basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2019

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages, 1 information sheet and an answer book of 24 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The table below shows the monthly income (in rands) of 6 different people and the amount (in rands) that each person spends on the monthly repayment of a motor vehicle.

MONTHLY INCOME (IN RANDS)	9 000	13 500	15 000	16 500	17 000	20 000
MONTHLY REPAYMENT (IN RANDS)	2 000	3 000	3 500	5 200	5 500	6 000

1.1 Determine the equation of the least squares regression line for the data. (3)

1.2 If a person earns R14 000 per month, predict the monthly repayment that the person could make towards a motor vehicle. (2)

1.3 Determine the correlation coefficient between the monthly income and the monthly repayment of a motor vehicle. (1)

1.4 A person who earns R18 000 per month has to decide whether to spend R9 000 as a monthly repayment of a motor vehicle, or not. If the above information is a true representation of the population data, which of the following would the person most likely decide on:

A Spend R9 000 per month because there is a very strong positive correlation between the amount earned and the monthly repayment.

B NOT to spend R9 000 per month because there is a very weak positive correlation between the amount earned and the monthly repayment.

C Spend R9 000 per month because the point (18 000 ; 9 000) lies very near to the least squares regression line.

D NOT to spend R9 000 per month because the point (18 000 ; 9 000) lies very far from the least squares regression line. (2)

[8]

QUESTION 2

A survey was conducted among 100 people about the amount that they paid on a monthly basis for their cellphone contracts. The person carrying out the survey calculated the estimated mean to be R309 per month. Unfortunately, he lost some of the data thereafter. The partial results of the survey are shown in the frequency table below:

AMOUNT PAID (IN RANDS)	FREQUENCY
$0 < x \leq 100$	7
$100 < x \leq 200$	12
$200 < x \leq 300$	a
$300 < x \leq 400$	35
$400 < x \leq 500$	b
$500 < x \leq 600$	6

2.1 How many people paid R200 or less on their monthly cellphone contracts? (1)

2.2 Use the information above to show that $a = 24$ and $b = 16$. (5)

2.3 Write down the modal class for the data. (1)

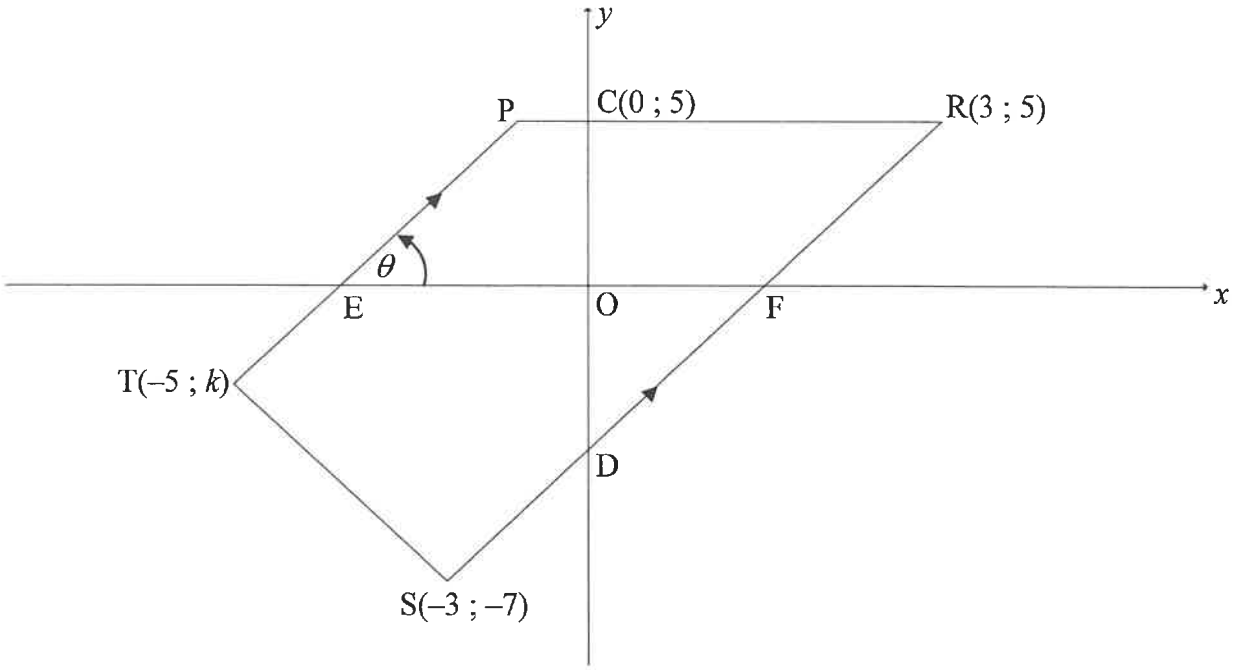
2.4 On the grid provided in the ANSWER BOOK, draw an ogive (cumulative frequency graph) to represent the data. (4)

2.5 Determine how many people paid more than R420 per month for their cellphone contracts. (2)

[13]

QUESTION 3

In the diagram, P, R(3 ; 5), S(–3 ; –7) and T(–5 ; k) are vertices of trapezium PRST and PT || RS. RS and PR cut the y-axis at D and C(0 ; 5) respectively. PT and RS cut the x-axis at E and F respectively. $P\hat{E}F = \theta$.



3.1 Write down the equation of PR. (1)

3.2 Calculate the:

3.2.1 Gradient of RS (2)

3.2.2 Size of θ (3)

3.2.3 Coordinates of D (3)

3.3 If it is given that $TS = 2\sqrt{5}$, calculate the value of k. (4)

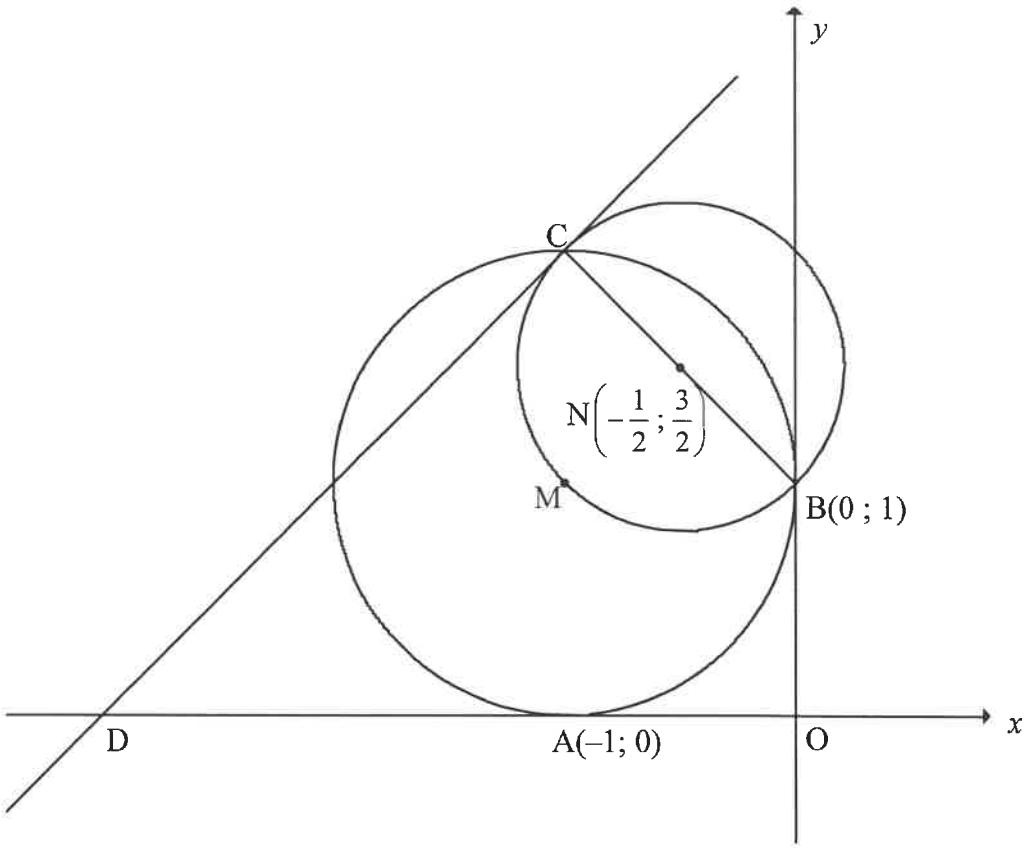
3.4 Parallelogram TDNS, with N in the 4th quadrant, is drawn. Calculate the coordinates of N. (3)

3.5 ΔPRD is reflected about the y-axis to form $\Delta P'R'D'$. Calculate the size of $R\hat{D}R'$. (3)

[19]

QUESTION 4

In the diagram, a circle having centre M touches the x–axis at A(–1 ; 0) and the y-axis at B(0 ; 1). A smaller circle, centred at $N\left(-\frac{1}{2};\frac{3}{2}\right)$, passes through M and cuts the larger circle at B and C. BNC is a diameter of the smaller circle. A tangent drawn to the smaller circle at C, cuts the x-axis at D.



4.1 Determine the equation of the circle centred at M in the form $(x-a)^2+(y-b)^2=r^2$ (3)

4.2 Calculate the coordinates of C. (2)

4.3 Show that the equation of the tangent CD is $y-x=3$. (4)

4.4 Determine the values of t for which the line $y=x+t$ will NOT touch or cut the smaller circle. (3)

4.5 The smaller circle centred at N is transformed such that point C is translated along the tangent to D. Calculate the coordinates of E, the new centre of the smaller circle. (3)

4.6 If it is given that the area of quadrilateral OBCD is $2a^2$ square units and $a>0$, show that $a=\frac{\sqrt{7}}{2}$ units. (5)

[20]

QUESTION 5

5.1 Simplify the following expression to ONE trigonometric term:

$$\frac{\sin x}{\cos x . \tan x} + \sin(180° + x)\cos(90° - x)$$ (5)

5.2 **Without using a calculator**, determine the value of: $\frac{\sin^2 35° - \cos^2 35°}{4\sin 10° \cos 10°}$ (4)

5.3 Given: $\cos 26° = m$

Without using a calculator, determine $2\sin^2 77°$ in terms of m. (4)

5.4 Consider: $f(x) = \sin(x + 25°)\cos 15° - \cos(x + 25°)\sin 15°$

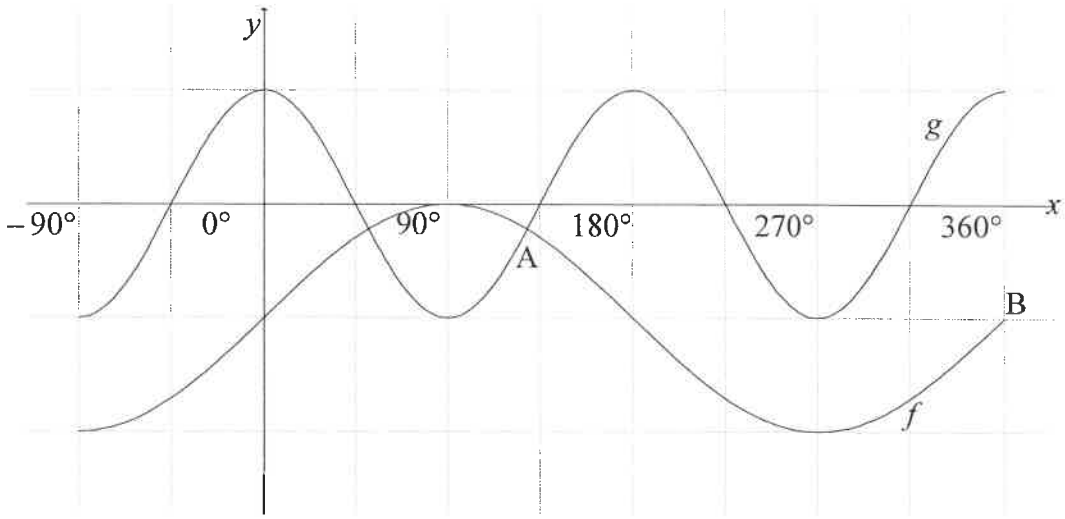
5.4.1 Determine the general solution of $f(x) = \tan 165°$ (6)

5.4.2 Determine the value(s) of x in the interval $x \in [0°; 360°]$ for which $f(x)$ will have a minimum value. (3)

[22]

QUESTION 6

In the diagram, the graphs of $f(x) = \sin x - 1$ and $g(x) = \cos 2x$ are drawn for the interval $x \in [-90°; 360°]$. Graphs f and g intersect at A. B(360° ; –1) is a point on f.



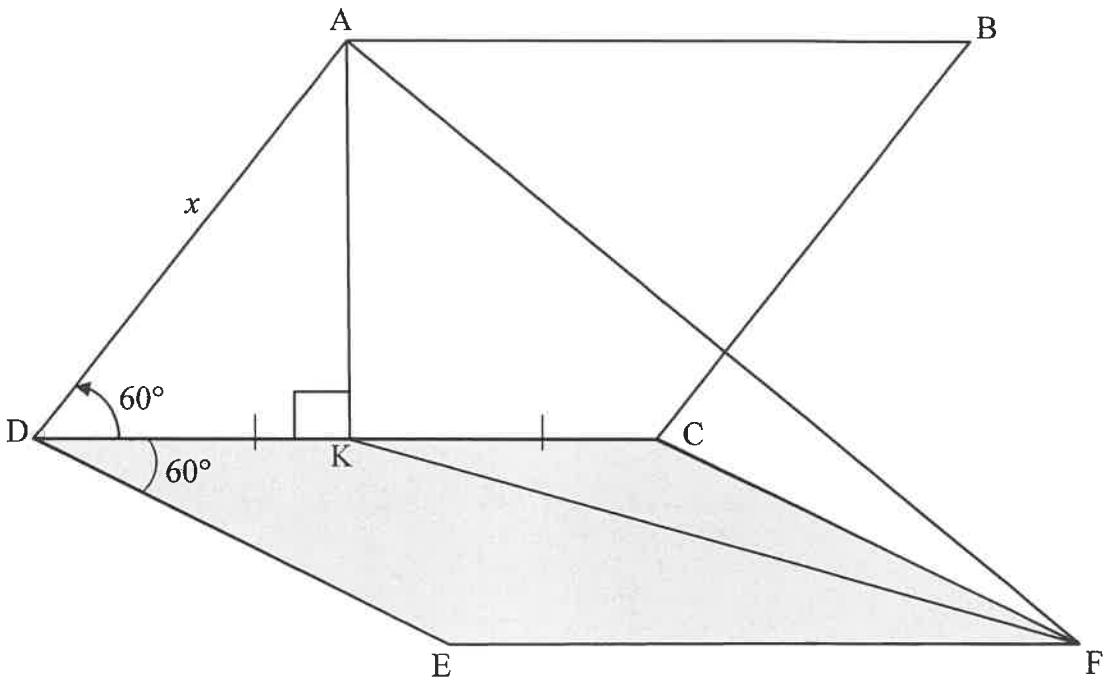
6.1 Write down the range of f. (2)

6.2 Write down the values of x in the interval $x \in [-90°; 360°]$ for which graph f is decreasing. (2)

6.3 P and Q are points on graphs g and f respectively such that PQ is parallel to the y-axis. If PQ lies between A and B, determine the value(s) of x for which PQ will be a maximum. (6)
[10]

QUESTION 7

The diagram below shows a solar panel, ABCD, which is fixed to a flat piece of concrete slab EFCD. ABCD and EFCD are two identical rhombuses. K is a point on DC such that $DK = KC$ and $AK \perp DC$. AF and KF are drawn. $A\hat{D}C = C\hat{D}E = 60°$ and $AD = x$ units.



