



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

JUNE 2019

MATHEMATICS P2

MARKS: 150

TIME: 3 hours



This question paper consists of 14 pages, including an information sheet and a special answer book of 19 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of TEN (10) questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

Eastern High School compared the Term 1 percentages of 20 Grade 12 learners consisting of 10 boys and 10 girls. The following data was recorded:

Boys' marks	41	30	24	65	72	15	83	52	60	38
-------------	----	----	----	----	----	----	----	----	----	----

Girls' mean mark = 51

Girls' standard deviation = 15,95

- 1.1 Calculate the mean mark for boys. (1)
- 1.2 Calculate the standard deviation for the boys' marks. (2)
- 1.3 Did boys or girls perform better? Give a reason for your answer. (2)
- 1.4 By what percentage must each of the boys' marks be adjusted so that the mean of boys can be the same as that of the girls? (1)
- 1.5 Will the boys' standard deviation increase, decrease or remain the same after the adjustment in QUESTION 1.4 above? (1)

[7]

QUESTION 2

The ages of people who were registering to vote at a voting station were recorded in the frequency table below:

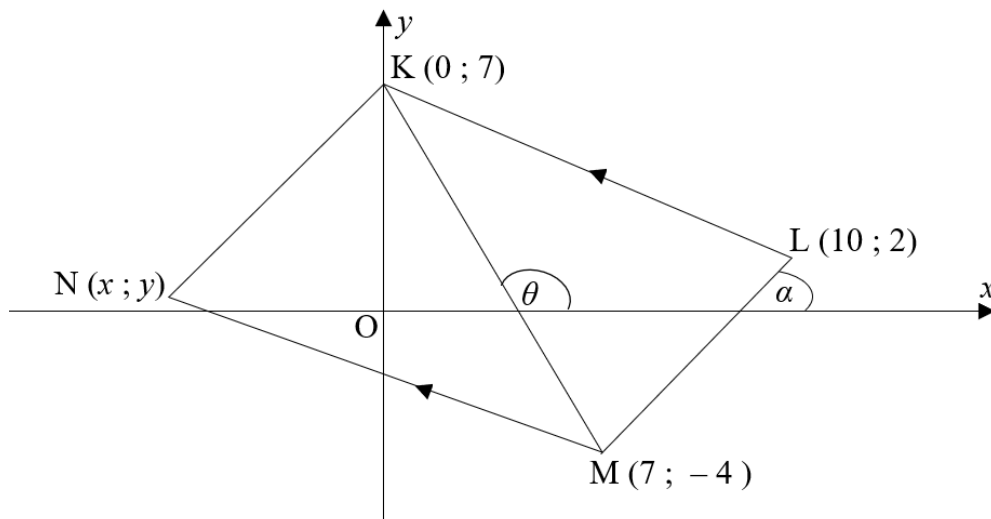
Ages (in years)	Frequency	Cumulative Frequency
$18 \leq x < 28$		4
$28 \leq x < 38$		14
$38 \leq x < 48$		28
$48 \leq x < 58$	17	
$58 \leq x < 68$	12	
$68 \leq x < 78$	3	

- 2.1 Complete the frequency table. (2)
- 2.2 Draw the cumulative frequency graph (ogive). (3)
- 2.3 Write down the modal class. (1)
- 2.4 People who are 60 years and older are regarded as senior citizens and must not queue but be taken to the front. Estimate the number of senior citizens. (2)
- 2.5 Write down the lower (Q_1), middle (Q_2) and upper (Q_3) quartiles. (3)
- 2.6 Draw a box and whisker diagram to represent the ages of the voters. (2)

[13]

QUESTION 3

In the diagram below, $K(0 ; 7)$, $L(10 ; 2)$, $M(7 ; -4)$ and $N(x ; y)$ are vertices of quadrilateral $KLMN$ with $MN \parallel KL$. θ and α are the angles formed by KM and ML with the x -axis respectively.

