \Delta SPR}{\text{Area of } \Delta RPQ} / \frac{\text{Oppervlakte van } \Delta SPR}{\text{Oppervlakte van } \Delta RPQ}$ $= \frac{21}{35}$ $= \frac{3}{5}$</p>	<ul style="list-style-type: none"> ✓ length of SP / <i>lengte van SP</i> ✓ length of RQ / <i>lengte van RQ</i> ✓ Area of / <i>Oppervlakte van ΔSPR</i> ✓ Area of / <i>Oppervlakte van ΔRPQ</i> ✓ answer / <i>antwoord</i> 	(5)
			[22]

QUESTION 4/VRAAG 4



4.1.1	$x^2 + y^2 + 6x - 6y + 9 = 0$ $x^2 + 6x + 9 + y^2 - 6y + 9 = -9 + 9 + 9$ $\therefore (x+3)^2 + (y-3)^2 = 9$ L(-3;3) and/en $r=3$ units/eenhede	<ul style="list-style-type: none"> ✓ method / metode ✓ $(x+3)^2 + (y-3)^2 = 9$ ✓ coordinates of L koördinate van L ✓ value of r / waarde van r 	(4)
4.1.2	$-3 = \frac{-5 + x_M}{2} \quad 3 = \frac{5 + y_M}{2}$ $x_M = -1 \quad y_M = 1$ $\therefore M(-1;1)$	<ul style="list-style-type: none"> ✓ method / metode ✓ value of x / waarde van x ✓ value of y / waarde van y 	(3)
4.1.3	$m_{KL} = \frac{5-1}{-5+1} = -1 \quad \text{OR} \quad m_{LM} = \frac{3-1}{-3+1} = -1$ $m_{\text{tangent}} = 1$ $y - 1 = 1(x - (-1))$ $\therefore y = x + 2$	<ul style="list-style-type: none"> ✓ m_{LM} OR m_{KM} ✓ m_{tangent} ✓ substitution / vervanging ✓ equation / vergelyking 	(4)
4.1.4	$(x+3)^2 + (y-3)^2 = 9$ $(0+3)^2 + (y-3)^2 = 9$ $(y-3)^2 = 0$ $y = 3$ N(0;3)	<ul style="list-style-type: none"> ✓ value of x / waarde van x ✓ value of y / waarde van y 	(2)

4.2.1	$L(-3; 3)$ $L'(2; -4)$	✓ value of x / <i>waarde van x</i> ✓ value of y / <i>waarde van y</i> (2)
4.2.2	$m_{ML'} = \frac{-4-1}{2+1} = -\frac{5}{3}$ $y - (1) = -\frac{5}{3}(x+1)$ $\therefore y = -\frac{5}{3}x - \frac{2}{3}$ Not passing through the origin <i>Gaan nie deur die oorsprong nie</i>	✓ $m_{ML'}$ ✓ substitution / <i>vervanging</i> ✓ equation / <i>vergelyking</i> ✓ conclusion / <i>gevolgtrekking</i> (4)
		[19]