7.1 Determine AK in terms of x. (2)

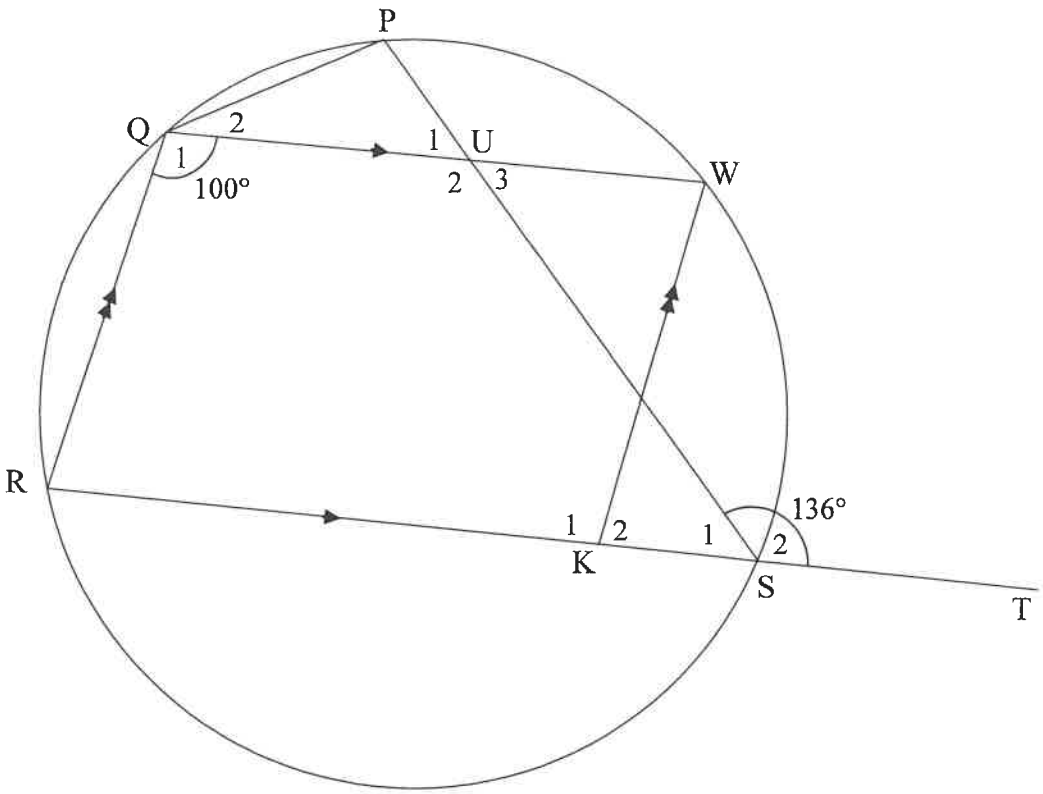
7.2 Write down the size of $K\hat{C}F$. (1)

7.3 It is further given that $A\hat{K}F$, the angle between the solar panel and the concrete slab, is y. Determine the area of ΔAKF in terms of x and y. (7)

[10]

QUESTION 8

8.1 In the diagram, PQRS is a cyclic quadrilateral. Chord RS is produced to T. K is a point on RS and W is a point on the circle such that QRKW is a parallelogram. PS and QW intersect at U. $P\hat{S}T = 136°$ and $\hat{Q}_1 = 100°$.



Determine, with reasons, the size of:

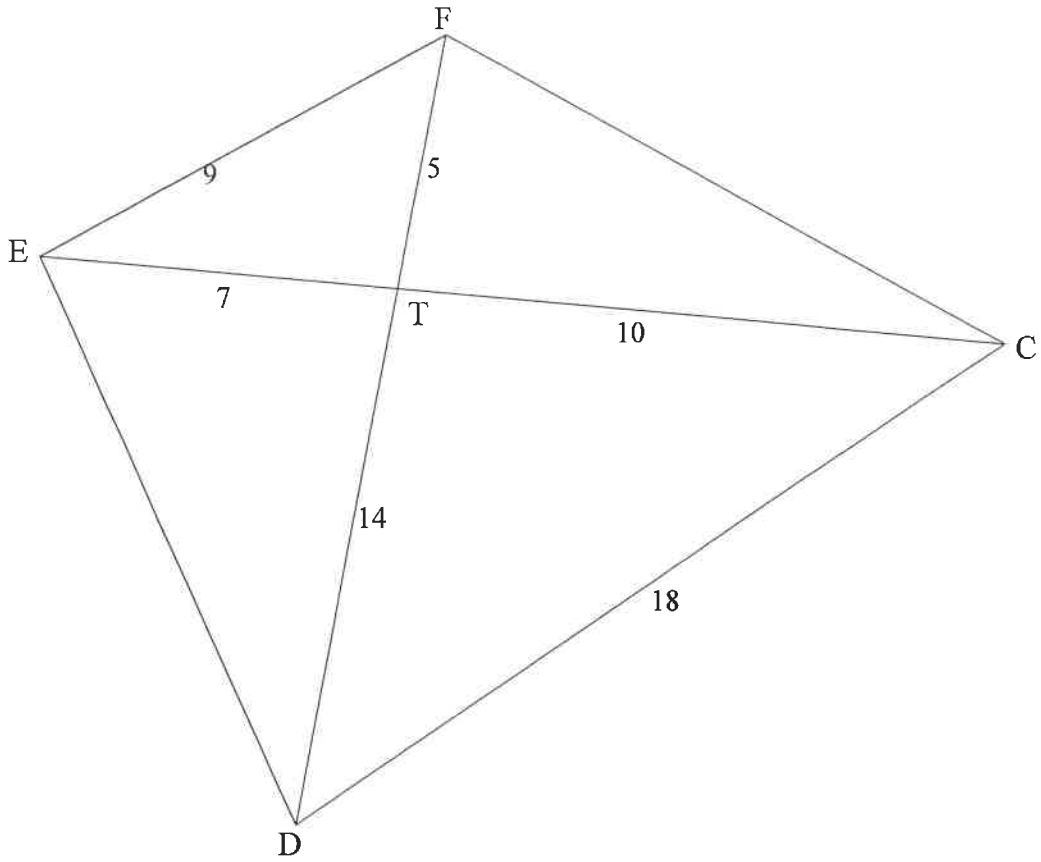
8.1.1 $\hat{R}$ (2)

8.1.2 $\hat{P}$ (2)

8.1.3 $P\hat{Q}W$ (3)

8.1.4 $\hat{U}_2$ (2)

8.2 In the diagram, the diagonals of quadrilateral CDEF intersect at T.
EF = 9 units, DC = 18 units, ET = 7 units, TC = 10 units, FT = 5 units and TD = 14 units.



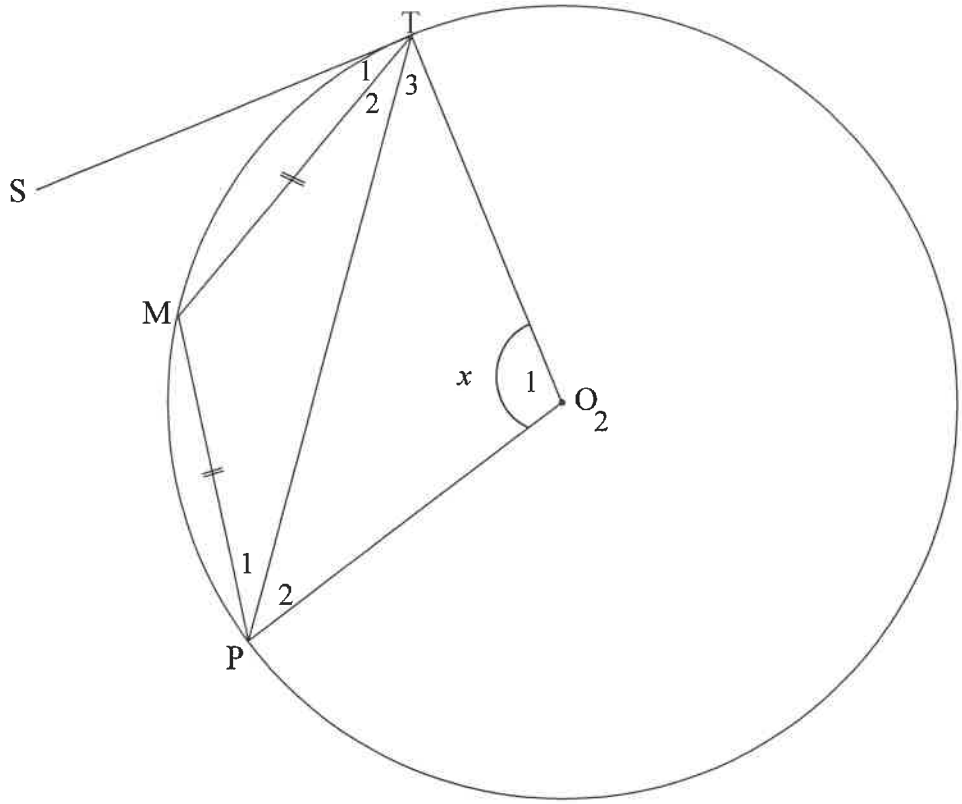
Prove, with reasons, that:

8.2.1 $E\hat{F}D = E\hat{C}D$ (4)

8.2.2 $D\hat{F}C = D\hat{E}C$ (3)
[16]

QUESTION 9

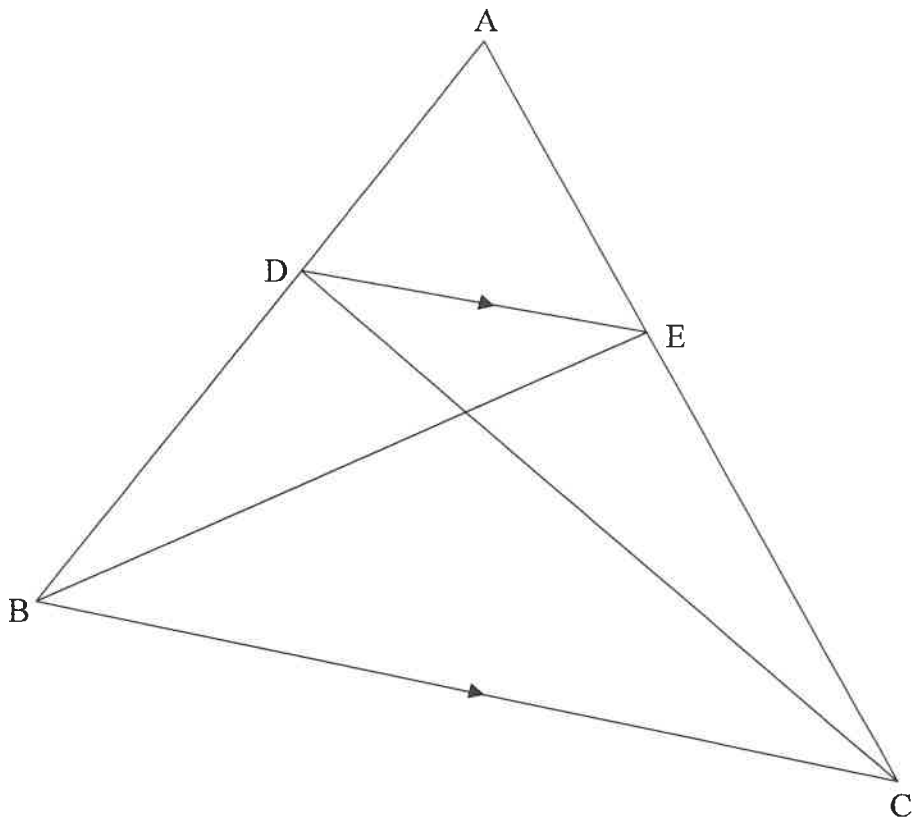
In the diagram, O is the centre of the circle. ST is a tangent to the circle at T. M and P are points on the circle such that TM = MP. OT, OP and TP are drawn. Let $\hat{O}_1 = x$.



Prove, with reasons, that $S\hat{T}M = \frac{1}{4}x$. **[7]**

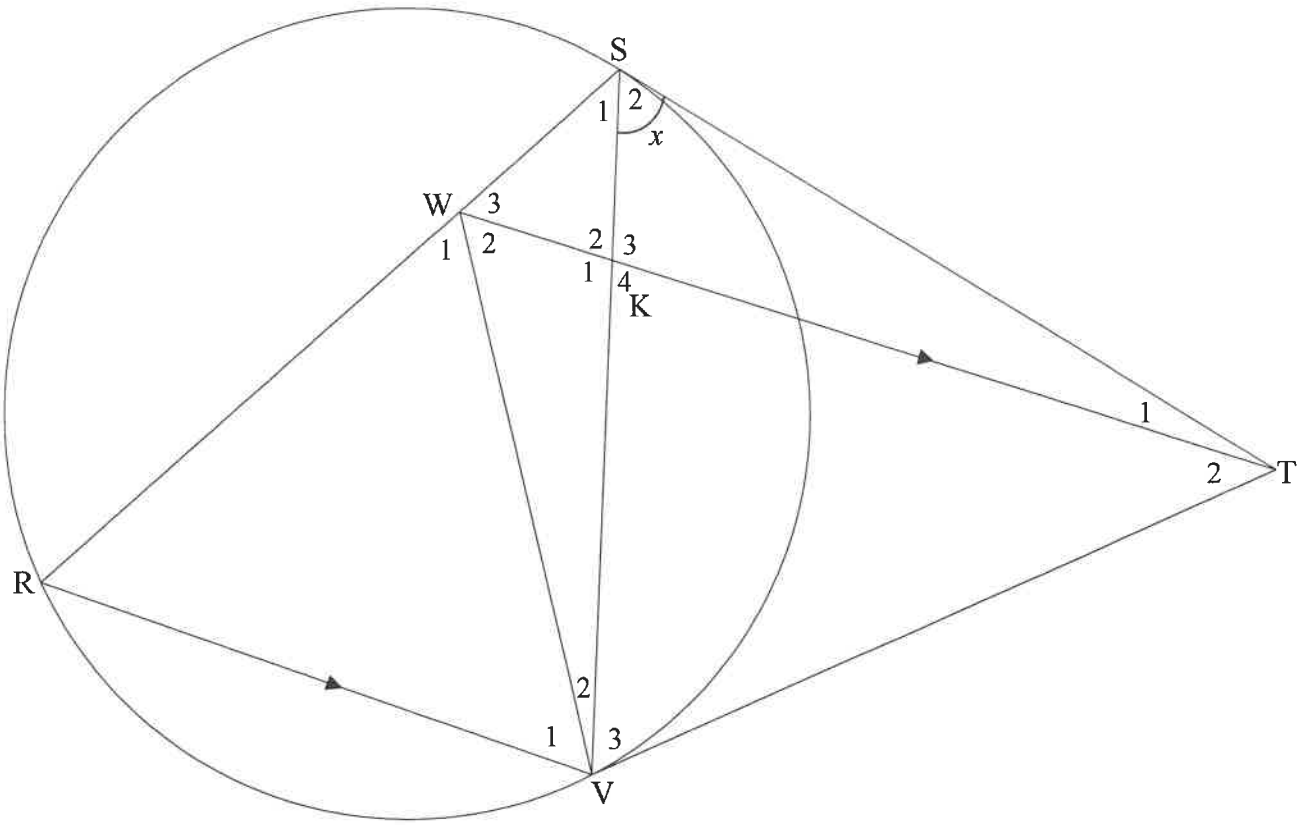
QUESTION 10

10.1 In the diagram, ΔABC is drawn. D is a point on AB and E is a point on AC such that DE || BC. BE and DC are drawn.



Use the diagram to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, in other words prove that $\frac{AD}{DB} = \frac{AE}{EC}$ (6)

10.2 In the diagram, ST and VT are tangents to the circle at S and V respectively. R is a point on the circle and W is a point on chord RS such that WT is parallel to RV. SV and WV are drawn. WT intersects SV at K. Let $\hat{S}_2 = x$.