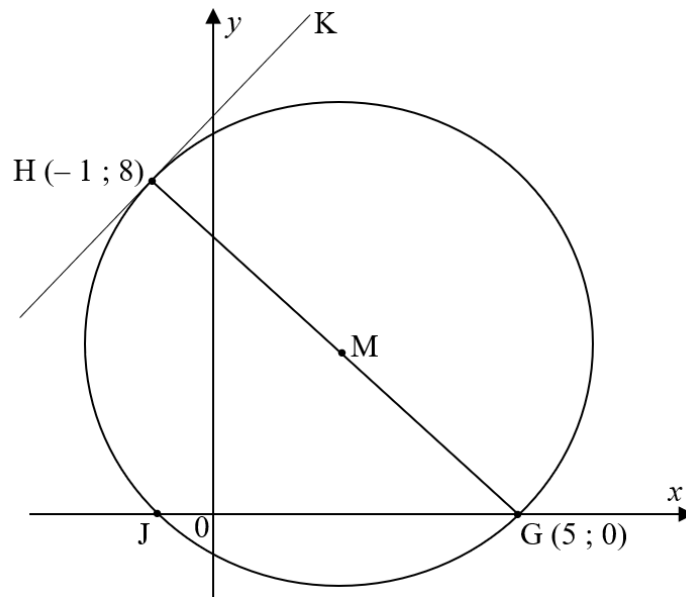


- 3.1 Determine:
- 3.1.1 The length of KL . Leave your answer in simplified surd form (2)
- 3.1.2 The gradient of KM (2)
- 3.1.3 The size of α , the angle of inclination of LM (3)
- 3.1.4 The size of \widehat{LMK} (4)
- 3.2 Determine the coordinates of N if $KLMN$ is a parallelogram. Show ALL calculations. (4)
- 3.3 Is \widehat{LMN} a right angle or not? Justify your answer by calculation(s). (2)
- 3.4 Determine the area of $\triangle KNM$. (5)

[22]

QUESTION 4

In the diagram below, circle with centre M, diameter GH with G(5 ; 0) and tangent HK with point of contact at H(-1 ; 8) is given.



- 4.1 Write down the coordinates of M. (2)
- 4.2 Determine the equation of the circle in the form $(x - a)^2 + (y - b)^2 = r^2$ (3)
- 4.3 Determine the equation of the tangent HK. (4)
- 4.4 Determine the coordinates of J. (3)
- 4.5 Find the new coordinates of J if the circle is rotated 180° around the centre M. (2)
- 4.6 The equation of another circle is given as $x^2 + y^2 - 12x - 2y + 17 = 0$.
Does the centre of the new circle lie on, inside or outside the originally given circle?
Justify your answer with relevant calculations. (5)

[19]

QUESTION 5

5.1 If $\sin 42^\circ = k$, determine the following in terms of k .

5.1.1 $\tan 42^\circ$ (2)

5.1.2 $\sin 84^\circ$ (3)

5.1.3 $\sin 3^\circ$ (4)

5.2 Simplify to a single trigonometric ratio:

$$\frac{\sin(x-450^\circ) \cdot \tan(180^\circ+x) \cdot \sin(90^\circ-x)}{\cos(-x)} \quad (6)$$

5.3 Consider the identity: $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

5.3.1 Complete: $\cos(A+B) = \dots$ (1)

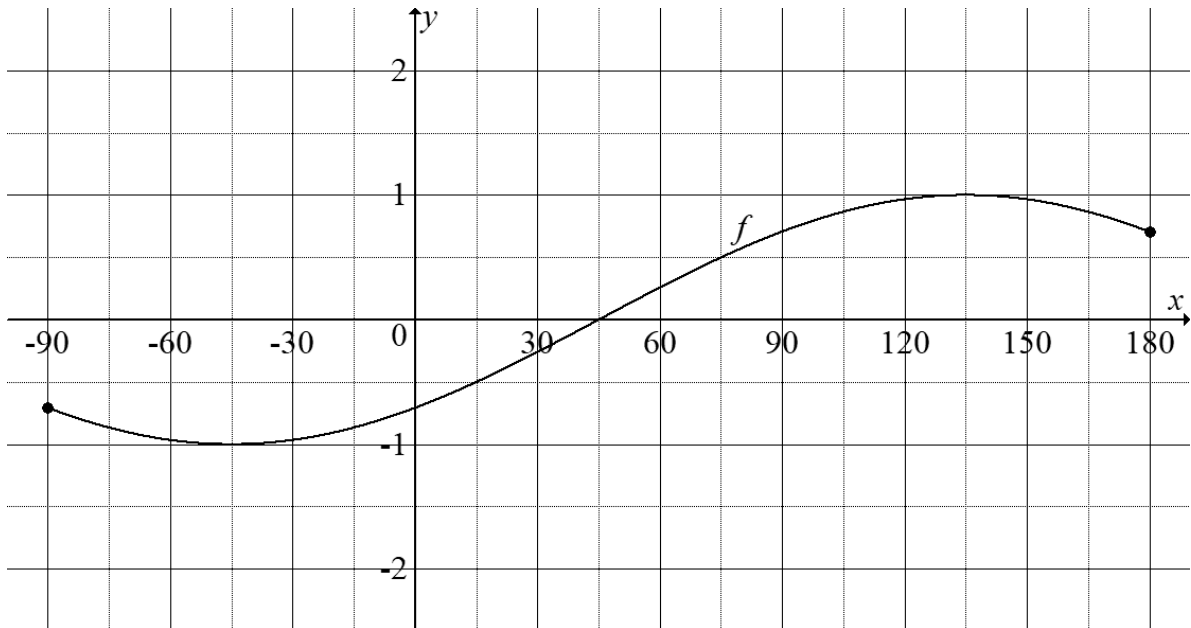
5.3.2 Prove the identity: $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ (4)