QUESTION/VRAAG 5

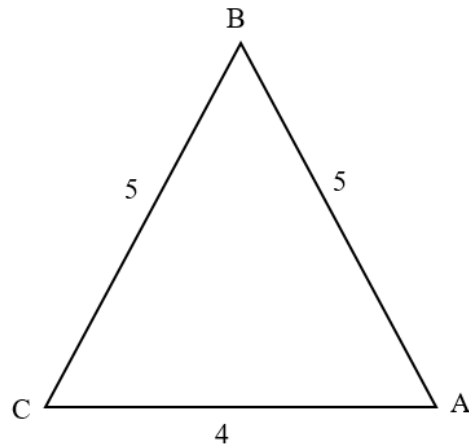
5.1.1	$\sin \alpha = -\frac{5}{13} \quad \text{and/en} \quad \tan \beta = -\frac{3}{4}$ $x = -12 \quad r = 5$ $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$ $= \left(\frac{-5}{13}\right)\left(\frac{-4}{5}\right) + \left(\frac{-12}{13}\right)\left(\frac{3}{5}\right)$ $= -\frac{16}{65}$	<ul style="list-style-type: none"> ✓ value of x / waarde van x ✓ value of r / waarde van r ✓ expansion / uitbreiding ✓ substitution / vervanging ✓ answer / antwoord <p style="text-align: right;">(5)</p>
5.1.2	$\cos 2\beta = 1 - 2\sin^2 \beta$ $= 1 - 2\left(\frac{3}{5}\right)^2$ $= \frac{7}{25}$	<ul style="list-style-type: none"> ✓ expansion / uitbreiding ✓ substitution / vervanging ✓ answer / antwoord <p style="text-align: right;">(3)</p>
5.1.3	$\tan(-\alpha - 180^\circ) = -\tan(180^\circ + \alpha)$ $= -\tan \alpha$ $= -\left(\frac{-5}{-12}\right) = -\frac{5}{12}$	<ul style="list-style-type: none"> ✓ reduction / reduksie ✓ substitution / vervanging <p style="text-align: right;">(2)</p>
5.2.1	$1 - \cos \theta = 0 \text{ or } \sin \theta = 0$ $\cos \theta = 1 \text{ or } \sin \theta = 0$ $\therefore \theta = 180 \cdot k \quad (k \in \mathbb{Z})$ <p style="text-align: center;">OR / OF</p> $\theta = 360 \cdot k \quad \text{or } \theta = 180^\circ + 360 \cdot k \quad (k \in \mathbb{Z})$	<ul style="list-style-type: none"> ✓ method / metode ✓ answer / antwoord <p style="text-align: right;">(2)</p>
5.2.2	$\text{LHS/LK} = \frac{\sin \theta}{1 - \cos \theta} - \frac{\cos \theta}{\sin \theta}$ $= \frac{\sin^2 \theta - \cos(1 - \cos \theta)}{\sin \theta(1 - \cos \theta)}$ $= \frac{\sin^2 \theta - \cos + \cos^2 \theta}{\sin \theta(1 - \cos \theta)}$ $= \frac{1 - \cos \theta}{\sin \theta(1 - \cos \theta)}$ $= \frac{1}{\sin \theta}$ $= \text{RHS/RK}$	<ul style="list-style-type: none"> ✓ common denominator gemene noemer ✓ simplification / vereenvoudiging ✓ identity / identiteit ✓ simplification / vereenvoudiging <p style="text-align: right;">(4)</p>

5.3	$\begin{aligned} \text{LHS/LK} &= \frac{\sin(x-y)}{\cos x \cdot \cos y} \\ &= \frac{\sin x \cos y - \cos x \sin y}{\cos x \cdot \cos y} \\ &= \frac{\sin x \cos y}{\cos x \cdot \cos y} - \frac{\cos x \sin y}{\cos x \cdot \cos y} \\ &= \tan x - \tan y \\ &= 3k - 2k \\ &= k \end{aligned}$	<p>✓ identity / <i>identiteit</i></p> <p>✓ method / <i>metode</i></p> <p>✓ simplification / <i>vereenvoudiging</i></p> <p>✓ substitution / <i>vervanging</i></p> <p style="text-align: right;">(4)</p>
		[20]