10.2.1 Write down, with reasons, THREE other angles EACH equal to x. (6)

10.2.2 Prove, with reasons, that:

(a) WSTV is a cyclic quadrilateral (2)

(b) ΔWRV is isosceles (4)

(c) $\Delta WRV \,|||\, \Delta TSV$ (3)

(d) $\dfrac{RV}{SR} = \dfrac{KV}{TS}$ (4)

[25]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1};\ r \neq 1 \qquad S_\infty = \frac{a}{1-r};\ -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$\text{area } \Delta\text{ABC} = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})} \qquad \mathrm{P(A\ or\ B)} = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A\ and\ B)}$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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Candidate Barcode label/*Stafieskodeplakker*

NSC Answer Book
NSS Antwoordeboek

National Senior Certificate/*Nasionale Senior Sertifikaat* (Grade 12/*Graad 12*)

CENTRE NUMBER *SENTRUMNOMMER*	
EXAMINATION NUMBER *EKSAMENNOMMER*	
DATE *DATUM*	
BOOK NUMBER *BOEKNOMMER*	OF *VAN* BOOKS *BOEKE*
SUBJECT CODE *VAKKODE*	
PAPER NUMBER *VRAESTELNOMMER*	2
SUBJECT NAME *VAKNAAM*	**MATHEMATICS/*WISKUNDE***

MARKER/*NASIENER*			MODERATOR'S INITIALS IN RELEVANT BLOCK *MODERATOR SE VOORLETTERS IN RELEVANTE BLOKKIE*									
Question *Vraag*	Marks *Punte*	Marker's Code & Initials *Nasiener se kode & Voorletters*	Marks *Punte*	SM	Marks *Punte*	DCM *AHN*	Marks *Punte*	CM *HN*	Marks *Punte*	IM	Marks *Punte*	EM
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CONTROLLED AND CERTIFIED CORRECT (SURNAME AND INITIALS OF EA)
GEKONTROLEER EN AS KORREK GESERTIFISEER (VAN EN VOORLETTERS VAN EA)

READ INSTRUCTIONS ON THE NEXT PAGE.
LEES INSTRUKSIES OP VOLGENDE BLADSY.

This answer book consists of 24 pages./*Hierdie antwoordeboek bestaan uit 24 bladsye.*

FOLLOW THESE INSTRUCTIONS CAREFULLY	*VOLG HIERDIE INSTRUKSIES NOUKEURIG*
1. Clearly write your examination number and centre number in the space provided and attach your barcode label in the space provided.	*1. Skryf jou eksamennommer en sentrumnommer duidelik in die ruimtes verskaf en plak jou stafieskodeplakker in die ruimte verskaf.*
2. Remember that your own name (or the name of your school) may not appear anywhere on or in this answer book.	*2. Onthou dat jou eie naam (of die naam van jou skool) NIE op of in hierdie antwoordeboek mag voorkom NIE.*
3. Answer ALL questions in the spaces provided.	*3. Beantwoord ALLE vrae in die ruimtes wat verskaf is.*
4. No pages may be torn from this answer book.	*4. GEEN bladsye mag uit hierdie antwoordeboek geskeur word NIE.*
5. Read the instructions printed on your timetable carefully as well as any other instructions which may be given in each examination paper.	*5. Lees die instruksies, wat op jou eksamenrooster gedruk is, sorgvuldig deur, asook enige ander instruksies wat in elke vraestel gegee word.*
6. Candidates may not retain an answer book or remove it from the examination room.	*6. GEEN antwoordeboek mag deur die kandidaat behou of uit die eksamenlokaal verwyder word NIE.*
7. Answers must be written in black/blue ink as distinctly as possible. Do not write in the margins.	*7. Skryf die antwoorde so duidelik moontlik met swart/blou ink. Laat die kantlyne oop.*
8. Write the numbers of the questions you have answered on the front cover of the answer book where marks are to be recorded.	*8. Skryf die nommers van die vrae wat jy beantwoord het op die voorblad van die antwoordeboek waar die punte aangebring word.*
9. If you require additional space for your answers: 9.1 Use the additional space provided at the end of the answer book. 9.2 When answering a question in the additional space, indicate clearly the question number in the column on the LHS. 9.3 Rule off after each answer.	*9. In geval jy bykomende ruimte benodig vir jou antwoorde:* *9.1 Gebruik die bykomende ruimte wat aan die einde van die antwoordeboek verskaf word.* *9.2 As 'n vraag in die bykomende ruimte beantwoord word, dui duidelik die vraagnommer in die kolom aan die LK aan.* *9.3 Trek 'n lyn na elke antwoord.*
10. Draw a neat line through any work/rough work that must not be marked.	*10. Trek 'n netjiese lyn deur enige werk/rofwerk wat nie nagesien moet word nie.*

QUESTION/*VRAAG* 1

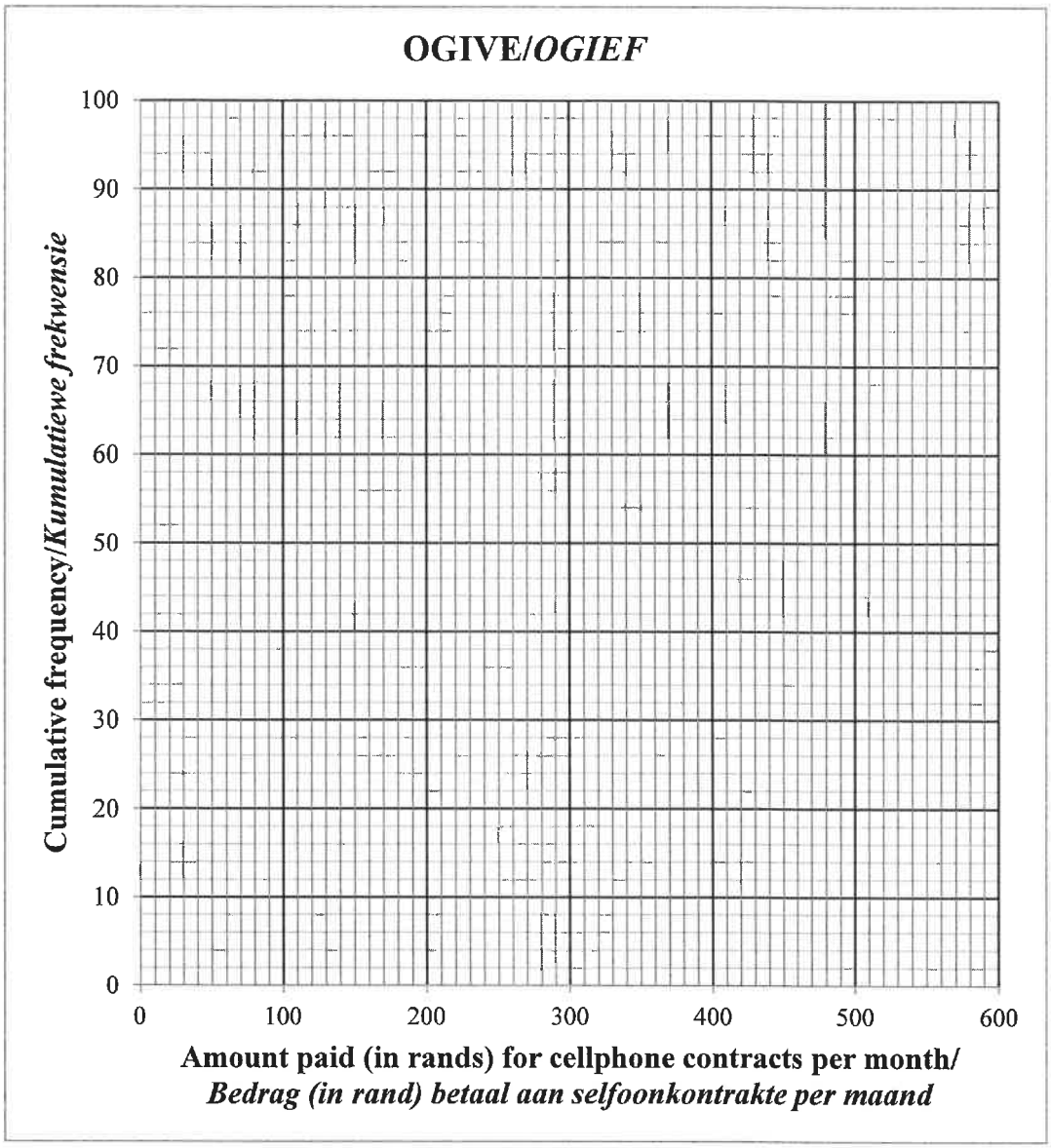
MONTHLY INCOME (IN RANDS) MAANDELIKSE INKOMSTE (IN RAND)	9 000	13 500	15 000	16 500	17 000	20 000
MONTHLY REPAYMENT (IN RANDS) MAANDELIKSE PAAIEMENT (IN RAND)	2 000	3 000	3 500	5 200	5 500	6 000

	Solution/*Oplossing*	**Marks *Punte***
1.1		(3)
1.2		(2)
1.3		(1)
1. 4		(2)
		[8]

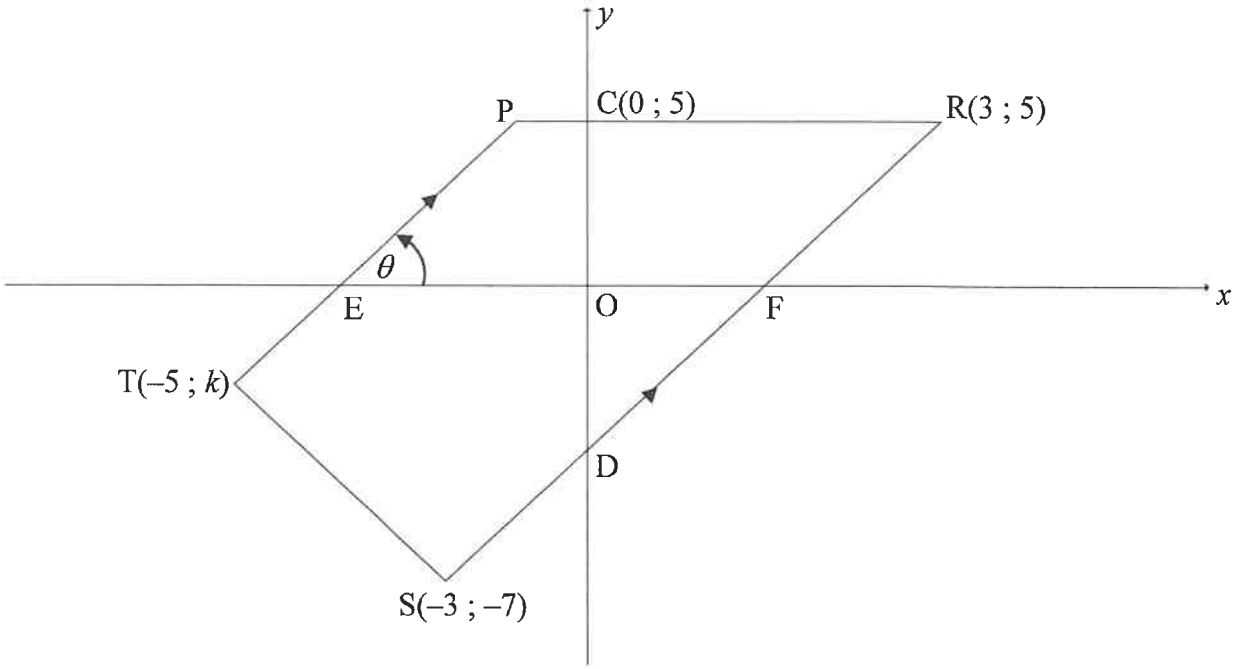
QUESTION/*VRAAG* 2

AMOUNT PAID (IN RANDS) BEDRAG BETAAL (IN RAND)	FREQUENCY FREKWENSIE
$0 < x \leq 100$	7
$100 < x \leq 200$	12
$200 < x \leq 300$	a
$300 < x \leq 400$	35
$400 < x \leq 500$	b
$500 < x \leq 600$	6

	Solution/*Oplossing*	**Marks** ***Punte***
2.1		(1)
2.2		(5)
2.3		(1)

	Solution/*Oplossing*	Marks *Punte*
2.4		(4)
2.5		(2)
		[13]

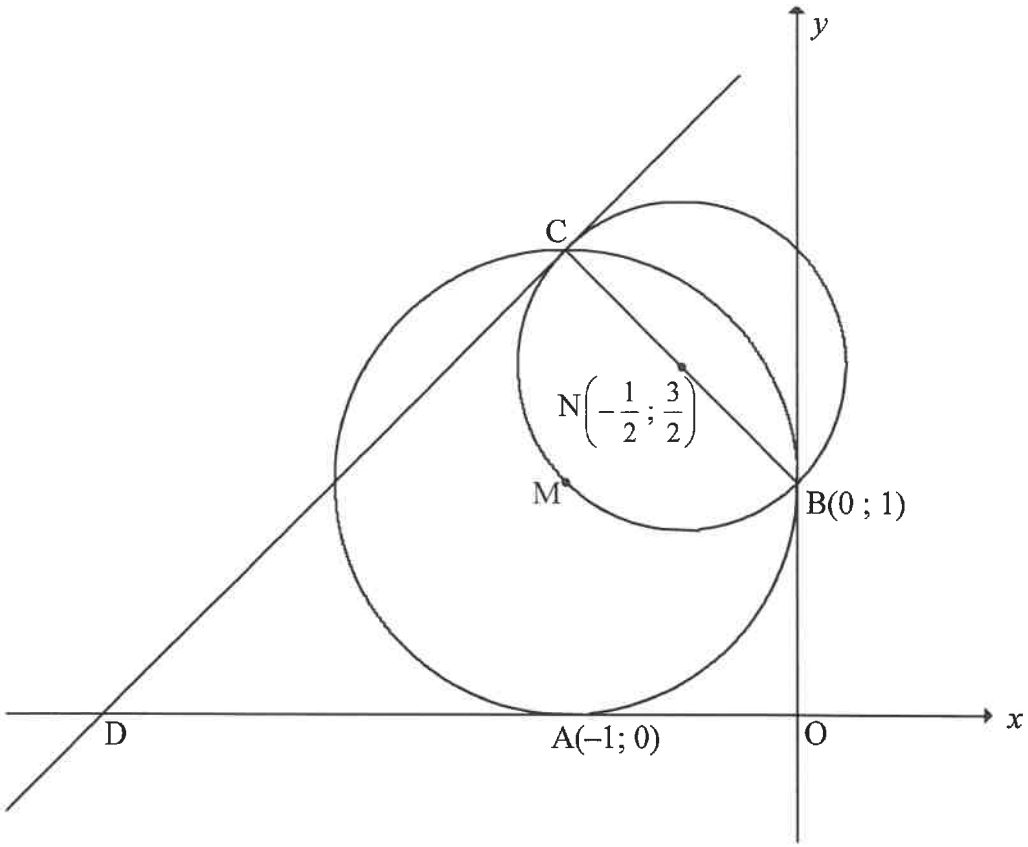
QUESTION/*VRAAG* 3



	Solution/*Oplossing*	**Marks** ***Punte***
3.1		(1)
3.2.1		(2)
3.2.2		(3)

	Solution/*Oplossing*	**Marks** ***Punte***
3.2.3		(3)
3.3		(4)
3.4		(3)
3.5		(3)
		[19]

QUESTION/*VRAAG* 4



	Solution/*Oplossing*	**Marks** ***Punte***
4.1		(3)
4.2		(2)

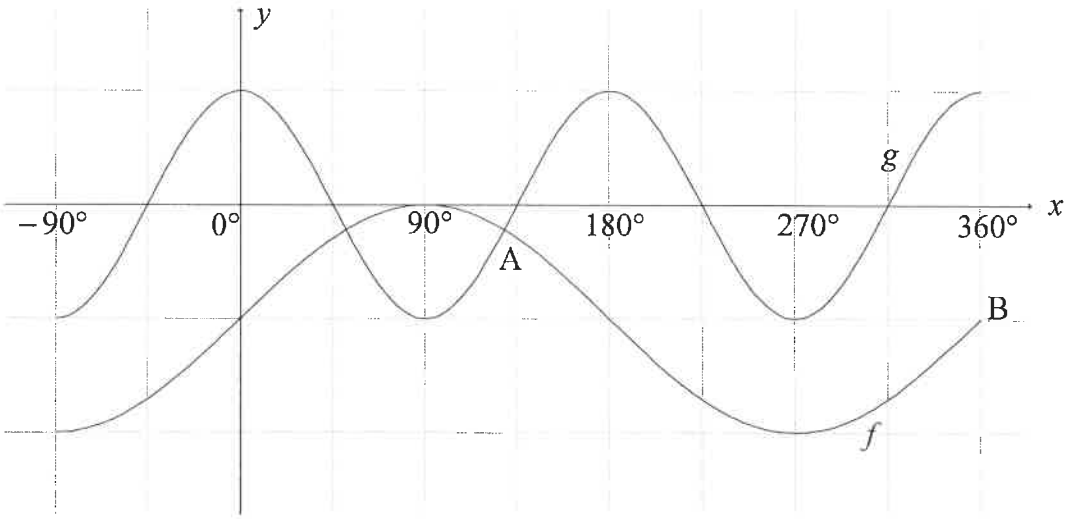
	Solution/*Oplossing*	**Marks** ***Punte***
4.3		(4)
4.4		(3)
4.5		(3)
4.6		(5)
		[20]