5.4 If $\cos\theta = 2p$ and $\cos 2\theta = 7p$, determine the possible value(s) of p . (5)

[25]

QUESTION 6

Given below is the graph of $f(x) = \sin(x - 45^\circ)$, for $x \in [-90^\circ ; 180^\circ]$.

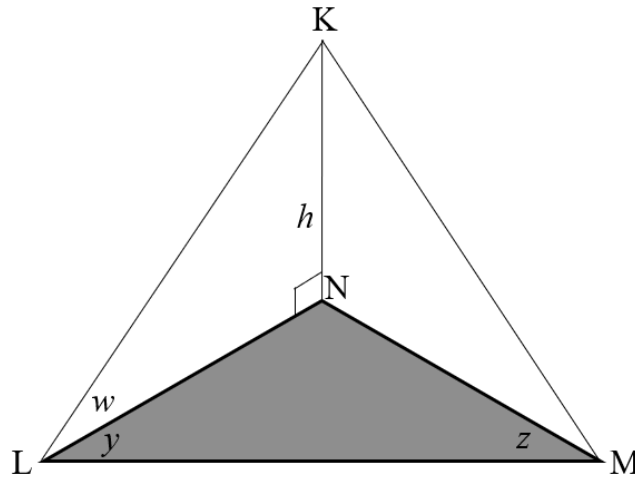


- 6.1 Write down the range of f . (1)
- 6.2 On the same set of axes, sketch the graph of $g(x) = \tan x$ for $x \in [-90^\circ ; 180^\circ]$ in the SPECIAL ANSWER BOOK. Show ALL intercepts with the axes as well as asymptotes and end points. (3)
- 6.3 Write down the period of g . (1)
- 6.4 Write down the value(s) of x for which $f(x) = g(x)$ for $x \in [-90^\circ ; 180^\circ]$. (1)
- 6.5 For which value(s) of x is $f(x) \cdot g(x) \geq 0$ for $x \in [0^\circ ; 180^\circ]$? (2)
- 6.6 Write down the equation of $h(x)$ if $h(x)$ is a result of shifting $f(x)$ such that its minimum value is zero. (1)

[9]

QUESTION 7

In the diagram KN represents a vertical tower, of height h metres, standing on a horizontal plane LMN. The angle of elevation of K, as seen from L, is w . $\hat{NLM} = y$ and $\hat{NML} = z$. (NOTE: all angles are measured in degrees).



7.1 Show that $LN = \frac{h}{\tan w}$ (1)

7.2 Hence, prove that $LM = \frac{h \sin(y+z)}{\tan w \sin z}$ (4)

7.3 Calculate LM if $h = 38\text{ m}$, $w = 21^\circ$, $y = 52^\circ$ and $z = 59^\circ$. (2)

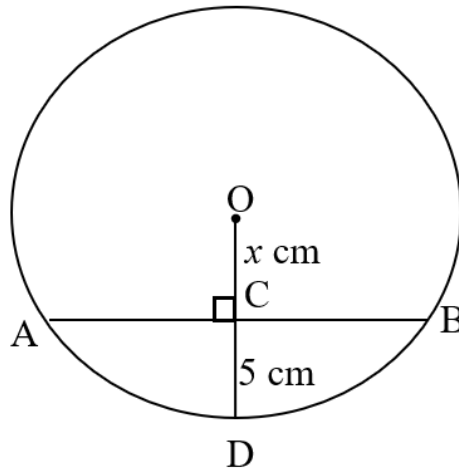
[7]

Give reasons for your statements in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 Complete:
The perpendicular bisector of a chord passes through ... (1)

8.2 In the diagram below, O is the centre of the circle, AB is a chord and $OC \perp AB$.
OC produced, intersects the circle at D. $AB = 20$ cm, $CD = 5$ cm and $OC = x$ cm.