QUESTION 6/VRAAG 6

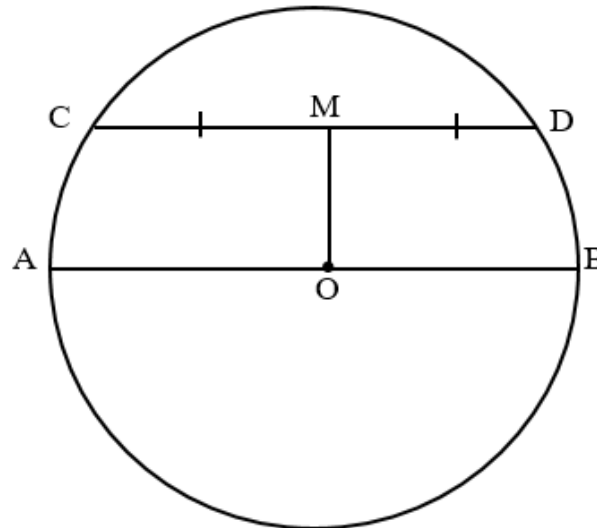
6.1.1	1	✓ 1 (1)
6.1.2	120°	✓ 120° (1)
6.2	$f(x) = g(x)$ $\cos(x - 60^\circ) = \sin 3x$ $\cos(x - 60^\circ) = \cos(90^\circ - 3x)$ $\pm(x - 60^\circ) = 90^\circ - 3x + k.360^\circ \quad (k \in \mathbb{Z})$ $x - 60^\circ = 90^\circ - 3x + k.360^\circ$ or / of $-x + 60^\circ = 90^\circ - 3x + k.360^\circ$ $4x = 150^\circ + k.360^\circ$ or / of $2x = 30^\circ + k.360^\circ$ $x = 37,5^\circ + k.90^\circ$ or / of $x = 15^\circ + k.180^\circ$ $x \in \{37,5^\circ, -52,5^\circ; 15^\circ; 127,5^\circ\}$	✓ $\cos(x - 60^\circ) = \cos(90^\circ - 3x)$ ✓ without cos <i>sonder cos</i> ✓ simplification <i>vereenvoudiging</i> ✓ $x = 30^\circ + k.90^\circ$ or / of $x = 15^\circ + k.180^\circ$ ✓ two values <i>twee waardes</i> ✓ two remaining values <i>twee oorblywende waardes</i> (6)
6.3		<i>f:</i> ✓ endpoints / <i>eindpunte</i> ✓ both intercepts <i>beide afsnitte</i> ✓ shape / <i>vorm</i> <i>g:</i> ✓ both intercepts <i>beide afsnitte</i> ✓ shape / <i>vorm</i> (5)
6.4	$x = -30^\circ$ or / of $x = 150^\circ$	✓ both values of x <i>beide waardes van x</i> (1)
6.5	$f(x) = \cos(x - 60^\circ + 15^\circ)$ $h(x) = \cos(x - 45^\circ)$	✓ $h(x) = \cos(x - 45^\circ)$ (1)
		[15]

QUESTION 7/VRAAG 7



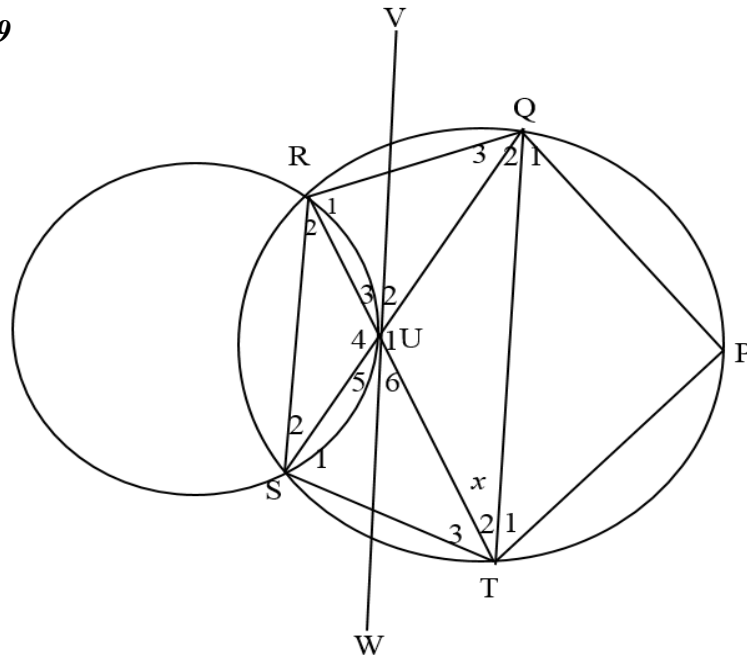
$5^2 = 4^2 + 5^2 - 2(4)(5) \cos A$ $\cos A = \frac{2}{5}$ $4^2 = 5^2 + 5^2 - 2(5)(5) \cos B$ $\cos B = \frac{17}{25}$ $\cos A - \cos B = \frac{2}{5} - \frac{17}{25}$ $= -\frac{7}{25} \approx -0,28$	<p>✓ substitution into cosine rule / <i>vervanging in die cosinusreël</i></p> <p>✓ value of cos A / <i>waarde van cos A</i></p> <p>✓ substitution into cosine rule / <i>vervanging in die cosinusreël</i></p> <p>✓ value of cos B / <i>waarde van cos B</i></p> <p>✓ value of cos A – cos B / <i>waarde van cos A – cos B</i></p> <p style="text-align: right;">(5)</p>
	[5]