QUESTION/*VRAAG* 5

	Solution/*Oplossing*	**Marks** ***Punte***
5.1		(5)
5.2		(4)
5.3		(4)

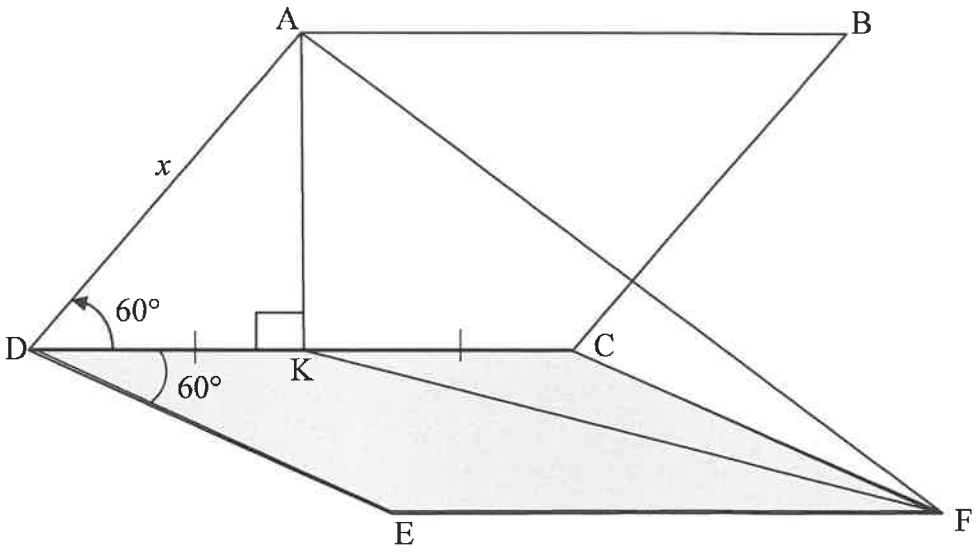
	Solution/*Oplossing*	**Marks** ***Punte***
5.4.1		(6)
5.4.2		(3)
		[22]

QUESTION/*VRAAG* 6



	Solution/*Oplossing*	Marks *Punte*
6.1		(2)
6.2		(2)
6.3		(6)
		[10]

QUESTION/*VRAAG* 7

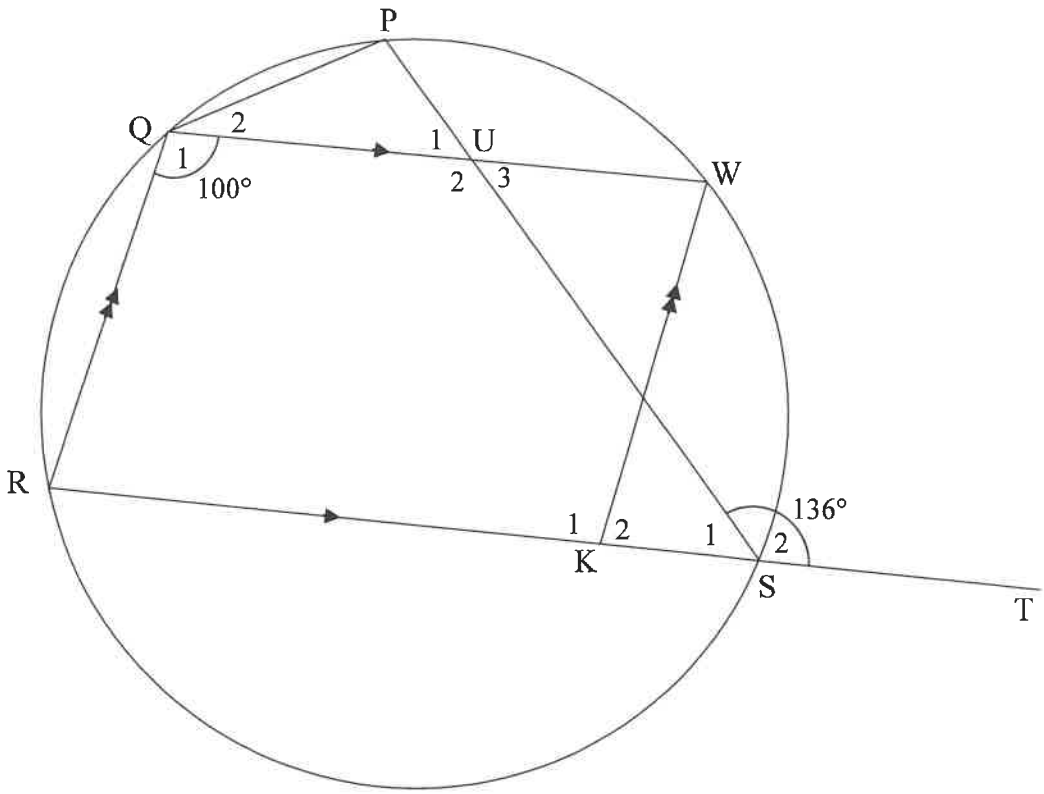


	Solution/*Oplossing*	Marks *Punte*
7.1		(2)
7.2		(1)
7.3		(7)
		[10]

Give reasons for your statements in QUESTIONS 8, 9 and 10.
Gee redes vir jou bewerings in VRAAG 8, 9 en 10.

QUESTION/*VRAAG* 8

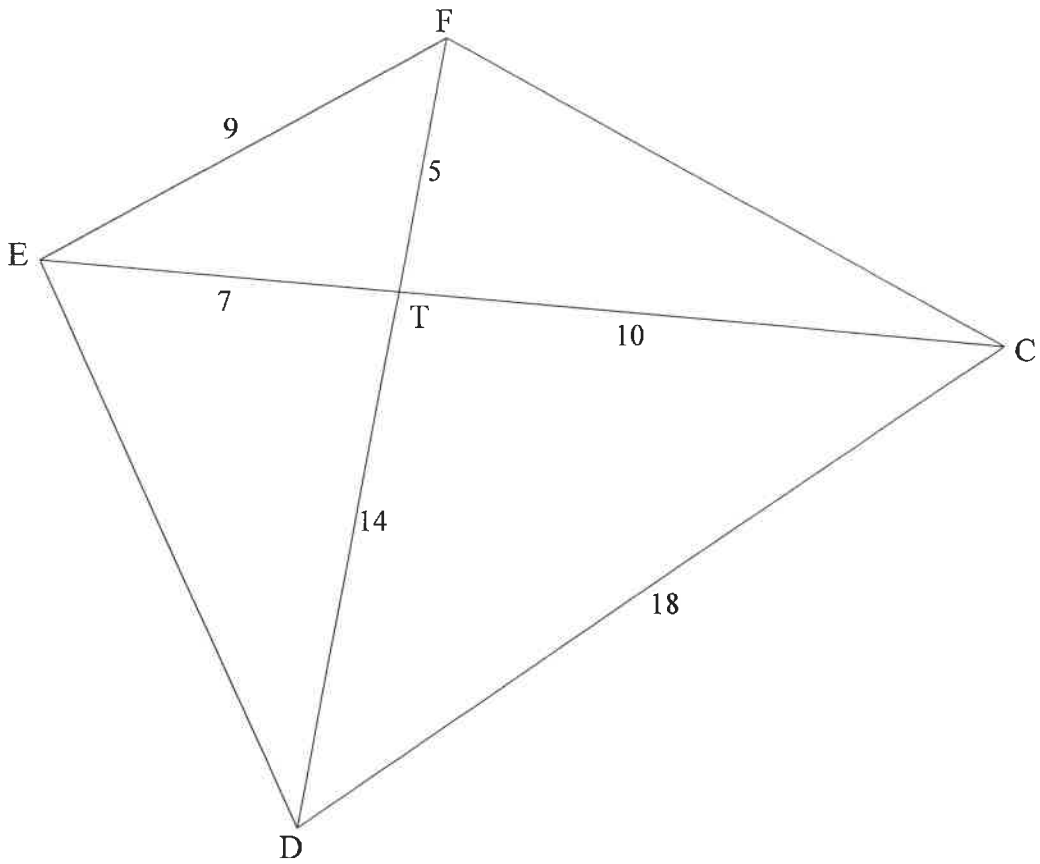
8.1



	Solution/*Oplossing*	**Marks** ***Punte***
8.1.1		(2)
8.1.2		(2)

	Solution/*Oplossing*	**Marks** ***Punte***
8.1.3		(3)
8.1.4		(2)

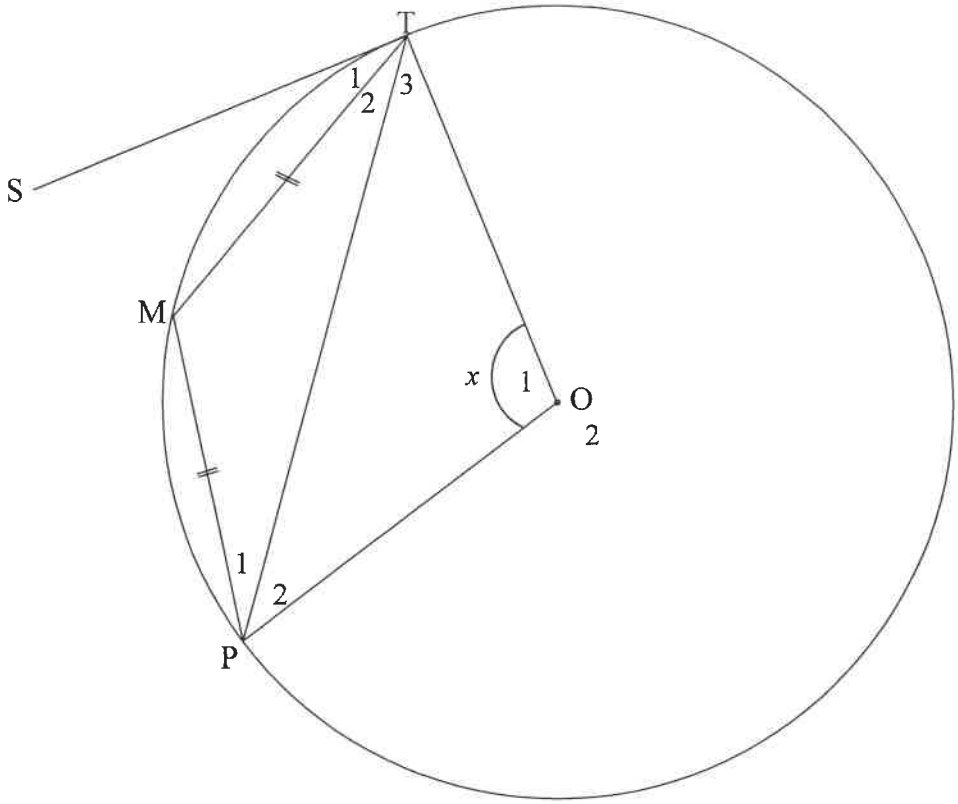
8.2



	Solution/*Oplossing*	**Marks** ***Punte***
8.2.1		(4)

	Solution/*Oplossing*	**Marks** ***Punte***
8.2.2		(3)
		[16]

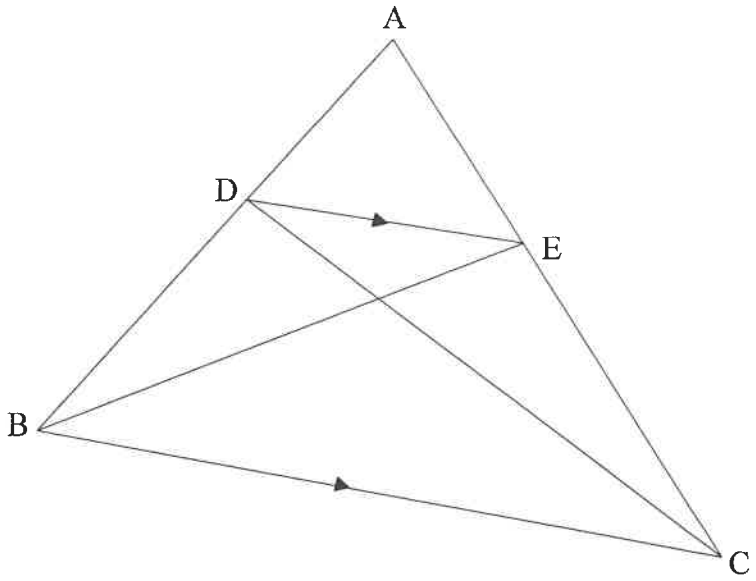
QUESTION/*VRAAG* 9



	Solution/*Oplossing*	**Marks** ***Punte***
		[7]

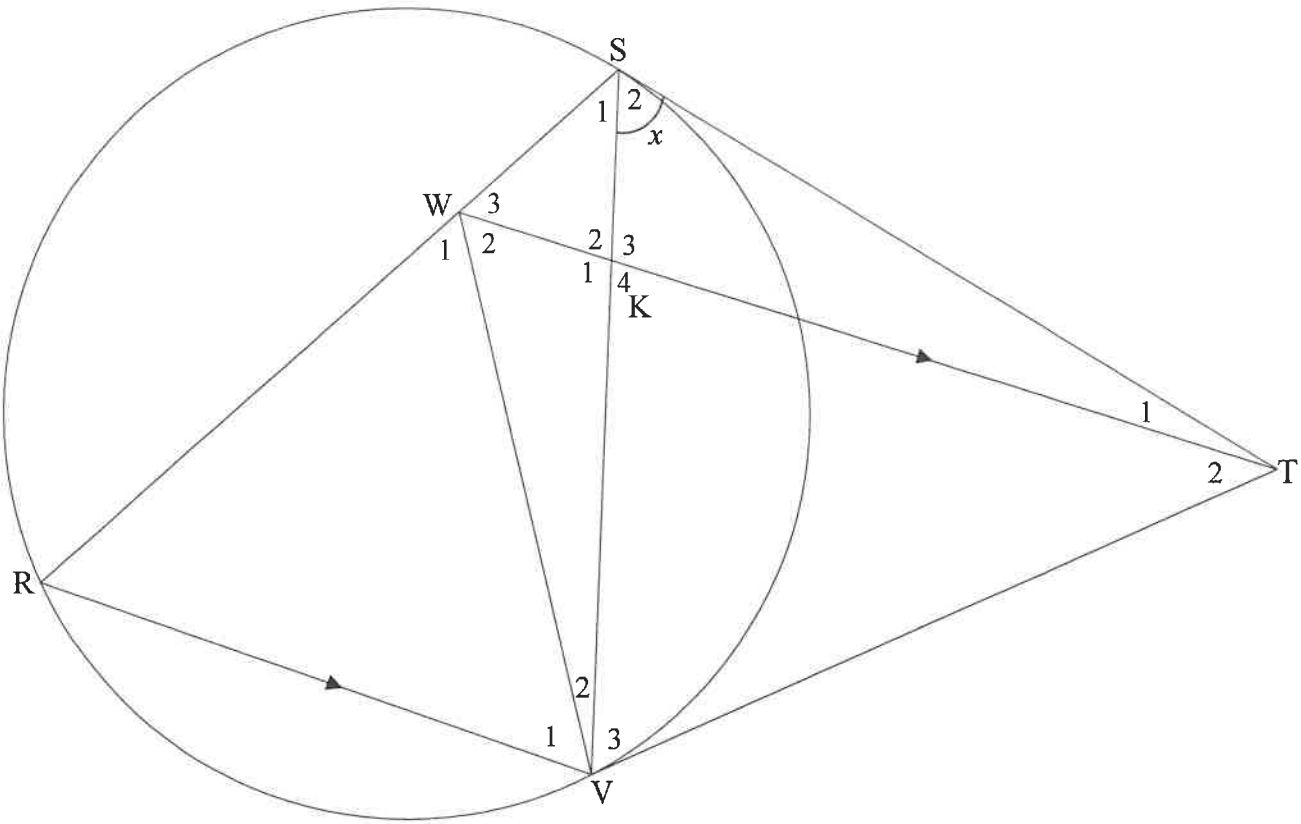
QUESTION/*VRAAG* 10

10.1



	Solution/*Oplossing*	**Marks** ***Punte***
		(6)

10.2



	Solution/*Oplossing*	**Marks** ***Punte***
10.2.1		(6)
10.2.2(a)		(2)

10.2.2(b)		(4)
10.2.2(c)		(3)
10.2.2(d)		(4)
		[25]

	Additional space/*Bykomende ruimte*	**Marks** ***Punte***

	Additional space/*Bykomende ruimte*	**Marks** ***Punte***

TOTAL/*TOTAAL*: 150

RE-MARK/RE-CHECK *HERMERK/HERSIEN*				
Question *Vraag*	**Marks** *Punte*			**Initials** *Voorletters*
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
TOTAL *TOTAAL*				
HASH TOTAL *KAF-TOTAAL*				

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2019

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 26pages.
Hierdie nasienriglyne bestaan uit 26 bladsye.