Determine, stating reasons:

8.2.1 The length of AC (2)

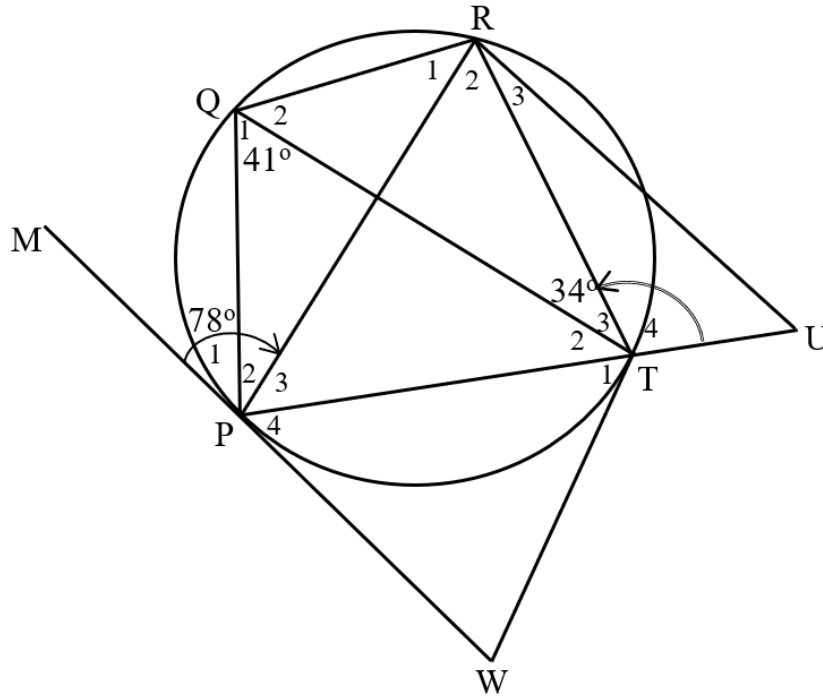
8.2.2 The radius of the circle (4)

[7]

QUESTION 9

9.1 Complete:
Exterior angle of a cyclic quadrilateral is equal to ... (1)

9.2 In the diagram below, points P, Q, R and T lie on the circumference of a circle. MW and TW are tangents to the circle at P and T respectively. PT is produced to meet RU at U. Furthermore, $\hat{MPR} = 78^\circ$, $\hat{PQT} = 41^\circ$ and $\hat{QTR} = 34^\circ$.



9.2.1 Write down, with reasons, THREE other angles that are each equal to 41° . (6)

9.2.2 Determine the following, stating reasons:

(a) \hat{T}_2 (2)

(b) \hat{Q}_2 (2)

(c) \hat{T}_4 (2)

(d) \hat{W} (2)

9.2.3 Determine, with reasons, whether:

(a) $QR \parallel PT$ or not (2)

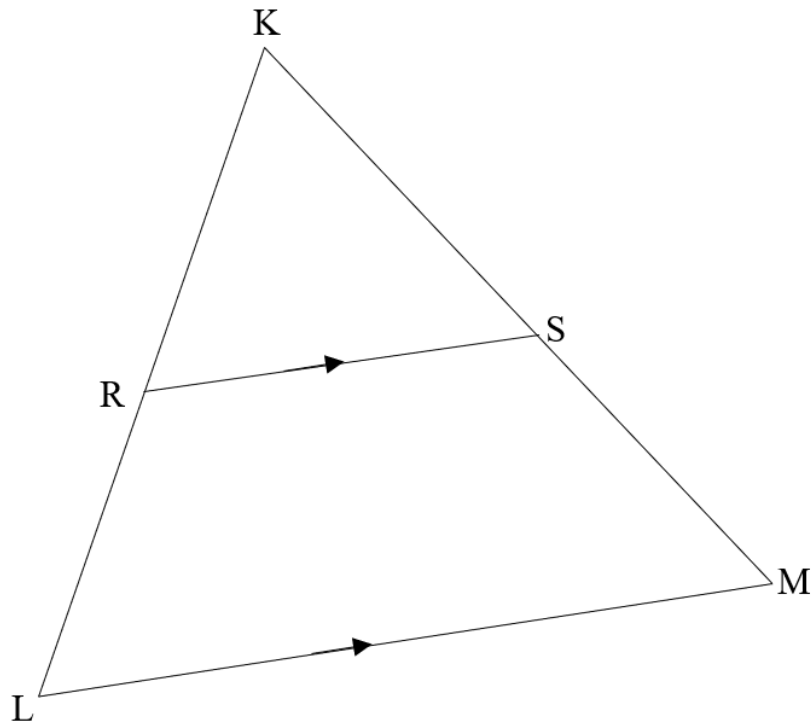
(b) PRTW is a cyclic quadrilateral or not (2)