QUESTION 8/VRAAG 8



<p>OM \perp CD (line from centre which bisects the chord) (lyn vanaf die middelpunt wat koord halveer)</p> <p>AO = OB = OD = 11 cm</p> <p>In $\triangle OMD$: $MD^2 + 7^2 = 11^2$ (Pythagoras theorem/stelling) $MD = 6\sqrt{2}$ $\therefore CD = 12\sqrt{2}$</p>	<p>✓ S/R</p> <p>✓ S</p> <p>✓ use of Pythagoras theorem gebruik van Pythagoras - stelling</p> <p>✓ length of MD/lengte van MD ✓ length of CD/lengte van CD</p> <p>(5)</p>
	[5]

QUESTION 9/VRAAG 9

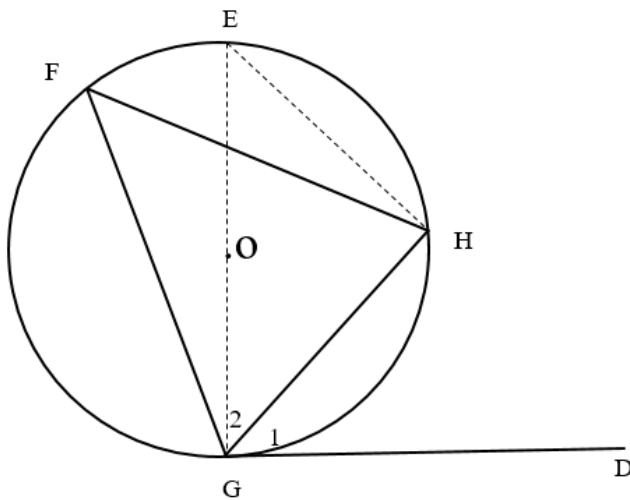


<p>9.1</p>	<p>$\hat{R}_2 = \hat{Q}_2$ (\angles in the same segment / \anglee in dieselfde segment) $\hat{U}_4 = \hat{U}_1$ (vert. opp. \angles / regoorst. \anglee) $\hat{S}_2 = \hat{T}_2$ (\angles in the same segment / \anglee in dieselfde segment) $\therefore \Delta RUS \parallel \Delta QUT$ ($\angle\angle\angle$)</p>	<p>✓ S/R ✓ S/R ✓ S/R (3)</p>
<p>9.2</p>	<p>$\hat{S}_2 = \hat{T}_2 = x$ (\angles in the same seg / \anglee in dieselfde segment) $\hat{U}_3 = \hat{S}_2 = x$ (tan-chord theorem / raaklyn-koord stelling) $\hat{U}_6 = \hat{U}_3 = x$ (vertically opp. \angles / regoorst. \anglee)</p>	<p>✓ S/R ✓ S ✓ R ✓ S/R (4)</p>
<p>9.3.1</p>	<p>$\hat{R}_1 + x + 90^\circ - x = 180^\circ$ (sum of \angles of Δ / som van die \anglee van Δ) $\therefore \hat{R}_1 = 90^\circ$ QT is a diameter (QT subtends a right angle) QT is 'n middellyn (QT onderspan 'n reghoek)</p>	<p>✓ S ✓ value of \hat{R}_1 / waarde van \hat{R}_1 ✓ QT is a diameter/ is 'n middellyn ✓ R (4)</p>
<p>9.3.2</p>	<p>$\hat{P} = 90^\circ$ (\angle in the semicircle / \angle in 'n semisirkel) OR / OF $\hat{P} + 90^\circ = 180^\circ$ (opp. \angles of a cyclic quad / teenoorst. \anglee van 'n koordevierhoek) $\therefore \hat{P} = 90^\circ$</p>	<p>✓ S ✓ R (2)</p>