NOTE:
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:
- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

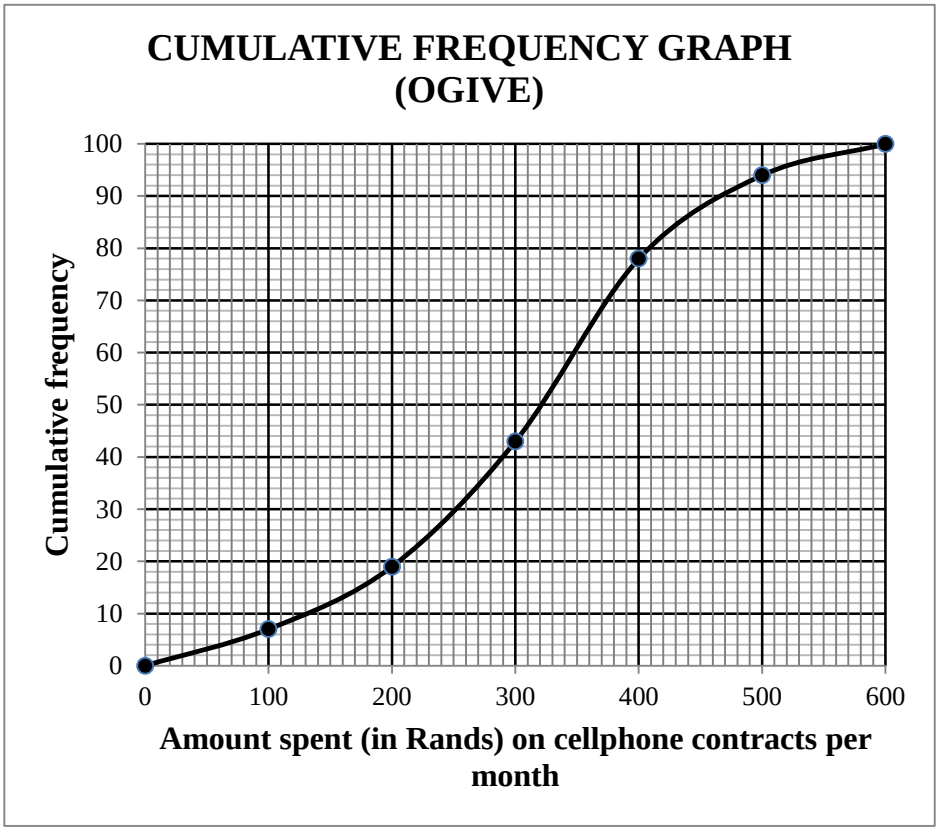
QUESTION/*VRAAG* 1

Monthly income (in rands) **Maandelikse inkomste (in rand)**	9 000	13 500	15 000	16 500	17 000	20 000
Monthly repayment (in rands) **Maandelikse paaiement (in rand)**	2 000	3 000	3 500	5 200	5 500	6 000

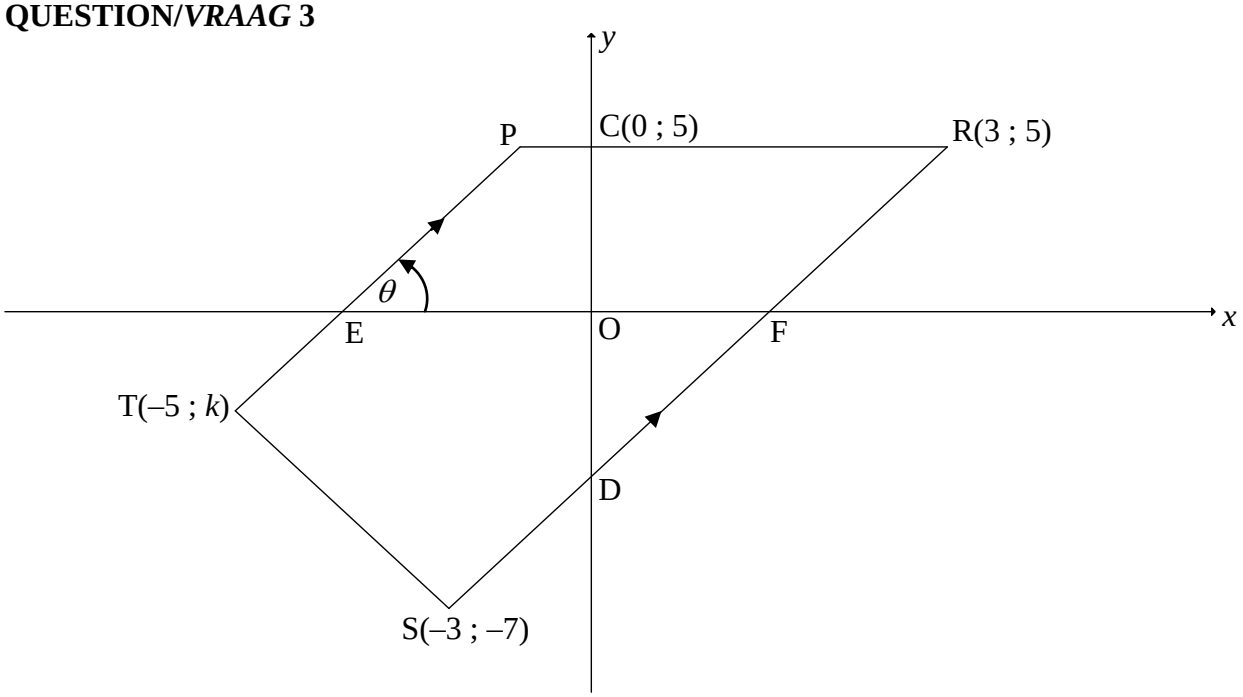
1.1	$a = -1946{,}875... = -1946{,}88$ $b = 0{,}41$ $\hat{y} = -1946{,}88 + 0{,}41x$ Answer only: Full marks	✓ $a = -1946{,}88$ ✓ $b = 0{,}41$ ✓ equation (3)
1.2	Monthly repayment $\approx$ R3 727,16 (calculator) *Maandelikse paaiement* $\approx$ R3 727,16 **OR** $\hat{y} = -1946{,}88 + 0{,}41(14000)$ $\approx$ R3 793,12	✓✓ answer (2) ✓ substitution ✓ answer (2)
1.3	r = 0,946 …. $\approx$ 0,95	✓ answer (1)
1.4	Not to spend R9 000 per month because the point (18 000 ; 9 000) lies very far from the least squares regression line. **OR** D *Spandeer nie R9 000 per maand nie, want die punt (18 000 ;9 000) lê baie ver van die kleinste-kwadrate regressielyn.* ***OF*** *D*	✓✓ answer (2)
		[8]

QUESTION/*VRAAG* 2

2.1	Number people paid R200 or less = 19 *Aantal mense wat R200 of minder betaal het = 19*	✓ answer (1)
2.2	$7+12+a+35+b+6=100$ $a=40-b$ $309=\frac{(50\times7)+(150\times12)+(250\times a)+(350\times35)+(450\times b)+(550\times6)}{100}$ $309=\frac{(50\times7)+(150\times12)+(250\times(40-b))+(350\times35)+(450\times b)+(550\times6)}{100}$ 350 + 1800 + 10000 – 250*b* + 12250 + 450*b* +3300 = 30900 200*b* = 3200 *b* = 16 *a* = 24 **OR/*OF*** $7+12+a+35+b+6=100$ $b=40-a$ $309=\frac{(50\times7)+(150\times12)+(250\times a)+(350\times35)+(450\times b)+(550\times6)}{100}$ $309=\frac{(50\times7)+(150\times12)+(250\times a)+(350\times35)+(450\times(40-a))+(550\times6)}{100}$ 350 + 1800 + 250*a* + 12250 + 1800 – 450a = 30900 200*a* = 4 800 *a* = 24 *b* = 16	✓ $\sum x=100$ ✓ $a=40-b$ ✓ $\sum fX$ ✓ $\sum\frac{fX}{n}=309$ ✓ 200*b* = 3200 (5) ✓ $\sum x=100$ ✓ $b=40-a$ ✓ $\sum fX$ ✓ $\sum\frac{fX}{n}=309$ ✓ 200*a* = 4 800 (5)
2.3	Modal class/*modale klas:* $300<x\leq400$	✓ answer (1)

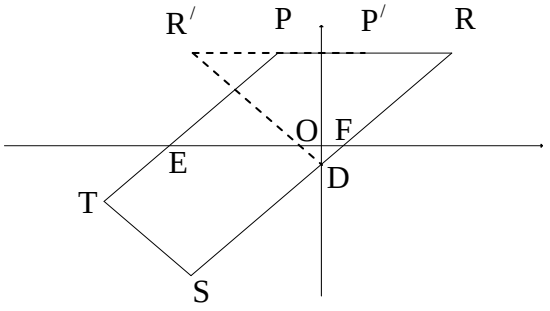
2.4		✓ grounded at (0 ; 0) ✓ (600 ; 100) ✓ cumulative frequencies for *y*-coordinates ✓ smooth shape (4)
2.5	Number of people/*Aantal mense* = 100 – 82 [accept 80 – 84 people] 18 people paid more than R420 per month/. [accept 16 – 20 people] *18 mense betaal meer as R420 per maand* Answer only: Full marks	✓ 82 ✓ answer (2)
		[13]

QUESTION/*VRAAG* 3

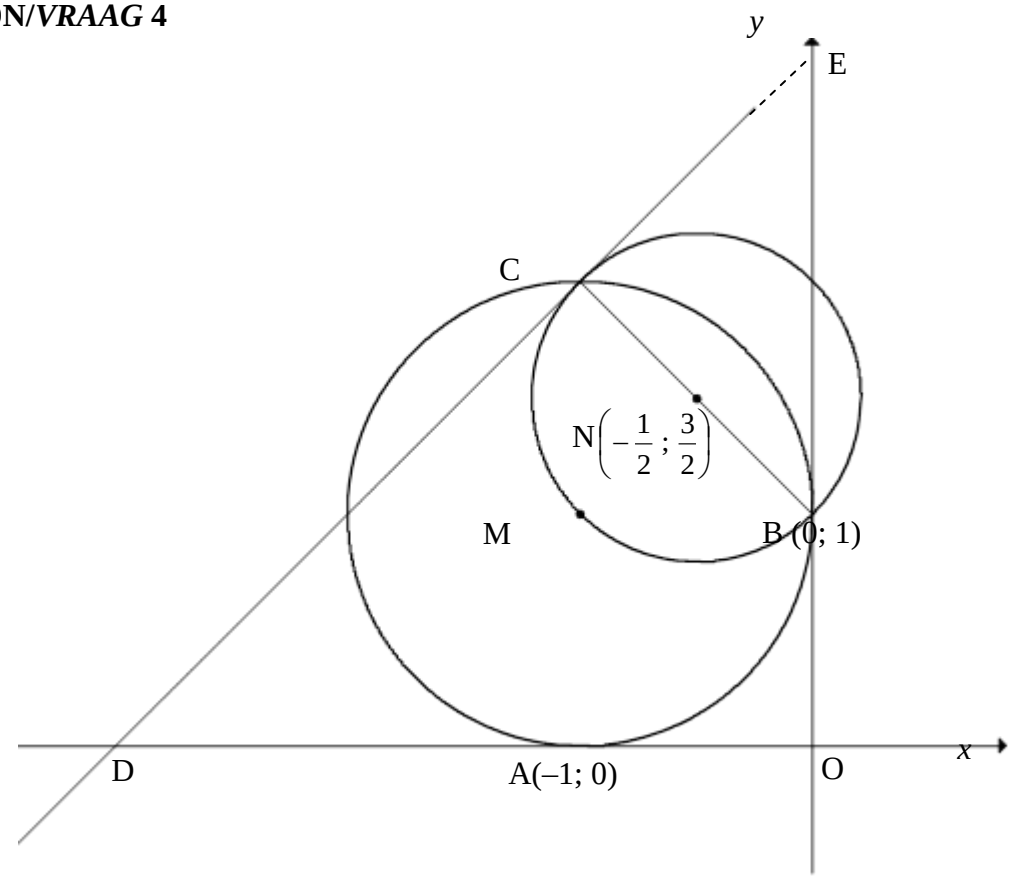


3.1	Equation of PR: $y = 5$	✓ answer (1)
3.2.1	$m_{RS} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{RS} = \frac{5-(-7)}{3-(-3)} = \frac{12}{6}$ $= 2$ Answer only: Full marks	✓ substitution of R & S into gradient formula ✓ answer (2)
3.2.2	$m_{RS} = m_{PT}$ [PT ‖ RS] $\tan\theta = 2$ $\theta = 63{,}43°$	✓ $m_{RS} = m_{PT}$ ✓ $\tan\theta = 2$ ✓ $\theta = 63{,}43°$ (3)
3.2.3	Equation of RS: $y-5 = 2(x-3)$ or $y-(-7) = 2(x-(-3))$ or $5 = 2(3)+c$ $y-5 = 2x-6$ $y+7 = 2x+6$ $c = -1$ $y = 2x-1$ $y = 2x-1$ $y = 2x-1$ $\therefore$ D(0 ; –1) **OR/OF** $m_{RS} = m_{RD} = m_{DS}$ $2 = \frac{5-y}{3-0} = \frac{y+7}{0-(-3)}$ $\therefore y = -1$ $\therefore$ D(0 ; –1) Answer only: Full marks	✓ substitution ✓ equation of RS ✓ coordinates of D (3) ✓ equating gradients ✓ value of y ✓ coordinates of D (3)

3.3	$ST = 2\sqrt{5} = \sqrt{[-5-(-3)]^2 + (k-(-7))^2}$ $20 = 4 + (k+7)^2$ $(k+7)^2 = 16$ $k+7 = \pm 4$ $k = -11$ *or* $k = -3$ $\therefore k = -3$ **OR** $ST = 2\sqrt{5} = \sqrt{[-5-(-3)]^2 + (k-(-7))^2}$ $20 = 4 + k^2 + 14k + 49$ $k^2 + 14k + 33 = 0$ $(k+11)(k+3) = 0$ $k = -11$ or $k = -3$ $\therefore k = -3$	✓ substitute S and T into distance formula ✓ isolate square ✓ square root both sides ✓ answer (4) ✓ substitute S and T into distance formula ✓ standard form ✓ factors ✓ answer (4)
3.4	Method: translation T→S: $(x\,;y) \rightarrow (x+2\,;y-4)$ $\therefore$ by symmetry: D→N: D(0 ; –1) → N(0 + 2 ; –1 – 4) $\therefore$ N(2 ; –5) Answer only: Full marks **OR** Midpoint of TN = Midpoint of SD $\frac{x+(-5)}{2} = \frac{-3+0}{2}$ and $\frac{y+(-3)}{2} = -\frac{-7+(-1)}{2}$ $x = 2$ and $y = -5$ $\therefore$ N(2 ; – 5) Answer only: Full marks	✓ method ✓ *x*-coordinate ✓ *y*-coordinate (3) ✓ method: midpoint of diagonals ✓ *x*-coordinate ✓ *y*-coordinate (3)