(c) TQ is a diameter or not (2)

[21]

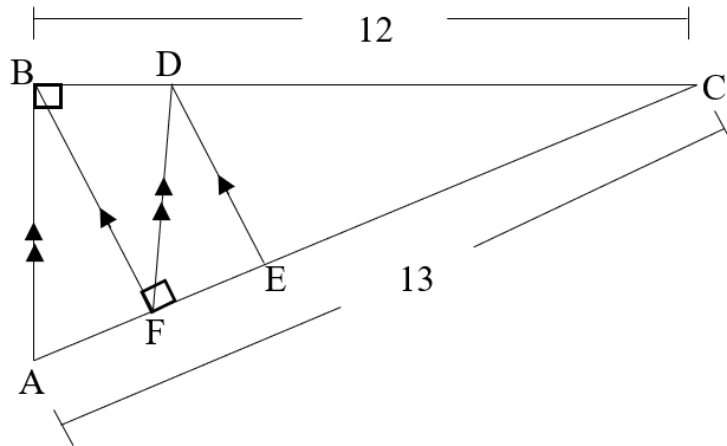
QUESTION 10

- 10.1 In the diagram below, $\triangle KLM$ is given with R and S on KL and KM respectively such that $RS \parallel LM$.



Prove the theorem which states that $\frac{KR}{RL} = \frac{KS}{SM}$ (5)

10.2 In the diagram below, $\triangle ABC$ is drawn with D on BC and F and E on AC such that $AB \parallel FD$, $BF \parallel DE$, $AB \perp BC$ and $BF \perp CA$. Furthermore, $CA = 13$ units and $CB = 12$ units.



10.2.1 Write down the length of AB (1)

10.2.2 Prove, stating reasons, that:

(a) $\triangle CBA \parallel \triangle CFB$ (3)

(b) $CF = \frac{CB^2}{CA}$ (3)

10.2.3 Hence, determine the length of CF, correct to the nearest unit. (2)

10.2.4 Give the length of AF. (1)

10.2.5 Determine the length of FE.

Leave your answer in the form $\frac{a}{b}$. (5)
[20]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In ΔABC :

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



Province of the
EASTERN CAPE
EDUCATION

LEARNER'S NAME: <i>LEERDERNAAM:</i>	
--	--

GRADE 12 <i>GRAAD 12</i>	
-----------------------------	--

**NATIONAL SENIOR
CERTIFICATE/NASIONALE
SENIOR SERTIFIKAAT**

GRADE 12/GRAAD 12

JUNE/JUNIE 2019

**MATHEMATICS P2/WISKUNDE V2
SPECIAL ANSWER BOOK/SPEZIALE ANTWOORDEBOEK**

Marker/Merker			Moderator's Initials / Moderator se paraaf							
Question <i>Vraag</i>	Mark <i>Punt</i>	Initial <i>Parafeer</i>	Marks <i>Punte</i>	S M	Marks <i>Punte</i>	D M	Marks <i>Punte</i>	P M	Marks <i>Punte</i>	N M
1										
2										
3										
4										
5										
6										
7										
8										
9										
10										
TOTAL TOTAAL										