9.4.1	$\hat{Q}_2 = \hat{T}_2 = x$ (\angle s opp. equal sides / \angle teenoor gelyke sye) $\therefore \hat{Q}_2 = \hat{S}_2$ $\therefore RS \parallel QT$ (Alt. \angle s are equal / <i>Verw. \anglee is gelyk</i>)	✓ S/R ✓ R (2)
9.4.2	$\hat{U}_2 = \hat{Q}_2 = x$ VW is a tangent to circle passing through QUT (Converse of tan-chord theorem) VW is 'n raaklyn aan die sirkel wat deur QUT gaan (<i>Omgekeerde van die raaklyn – koord stelling</i>)	✓ S ✓ R (2)
		[17]

QUESTION 10/VRAAG 10

10.1



Construction: Draw diameter GOE. Join EH
 Konstruksie: Trek middellyn GOE. Verbind EH

Proof/Bewys:

$$\hat{G}_1 + \hat{G}_2 = 90^\circ \text{ (tangent } \perp \text{ diameter) /}$$

$$\text{(raaklyn } \perp \text{ radius)}$$

$$\hat{EHG} = 90^\circ \text{ (}\angle \text{ in the semi circle) /}$$

$$\text{(}\angle \text{ in semi - sirkel)}$$

$$\hat{G}_2 + \hat{E} = 90^\circ \text{ (sum of } \angle \text{s of } \Delta) /$$

$$\text{(som van die } \angle \text{e van 'n } \Delta)$$

$$\therefore \hat{G}_1 + \hat{G}_2 = \hat{G}_2 + \hat{E}$$

$$\therefore \hat{G}_1 = \hat{E}$$

But/Maar: $\hat{E} = \hat{F}$ (\angle s in the same segment) /
 (\angle e in dieselfde segment)