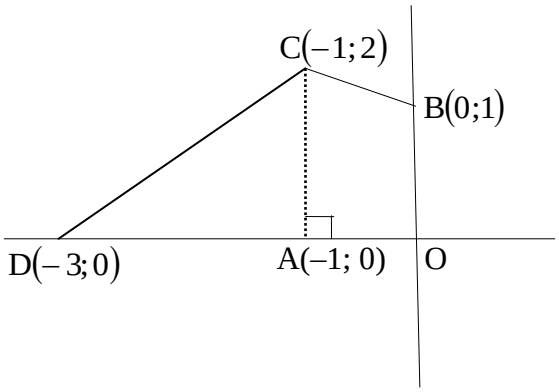
3.5		
	β is the inclination of RS $\quad\therefore\ \beta = 63{,}434...°$ $\hat{OFD} = 63{,}434...°$ [vert opp ∠s] $\hat{ODF} = 90° - 63{,}434...° = 26{,}565...°$ $R\hat{D}R' = 2(26{,}565...°) = 53{,}13°$	✓ $\beta = 63{,}43°$ ✓ $\hat{ODF} = 26{,}57°$ ✓ answer (3)
	OR PEFR is a ‖m [both pairs of opp sides ‖] $\therefore \hat{R} = \theta = 63{,}434...°$ [opp ∠s of ‖m] $R\hat{R}'D = 63{,}434...°$ [∠s opp = sides:RD = R′D] $R\hat{D}R' = 180° - (63{,}43° + 63{,}43°)$ [sum of ∠s in Δ] $R\hat{D}R' = 53{,}13°$	✓ $\hat{R} = 63{,}43°$ ✓ $R\hat{R}'D = 63{,}43°$ ✓ answer (3)
	OR $\tan \hat{ODF} = \frac{3}{6}$ $\hat{ODF} = 26{,}565..°$ $R\hat{D}R' = 2(26{,}565...°) = 53{,}13°$	✓ trig ratio ✓ $\hat{ODF} = 26{,}565..°$ ✓ answer (3)
	OR $R'(-3;5)$ [reflection of R(3 ; 5) about the *y*-axis] $RD = \sqrt{(3-0)^2 + (5-(-1)^2}$ $RD = \sqrt{45}$ = R′/D or $3\sqrt{5}$ or 6,71 $(RR')^2 = (\sqrt{45})^2 + (\sqrt{45})^2 - 2(\sqrt{45})(\sqrt{45})(\cos R\hat{D}R')$ $6^2 = 45 + 45 - 2(45)(\cos R\hat{D}R')$ $\cos R\hat{D}R' = \frac{45+45-36}{2(45)}$ $\cos R\hat{D}R' = \frac{3}{5}$ $\therefore\ R\hat{D}R' = 53{,}13°$	✓ $R'(-3;5)$ **OR** $RD = \sqrt{45}$ = R′/D ✓ substitution into cosine rule ✓ answer (3)
		[19]

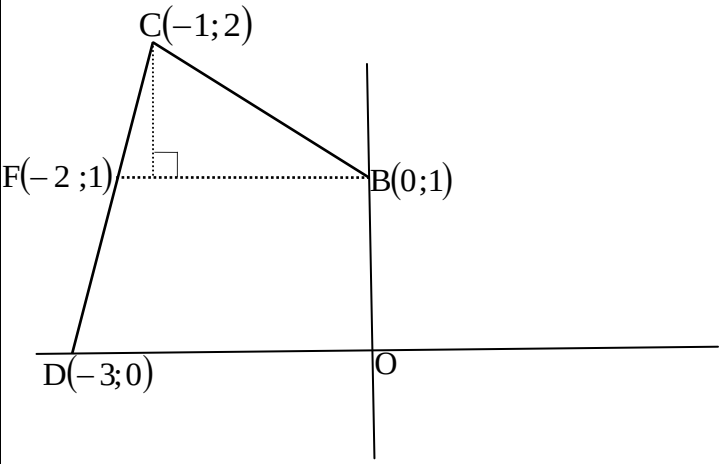
QUESTION/*VRAAG* 4



4.1	M(–1;1) $(x+1)^2+(y-1)^2=1$ Answer only: Full marks	✓ M(–1;1) ✓LHS ✓ RHS (3)
4.2	Midpoint of CB, N: (– 0,5 ; 1,5) $\therefore \frac{x_C+0}{2}=-\frac{1}{2}$ and $\frac{y_C+1}{2}=\frac{3}{2}$ Answer only: Full marks $\therefore$ C(– 1 ; 2) **OR** B→N: $(x;y)\rightarrow(x-0{,}5;y+0{,}5)$ N→C: Answer only: Full marks $(x;y)\rightarrow(x-0{,}5;y+0{,}5)$ $\therefore$ C(– 0,5 – 0,5 ; 1,5 + 0,5) $\therefore$ C(– 1 ; 2)	✓*x* value ✓ *y* value (2) ✓*x* value ✓ *y* value (2)

4.3	$m_{\text{radius}} = \frac{2-1}{-1-0}$ OR $\frac{2-(-\frac{1}{2})}{-1-\frac{3}{2}}$ OR $\frac{0-(-\frac{1}{2})}{1-\frac{3}{2}}$ $= -1$ $\therefore m_{\text{tangent}} = 1$ $y = mx + c$ $y = x + c$ $2 = 1(-1) + c$ $c = 3$ $\therefore y = x + 3$ $y - x = 3$ **OR** $m_{\text{radius}} = \frac{2-1}{-1-0}$ $= -1$ $\therefore m_{\text{tangent}} = 1$ $y - y_1 = m(x - x_1)$ $y - y_1 = 1(x - x_1)$ $y - 2 = 1(x - (-1))$ $y - 2 = x + 1$ $\therefore y = x + 3$ $y - x = 3$	✓ m_{radius} ✓ m_{tangent} ✓ substitute (– 1 ; 2) and *m* ✓ simplification (4) ✓ m_{radius} ✓ m_{tangent} ✓ substitute (– 1 ; 2) and *m* ✓ simplification (4)
4.4	Tangents to circle: $y = x + 3$ and $y = x + 1$ $\therefore t > 3$ or $t < 1$ Answers only: Full marks	✓ $y = x + 1$ ✓ $t > 3$ ✓ $t < 1$ (3)
4.5	Draw rectangle CNED: Midpt of DN $\left(-\frac{7}{4}; \frac{3}{4}\right)$ $\therefore$ E$(-\frac{5}{2}; -\frac{1}{2})$ C(–1 ; 2) N$\left(-\frac{1}{2}; \frac{3}{2}\right)$ D(–3 ; 0) E **OR/*OF*** D(– 3 ; 0) C→N: $(x; y) \rightarrow (x + 0{,}5; y - 0{,}5)$ D→E: D $(x; y) \rightarrow$ E$(x + 0{,}5; y - 0{,}5)$ $\therefore$ E(– 3 + 0,5 ; 0 – 0,5) $\therefore$ E(– 2,5 ; – 0,5) Answer only: Full marks	✓ midpt of DN ✓*x* value ✓ *y* value (3) ✓ coordinates of D ✓*x* value ✓ *y* value (3)

4.6	 area of trapezium AOBC = $\frac{1}{2}(1+2)(1)$ = $1\frac{1}{2}$ square units area of ΔACD = $\frac{1}{2}(2)(2)$ = 2 square units area of quadrilateral OBCD = $3\frac{1}{2}$ square units $\therefore 2a^2 = \frac{7}{2}$ $a^2 = \frac{7}{4}$ $a = \frac{\sqrt{7}}{2}$ **OR**	✓ substitution into area of trapezium form ✓ area of trapezium ✓ area of triangle ✓ area of OBCD ✓ equating area OBCD to $2a^2$ (5)



BM produced cuts the tangent at F.

area of $\Delta CFB = \frac{1}{2}(2)(1)$

$= 1$ square unit

area of trapezium BFDO $= \frac{1}{2}(2+3)(1)$

$= 2\frac{1}{2}$ square units

area of quadrilateral OBCD $= 3\frac{1}{2}$ square units

$\therefore 2a^2 = \frac{7}{2}$

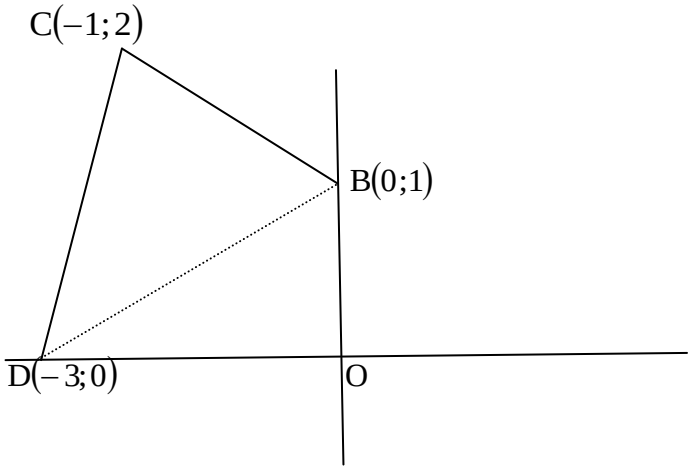
$a^2 = \frac{7}{4}$

$a = \frac{\sqrt{7}}{2}$

- ✓ area of triangle
- ✓ substitution into area of trapezium
- ✓ area of trapezium
- ✓ area of OBCD
- ✓ equating area OBCD to $2a^2$

(5)

OR



Join DB

area of $\Delta ODB = \frac{1}{2}(3)(1)$

$= \frac{3}{2}$ square unit ✓ area of Δ

area of $\Delta DCB = \frac{1}{2}\left(2\sqrt{2}\right)\left(\sqrt{2}\right)$ ✓ subst into area of Δ

= 2 square unit ✓ area of Δ

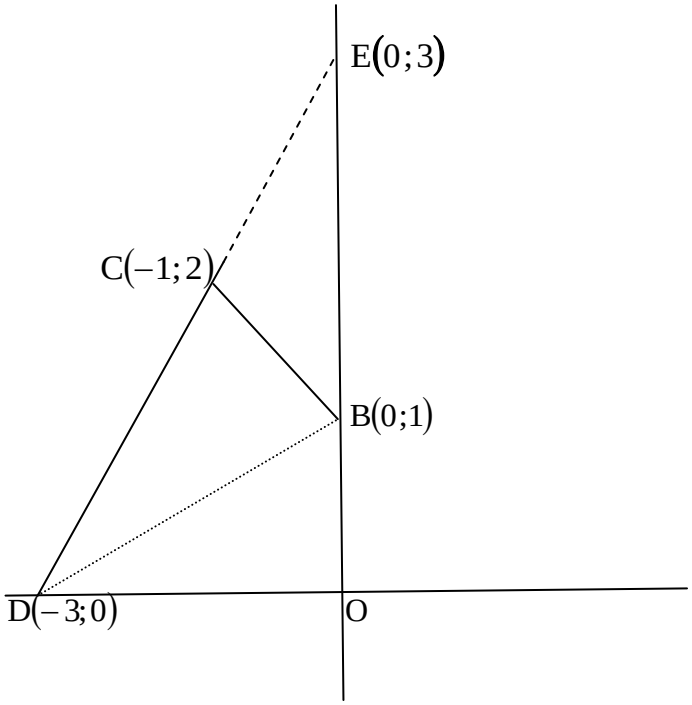
$\therefore$ area of OBCD = $\frac{3}{2} + 2$ = square units ✓ area of OBCD

$2a^2 = \frac{7}{2}$ ✓ equating area OBCD to $2a^2$

$a^2 = \frac{7}{4}$

$a = \frac{\sqrt{7}}{2}$ (5)

OR



Let E be the point of intersection of DC with the positive *y*–axis.

area of $\Delta DEO = \frac{1}{2}(3)(3)$

$= \frac{9}{2}$ square unit

area of $\Delta ECB = \frac{1}{2}(2)(1)$ or $\frac{1}{2}(\sqrt{2})(\sqrt{2})$

= 1 square unit

area of quadrilateral OBCD = $\frac{9}{2} - 1 = 3\frac{1}{2}$ square units

$\therefore 2a^2 = \frac{7}{2}$

$a^2 = \frac{7}{4}$

$a = \frac{\sqrt{7}}{2}$

✓ area of Δ

✓ subst into area of Δ

✓ area of Δ

✓ area of OBCD

✓ equating area OBCD to $2a^2$

(5)

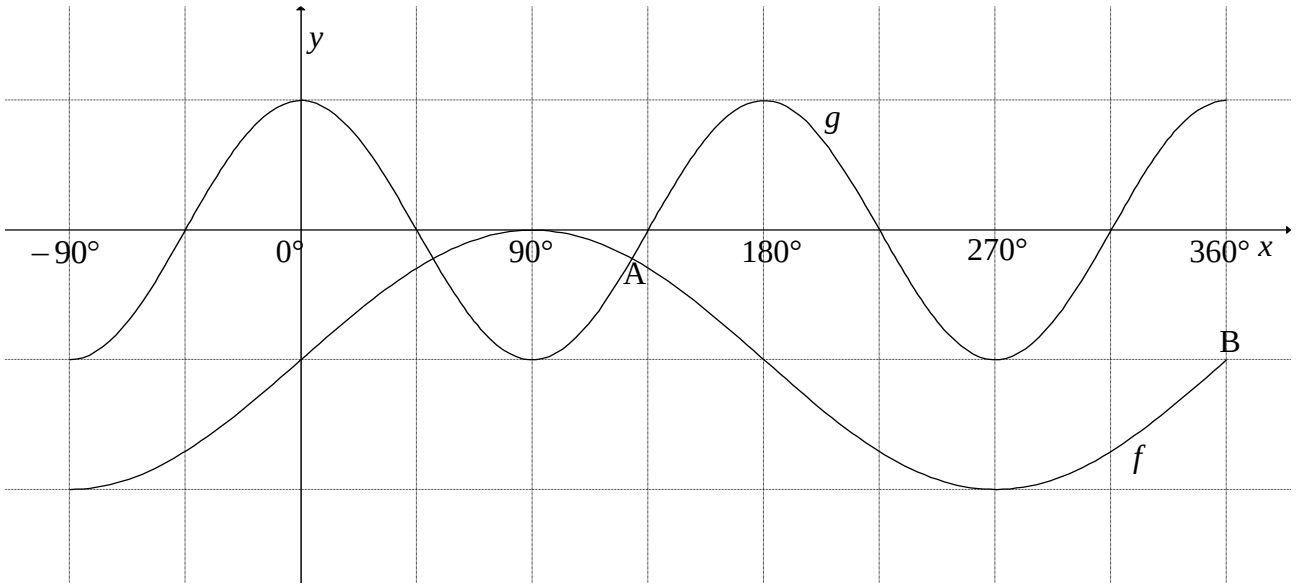
[20]

QUESTION/*VRAAG* 5

5.1	$\frac{\sin x}{\cos x.\tan x}+\sin(180°+x)\cos(90°-x)$ $=\frac{\sin x}{\cos x.\frac{\sin x}{\cos x}}+(-\sin x)\sin x$ $=1-\sin^2 x$ $=\cos^2 x$	✓ $-\sin x$ ✓ $\sin x$ ✓ $\tan x=\frac{\sin x}{\cos x}$ ✓ $1-\sin^2 x$ ✓ $\cos^2 x$ (5)
5.2	$\frac{\sin^2 35°-\cos^2 35°}{4\sin 10°\cos 10°}$ $=\frac{-\left(\cos^2 35°-\sin^2 35°\right)}{2(2\sin 10°\cos 10°)}$ $=\frac{-\cos 70°}{2\sin 20°}$ $=\frac{-\cos 70°}{2\cos 70°}$ **OR** $=\frac{-\sin 20°}{2\sin 20°}=-\frac{1}{2}$	✓ $-\left(\cos^2 35°-\sin^2 35°\right)$ ✓ $-\cos 70°$ ✓ $2\sin 20°$ ✓ answer (4)
5.3	$2\sin^2 77°=2\left[\sin(90°-13°)\right]^2$ $=2\cos^2 13°$ $=2\cos^2 13°-1+1$ $=\cos 26°+1$ $=m+1$ **OR** $1-2\sin^2 77°=\cos 154°$ $2\sin^2 77°=1-\cos 154°$ $=1-(-\cos 26°)$ $=1+m$	✓ using co-ratio ✓ reduction ✓ $2\cos^2 13°-1=\cos 26°$ ✓ answer (4) ✓ $1-2\sin^2 77°=\cos 154°$ ✓ $2\sin^2 77°=1-\cos 154°$ ✓ reduction ✓ answer (4)
5.4.1	$\sin(x+25°)\cos 15°-\cos(x+25°)\sin 15°=\tan 165°$ $\sin(x+25°-15°)=-0,2679...\ OR\ -2+\sqrt{3}$ $\sin(x+10°)=-0,2679...OR\ -2+\sqrt{3}$ $x+10°=195,54°+k.360°$ or $x+10°=344,46°+k.360°$ $x=185,54°+k.360°;\ k\in Z$ or $x=334,46°+k.360°;k\in Z$ **OR/*OF***	✓✓ $\sin(x+10°)$ ✓ $-0,2679...$ ✓ 195,54° & 344,46° ✓ 185,54° & 334,46° ✓ $+k.360°;\ k\in Z$ (6)

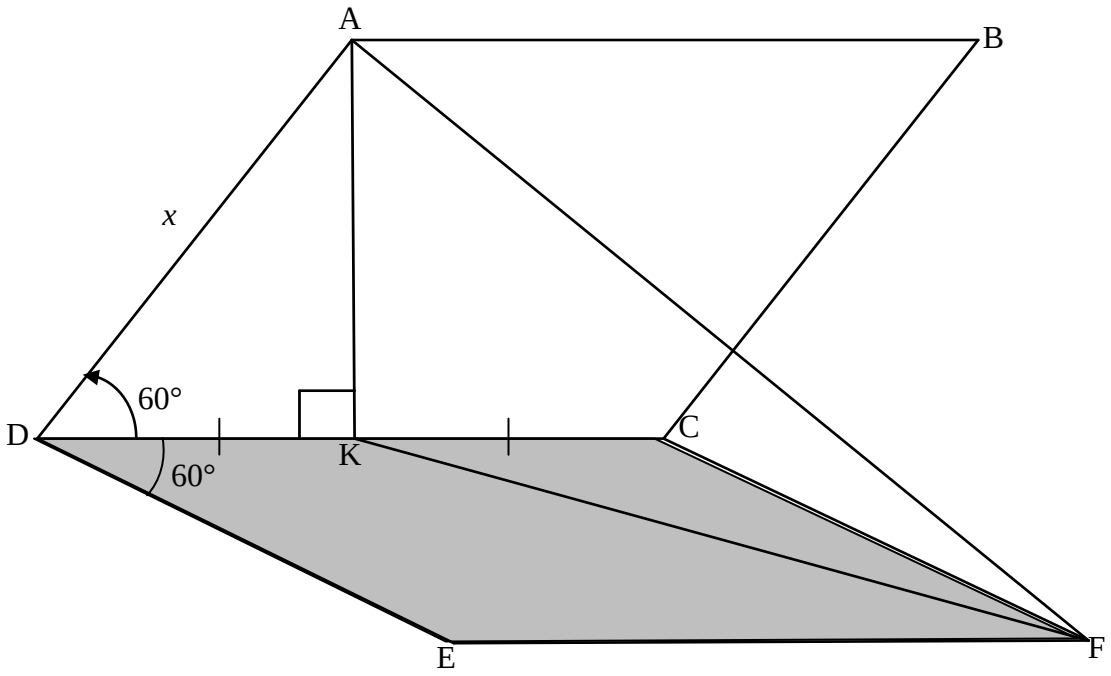
	$\sin(x+25°)\sin 75° - \cos(x+25°)\cos 75° = \tan 165°$ $-(\cos(x+25°)\cos 75° - \sin(x+25°)\sin 75°) = -0,2679...$ $\cos(x+100°) = 0,2679...$ ref. $\angle = 74.4577…°$ $x+100° = 74,46° + k.360°$ *or* $x+100° = 285,54° + k.360°$ $x = -25,54° + k.360°;\ k \in Z$ or $x = 185,54° + k.360°;\ k \in Z$	✓✓ $\cos(x+100°)$ ✓ $-0,2679...$ ✓ 74,46° & 285,54° ✓ $-25,54°$ & 185,54° ✓ $+ k.360°;\ k \in Z$ (6)
5.4.2	$f(x) = \sin(x+10°)$ Answers only: Full marks For minimum value of $\sin x$: $x = 270°$ For minimum value of $\sin(x+10°)$: $x = 260°$	✓ $f(x) = \sin(x+10°)$ ✓ 270° ✓ answer (3)
		[22]

QUESTION/*VRAAG* 6



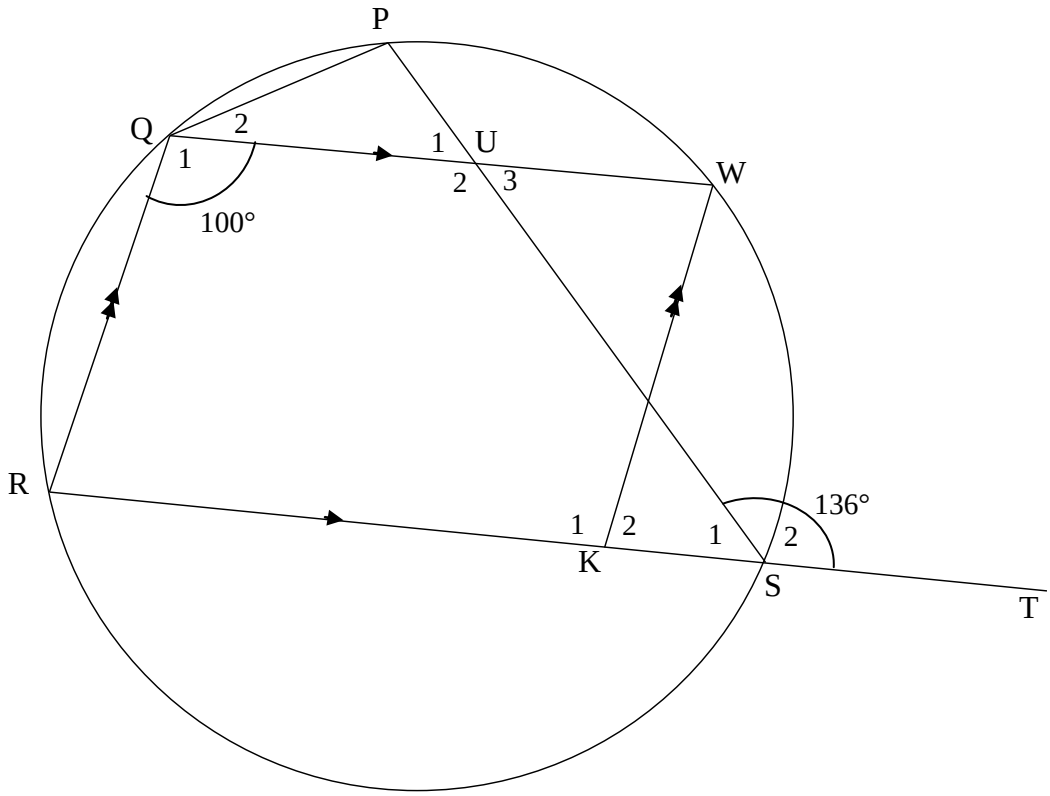
6.1	Range of f: $y \in [-2\,;0]$ **OR** $-2 \le y \le 0$	✓ critical values ✓ notation (2)
6.2	$x \in (90° ; 270°)$ OR $x \in [90° ; 270°]$	✓ critical values ✓ notation (2)
6.3	$PQ = \cos 2x - (\sin x - 1)$ $= 1 - 2\sin^2 x - \sin x + 1$ $= -2\sin^2 x - \sin x + 2$ $\sin x = -\frac{b}{2a}$ $= \frac{-(-1)}{2(-2)}$ $\sin x = -\frac{1}{4}$ $\therefore x = 194{,}48°$ or $x = 345{,}52°$	✓ $PQ = \cos 2x - (\sin x - 1)$ ✓ $\cos 2x = 1 - 2\sin^2 x$ ✓ substitution into formula ✓ $\sin x = -\frac{1}{4}$ ✓ 194,48° ✓ 345,52° (6)
		[10]

QUESTION/*VRAAG* 7



7.1	$\sin 60° = \frac{AK}{x}$ $AK = x\sin 60°$ or $\frac{\sqrt{3}}{2}x$ or $0{,}866x$	✓ trig ratio ✓ answer (2)
7.2	$K\hat{C}F = 120°$	✓ answer (1)
7.3	$KF^2 = CF^2 + CK^2 - 2CF.CK\cos K\hat{C}F$ $= x^2 + \left(\frac{x}{2}\right)^2 - 2x\left(\frac{x}{2}\right)\cos 120°$ $= x^2 + \frac{x^2}{4} - x^2\left(-\frac{1}{2}\right)$ $= \frac{7x^2}{4}$ $KF = \frac{\sqrt{7}x}{2}$ $A\hat{K}F = y$ Area $\Delta AKF = \frac{1}{2}.AK.KF\sin A\hat{K}F$ $= \frac{1}{2}.\frac{\sqrt{3}x}{2}.\frac{\sqrt{7}x}{2}\sin y$ $= \frac{x^2\sqrt{21}\sin y}{8}$	✓ correct use of cosine rule ✓substitution ✓ $\cos 120° = -\frac{1}{2}$ ✓ $KF = \frac{\sqrt{7}x}{2}$ ✓ correct use of area rule ✓substitution ✓ answer in terms of x and y (7)
		[10]

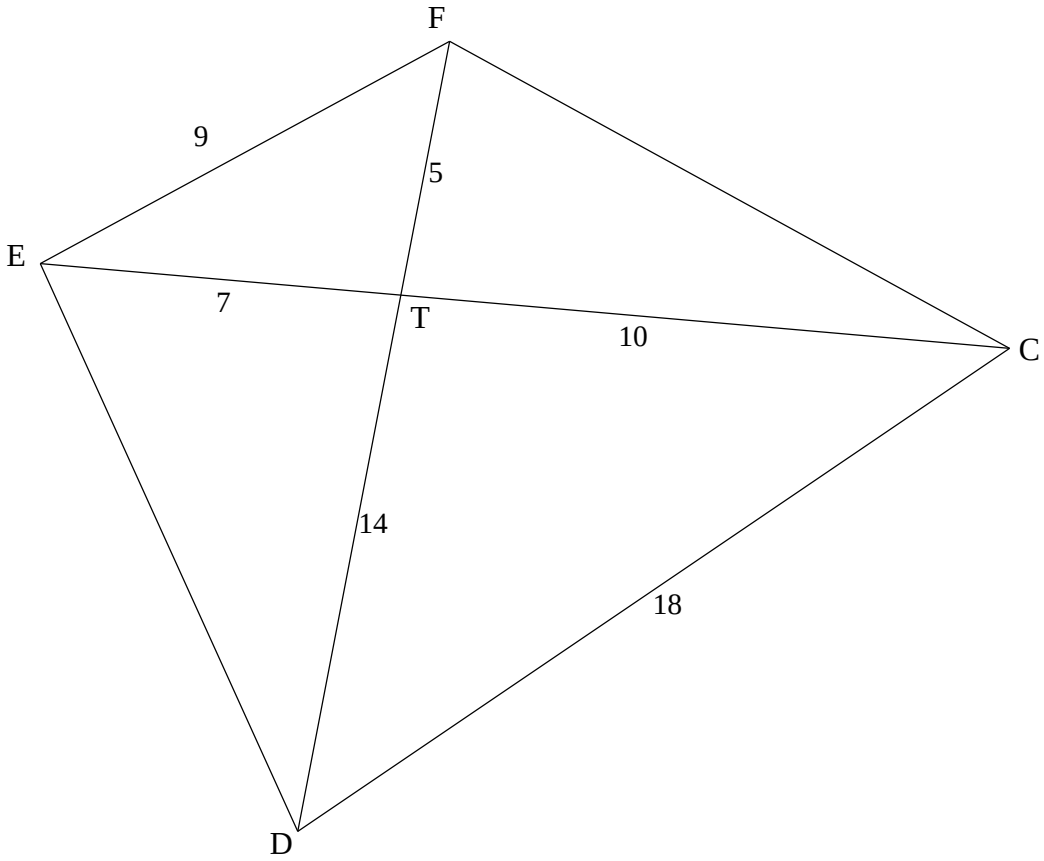
QUESTION/*VRAAG* 8



8.1.1	$\hat{R} = 80°$ [co-int ∠s/*ko-binne ∠e*; QW ‖ RK]	✓S ✓R (2)
8.1.2	$\hat{P} = 100°$ [opp ∠s of cyclic quad/*teenoorst ∠e v koordevh*]	✓S ✓R (2)
8.1.3	$P\hat{Q}R = 136°$ [ext ∠ of cyclic quad/*buite ∠ v koordevh*] $\hat{Q}_2 = 36°$ **OR** $P\hat{U}W = \hat{S}_2 = 136°$ [corresp ∠s/*ooreenkomstige ∠e*; QW ‖ RK] $P\hat{Q}W + \hat{P} = P\hat{U}W$ [ext ∠s of/*buite ∠ van* Δ QPU] $P\hat{Q}W + 100° = 136°$ $P\hat{Q}W = 36°$ **OR** $\hat{U}_3 = 180° - 136° = 44°$ [co-int ∠s/*ko-binne ∠e*; QW ‖ RK] $\hat{U}_1 = \hat{U}_3 = 44°$ [vert opp ∠s/*regoorstaande ∠e*] $P\hat{Q}W = 180° - (100 + 44°)$ [sum of ∠s in Δ/*som ∠e van* Δ] $P\hat{Q}W = 36°$	✓S ✓R ✓S (3) ✓S ✓R ✓S (3) ✓S ✓R ✓S (3)

8.1.4	$\hat{U}_2 = \hat{S}_2 = 136°$	[alt ∠s/*verwiss ∠e* ; QW || RK]	✓S ✓R (2)
	OR		
	$\hat{U}_2 = 100° + 36°$ $= 136°$	[ext ∠s of/*buite ∠ van* ΔQPU]	✓S ✓R (2)
	OR		
	$\hat{U}_2 = P\hat{U}W = 136°$	[vert opp ∠s/*regoorstaande ∠e*]	✓S ✓R (2)
	OR		
	$\hat{U}_2 = 180° - \hat{U}_3$ $= 180° - 44°$ $= 136°$	[∠s on a str line/*∠e op reguitlyn*]	✓S ✓R (2)