$$\therefore \hat{DGH} = \hat{F}$$

✓ construction / konstruksie

✓ S/R

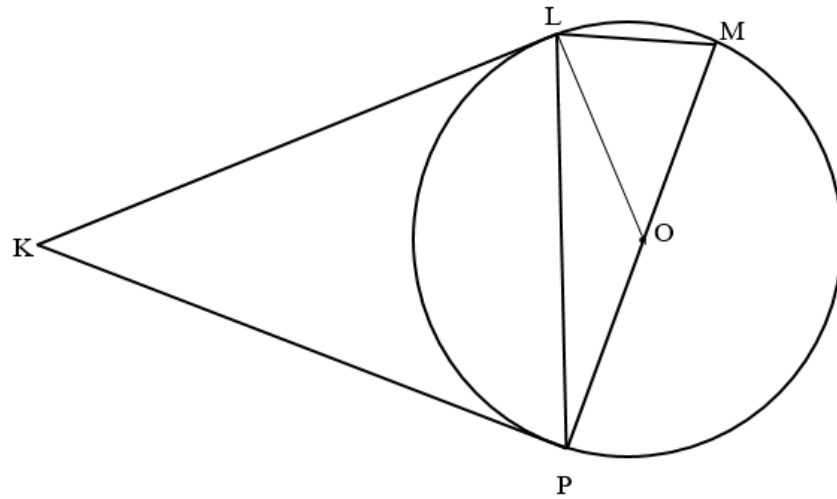
✓ S/R

✓ S

✓ S/R

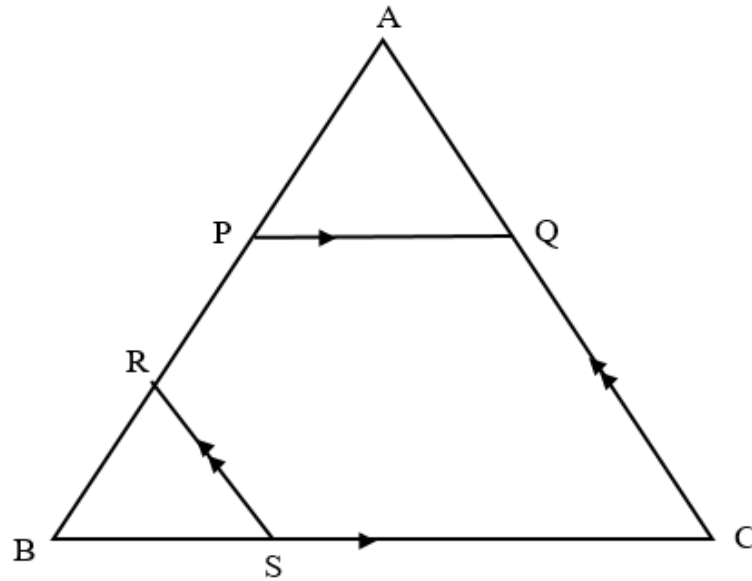
(5)

10.2



10.2.1	Kite / <i>Vlieër</i>	✓ answer / <i>antwoord</i> (1)
10.2.2	$\hat{KLO} = 90^\circ$ (tan \perp rad.) / (<i>raaklyn \perp radius</i>) $\hat{KPO} = 90^\circ$ (tan \perp rad.) / (<i>raaklyn \perp radius</i>) $\hat{MLP} = 90^\circ$ (\angle s in the semi circle) / (\angle in 'n semi-sirkel)	✓ S ✓ R ✓ S ✓ S ✓ R (5)
10.2.3	$\hat{KLO} + \hat{KPO} = 90^\circ + 90^\circ$ $= 180^\circ$ \therefore KLOP is a cyclic quad. (Opp. \angle s are supp.) <i>KLOP is 'n koordevierhoek (Teenoorst. \anglee is suppl.)</i>	✓ S ✓ R (2)
10.2.4	$\hat{K} + \hat{LOP} = 180^\circ$ (Opp. \angle s of cyclic quad.) But $\hat{M} = 67^\circ$ (\angle s opp. equal sides) $\therefore \hat{LOP} = 67^\circ + 67^\circ$ (Ext. \angle of Δ) $= 134^\circ$ $\therefore \hat{K} + 134^\circ = 180^\circ$ $\therefore \hat{K} = 46^\circ$ <p style="text-align: center;">OR/OF</p> $\hat{M} = 67^\circ$ (\angle s opp. = sides) $\hat{LOM} = 46^\circ$ (\angle s of Δ) $\therefore \hat{K} = 46^\circ$ (ext. \angle of cyclic quad.)	✓ S ✓ R ✓ S/R ✓ value of \hat{LOP} <i>waarde van \hat{LOP}</i> ✓ value of \hat{K} / <i>waarde van \hat{K}</i> <p style="text-align: center;">OR/OF</p> ✓ S ✓ R ✓ value of / <i>waarde van \hat{LOM}</i> ✓ value of / <i>waarde van \hat{K}</i> ✓ reason / <i>rede</i> (5)
		[18]

QUESTION 11/VRAAG 11



$\frac{AP}{PB} = \frac{3}{5}$ <p>(Prop. theorem; $PQ \parallel BC$) (Verhouding stelling; $PQ \parallel BC$)</p> $\frac{AP}{PR + BR} = \frac{3}{5}$ $\therefore 5AP = 3PR + 3BR$	<p>✓ S/R</p> <p>✓ S</p> <p>✓ simplification / vereenvoudiging</p>
$\frac{BR}{RA} = \frac{1}{3}$ <p>(Prop. theorem; $RS \parallel AC$) (Verhouding stelling; $RS \parallel AC$)</p> $\frac{AP}{AP + PR} = \frac{1}{3}$ $3BR = AP + PR$ $\therefore 5AP = 3PR + AP + PR$ $4AP = 4PR$ $\therefore AP = PR$	<p>✓ S/R</p> <p>✓ S</p> <p>✓ simplification / vereenvoudiging</p> <p>✓ substitution / vervanging</p>
	(7)
	[7]

TOTAL/TOTAAL: 150