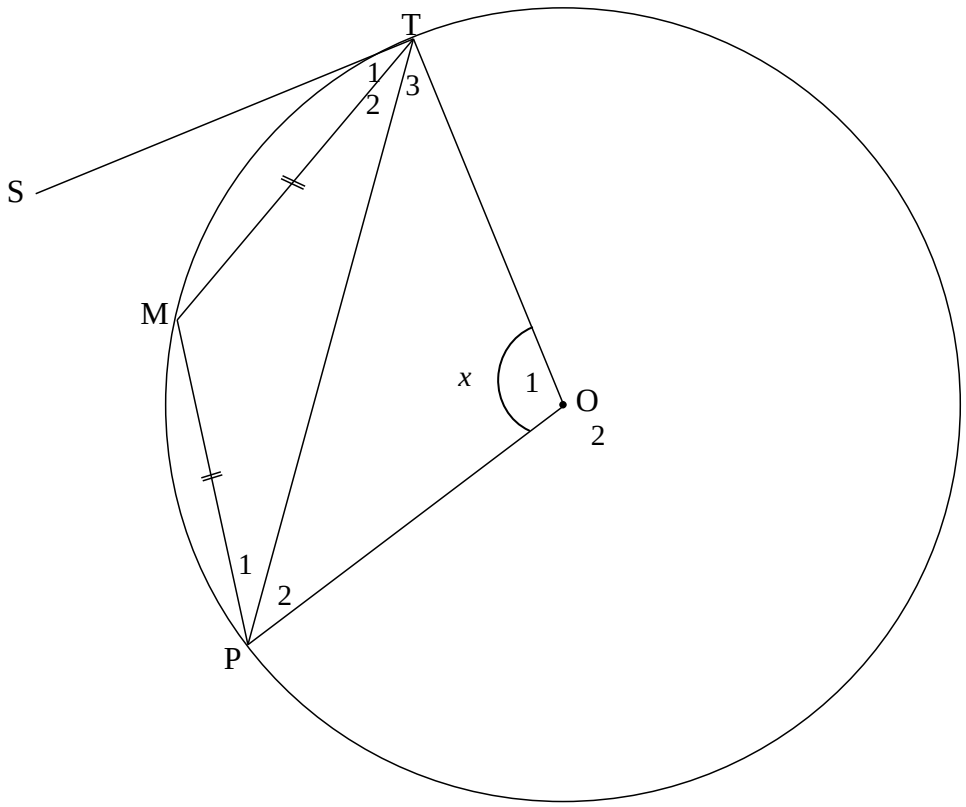
8.2



8.2.1	In ΔEFT and ΔDCT: $\frac{EF}{CD}=\frac{9}{18}=\frac{1}{2}$ $\frac{FT}{TC}=\frac{5}{10}=\frac{1}{2}$ $\frac{ET}{TD}=\frac{7}{14}=\frac{1}{2}$ $\therefore \Delta EFT \text{ ||| } \Delta DCT$ [Sides of Δ in prop/ *sye van Δ in dieselfde verh*] $\therefore E\hat{F}D = E\hat{C}D$ **OR** In ΔFET: $49 = 25 + 81 - 2(5)(9)\cos\hat{F}$ $\cos\hat{F} = \frac{19}{30}$ $\hat{F} = 50{,}7°$ In ΔTDC: $196 = 100 + 256 - 2(10)(18)\cos\hat{C}$ $\cos\hat{C} = \frac{19}{30}$ $\hat{C} = 50{,}7°$	✓✓ all 3 ratios = $\frac{1}{2}$ ✓ $\Delta EFT \text{ ||| } \Delta DCT$ ✓ R (4) ✓✓ $\hat{F} = 50{,}7°$ ✓✓ $\hat{C} = 50{,}7°$ (4)

8.2.2	$E\hat{F}D = E\hat{C}D$ [proved in 8.2.1] E, F, C and D are concyclic EFCD is a cyclic quad [converse ∠s in the same segment/ *omgekeerde ∠e in dies segment*] $\therefore D\hat{F}C = D\hat{E}C$ [∠s in the same segment/*∠e in dies segment*]	✓S ✓R ✓ R (3)
		[16]

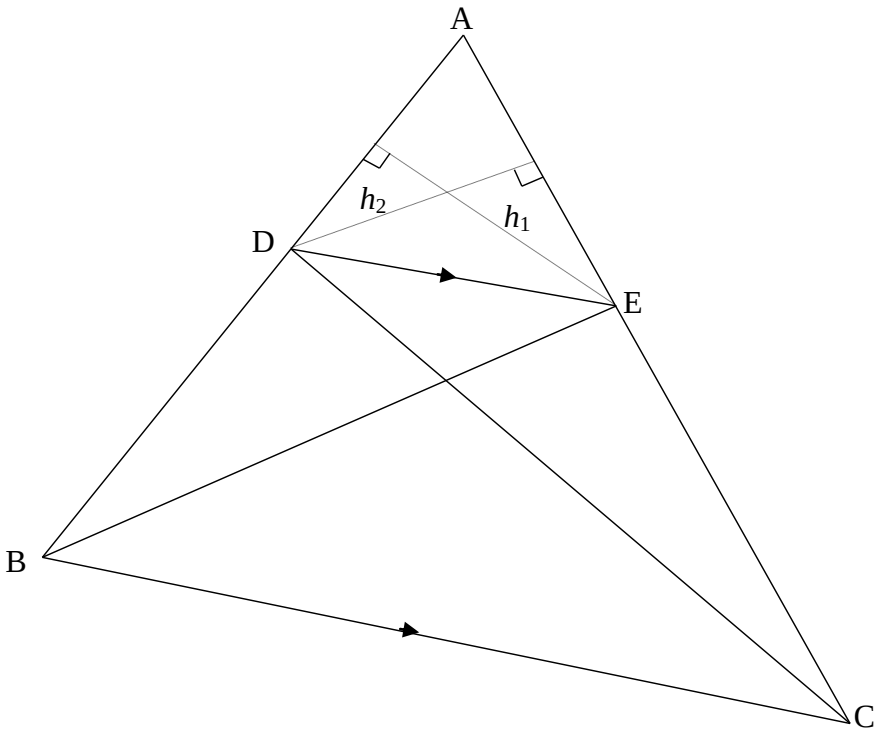
QUESTION/*VRAAG* 9



	$\hat{O}_2 = 360° - x$ [∠s round a pt/*∠e om ´n punt*]	✓ $\hat{O}_2 = 360° - x$
	$\therefore \hat{M} = 180° - \frac{1}{2}x$ [∠ at centre = 2 ×∠ at circumference/ *middelpunts∠ = 2 × omtreks∠*]	✓ $\hat{M} = 180° - \frac{1}{2}x$ ✓R
	$\therefore \hat{T}_2 + \hat{P}_1 = \frac{1}{2}x$ [sum of ∠s in Δ/*som ∠e van* Δ]	✓ $\hat{T}_2 + \hat{P}_1 = \frac{1}{2}x$
	$\therefore \hat{T}_2 = \hat{P}_1 = \frac{1}{4}x$ [∠s opp equal sides/*∠e teenoor gelyke sye*]	✓ $\hat{P}_1 = \frac{1}{4}x$ ✓R
	$\therefore S\hat{T}M = \hat{P}_1 = \frac{1}{4}x$ [tan chord theorem/*raaklyn koordstelling*]	✓R (7)
	OR/*OF*	
	$\hat{O}_2 = 360° - x$ [∠s round a pt/*∠e om ´n punt*]	✓ $\hat{O}_2 = 360° - x$
	$\therefore \hat{M} = \frac{1}{2}\hat{O}_2$ [∠ at centre = 2 ×∠ at circumference]	✓ S ✓R
	$\therefore \hat{T}_2 + \hat{P}_1 = 180° - \hat{M}$ [sum of ∠s in Δ/*som ∠e van* Δ]	✓S
	$\therefore \hat{T}_2 = \hat{P}_1$ [∠s opp equal sides/*∠e teenoor gelyke sye*]	✓R
	$= \frac{180° - \hat{M}}{2} = \frac{180° - \frac{1}{2}\hat{O}_2}{2} = \frac{180° - \frac{1}{2}(360° - x)}{2} = \frac{1}{4}x$	✓ S
	$\therefore S\hat{T}M = \frac{1}{4}x$ [tan chord theorem/*raaklyn koordstelling*]	✓R (7)
		[7]

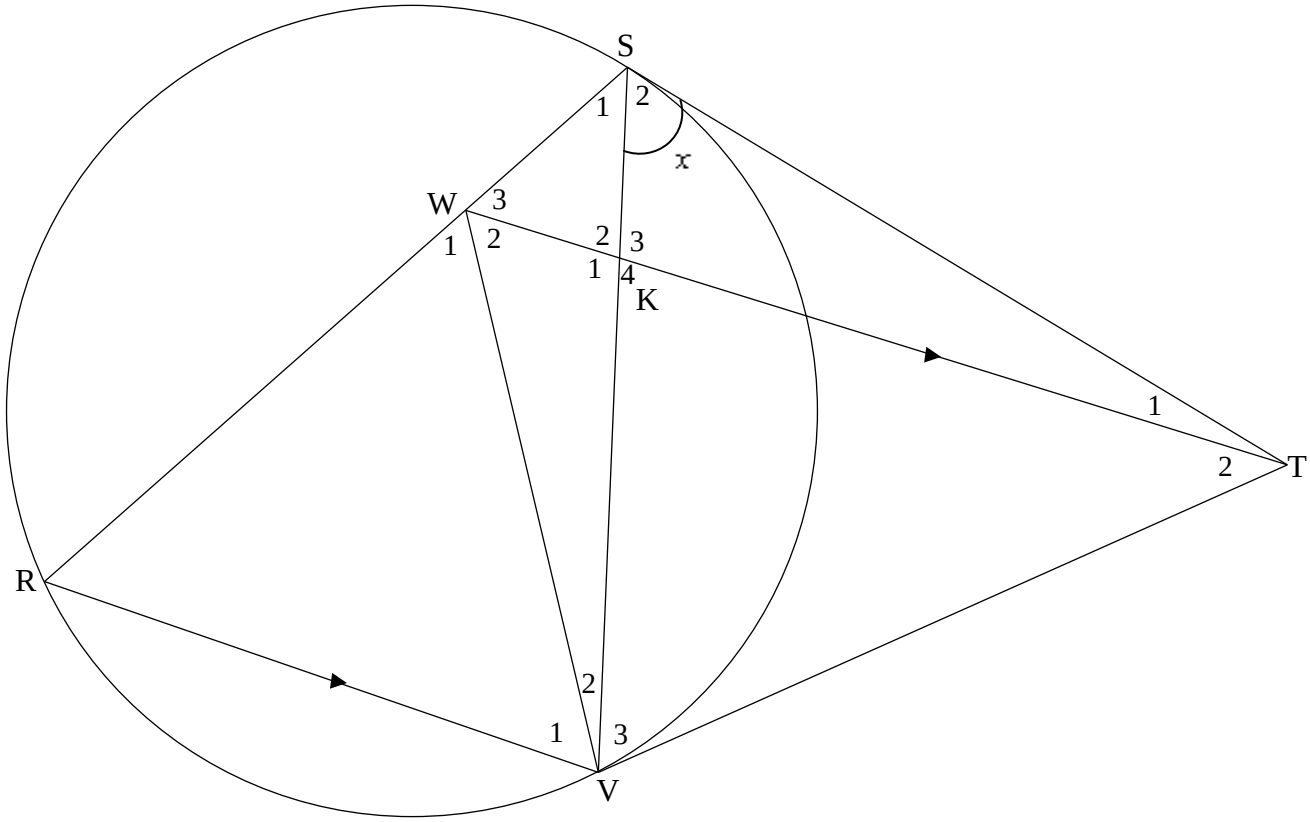
QUESTION/*VRAAG* 10

10.1



10.1	Constr: Draw h_1 from E $\perp$ AD and h_2 from D $\perp$ AE *Konstr: Trek* h_1 *vanaf* E $\perp$ AD *en* h_2 *vanaf* D $\perp$ AE Proof/*Bewys*: $\frac{\text{area } \Delta ADE}{\text{area } \Delta BDE} = \frac{\frac{1}{2}AD \times h_1}{\frac{1}{2}DB \times h_1} = \frac{AD}{DB}$ $\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \frac{\frac{1}{2}AE \times h_2}{\frac{1}{2}EC \times h_2} = \frac{AE}{EC}$ But area ΔBDE = area ΔDEC [same base & height or DE || BC/ *dies basis & hoogte*; of DE || BC] $\therefore \frac{\text{area } \Delta ADE}{\text{area } \Delta BDE} = \frac{\text{area } \Delta ADE}{\text{area } \Delta DEC}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$	✓ constr/*konstr* **OR reason: common vertex or same height** ✓ $\frac{\text{area } \Delta ADE}{\text{area } \Delta BDE} = \frac{\frac{1}{2}AD \times h_1}{\frac{1}{2}DB \times h_1}$ ✓ $\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \frac{AE}{EC}$ ✓ S ✓R ✓ S (6)

10.2



10.2.1	$\hat{V}_3 = x$ [Tans from same point/*raaklyne vanaf dieselfde pt*] $\hat{R} = x$ [tan chord theorem/*raaklyn koordstelling*] $\hat{W}_3 = x$ [corresp ∠s/*ooreenkomstige ∠e*; WT ‖ RV]	✓ S ✓ R ✓ S ✓ R ✓ S ✓ R (6)
10.2.2(a)	$\hat{V}_3 = \hat{W}_3 = x$ [proved in 10.2.1] W, S, T and V are concyclic/*is konsiklies* WSTV is a cyclic quad [converse ∠s in the same segment/ *Omgekeerde ∠e in dieselfde segment*]	✓ S ✓ R (2)
10.2.2(b)	$\hat{W}_2 = \hat{S}_2 = x$ [∠s in the same segment/∠e *in dies segment*] $\hat{V}_1 = \hat{W}_2 = x$ [alt ∠s/*verwiss ∠*e ; WT ‖ RV] But $\hat{R} = x$ [proved in 10.2.1] $\therefore \hat{R} = \hat{V}_1 = x$ ∴ WR = WV [sides opp equal ∠s/*sye teenoor gelyke ∠e*] ΔWRV is isosceles/*is gelykbenig* **OR/*OF***	✓ S ✓R ✓ S/ R ✓ S (4)

	$\hat{S}_2 = \hat{W}_2 = x$ [∠s in the same segment] $\hat{W}_2 = \hat{W}_3 = x$ $\hat{W}_2 + \hat{W}_3 = \hat{R} + \hat{V}_1$ [ext ∠ of Δ] $\therefore\ \hat{V}_1 = x = \hat{R}$ $\therefore$ WR = WV [sides opp equal ∠s/*sye teenoor gelyke ∠e*] ΔWRV is isosceles/*is gelykbenig*	✓ S ✓R ✓ S/ R ✓ S (4)
10.2.2(c)	In ΔWRV and/*en* ΔTSV $\hat{R} = \hat{S}_2 = x$ [proved **OR** tan chord theorem] $\hat{V}_1 = \hat{V}_3 = x$ [proved] $\therefore$ ΔWRV ||| ΔTSV [∠, ∠, ∠] **OR/*OF*** In ΔWRV and/*en* ΔTSV $\hat{R} = \hat{S}_2 = x$ [proved **OR** tan chord theorem] $\hat{V}_1 = \hat{V}_3 = x$ [proved] $\hat{W}_1 = S\hat{T}V = x$ [sum of ∠s in Δ/*∠e van* Δ] $\therefore$ ΔWRV ||| ΔTSV	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
10.2.2(d)	$\frac{RV}{SV} = \frac{WR}{TS}$ [ΔWRV ||| ΔTSV] $\therefore WR \times SV = RV \times TS$ $\frac{WR}{SR} = \frac{KV}{SV}$ [prop theorem/*eweredighst*; WT || RV] $\therefore WR \times SV = KV \times SR$ $\therefore RV \times TS = KV \times SR$ $\therefore \frac{RV}{SR} = \frac{KV}{TS}$ **OR/*OF*** In ΔRVS and/*en* ΔVKT $S\hat{V}R = \hat{K}_4$ [alt ∠s, WT || RV] $S\hat{R}V = \hat{V}_3$ [proven] ΔRVS ||| ΔVKT [∠, ∠, ∠] $\therefore \frac{RV}{SR} = \frac{KV}{VT}$ but VT = ST [tans from same point] $\therefore \frac{RV}{SR} = \frac{KV}{TS}$	✓ correct ratios ✓ $\frac{WR}{SR} = \frac{KV}{SV}$ ✓ R ✓ equating $WR \times SV$ (4) ✓ identifying correct Δs ✓ proving ||| ✓ correct ratio ✓ S (4)
		[25]

TOTAL/*TOTAAL*: 150