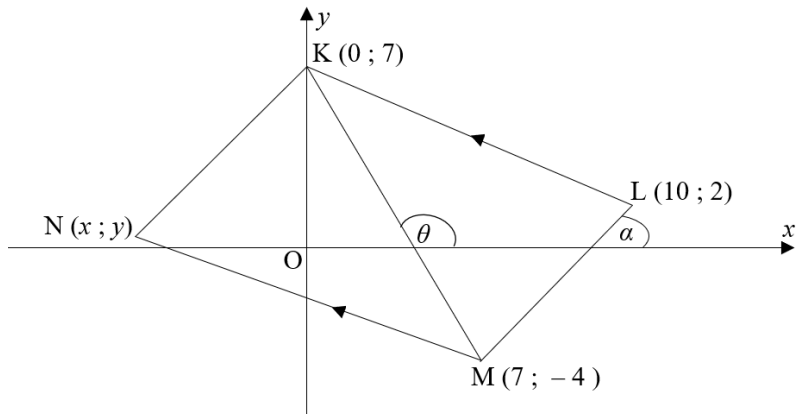
This special answer book consists of 19 pages./
Hierdie spesiale antwoordeboek bestaan uit 19 bladsye.



QUESTION 1/VRAAG 1		
1.1		(1)
1.2		(2)
1.3		(2)
1.4		(1)
1.5		(1)
		[7]

QUESTION 2/VRAAG 2					
2.1	Ages (in years) <i>Ouderdom (in jare)</i>	Frequency <i>Frekwensie</i>	Cumulative Frequency <i>Kumulatiewe Frekwensie</i>		
	$18 \leq x < 28$		4		
	$28 \leq x < 38$		14		
	$38 \leq x < 48$		28		
	$48 \leq x < 58$	17			
	$58 \leq x < 68$	12			
	$68 \leq x < 78$	3			
				(2)	
2.2	<p style="text-align: center;">Ogive – Ages of people registering to vote <i>Ogief – Ouderdomme van mense wat registreer om te stem</i></p>				(3)
2.3				(1)	
2.4				(2)	
2.5				(3)	
2.6				(2)	
				[13]	

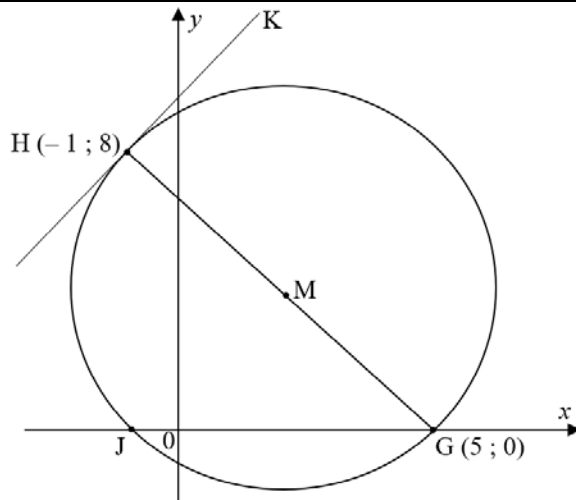
QUESTION 3/VRAAG 3



3.1.1		(2)
3.1.2		(2)
3.1.3		(3)
3.1.4		(4)

3.2		
		(4)
3.3		
		(2)
3.4		
		(5)
	[22]	

QUESTION 4/VRAAG 4



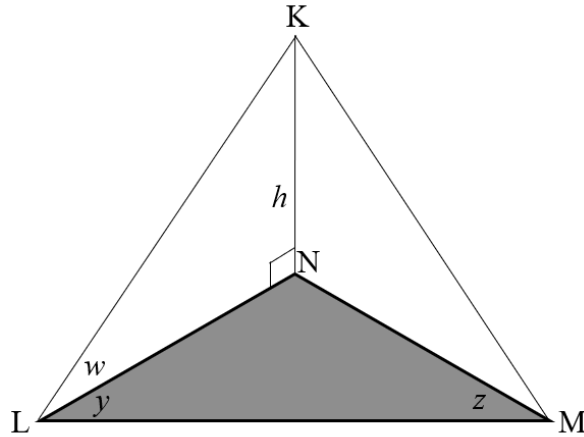
4.1		(2)
4.2		(3)
4.3		(4)

QUESTION 5/VRAAG 5		
5.1.1		(2)
5.1.2		(3)
5.1.3		(4)
5.2		(6)

5.3.1		
		(1)
5.3.2		(4)
5.4		(5)
		[25]

QUESTION 6/VRAAG 6		
6.1		(1)
6.2		(3)
6.3		(1)
6.4		(1)
6.5		(2)
6.6		(1)
		[9]

QUESTION 7/VRAAG 7



7.1

(1)

7.2

(4)

7.3

(2)

[7]

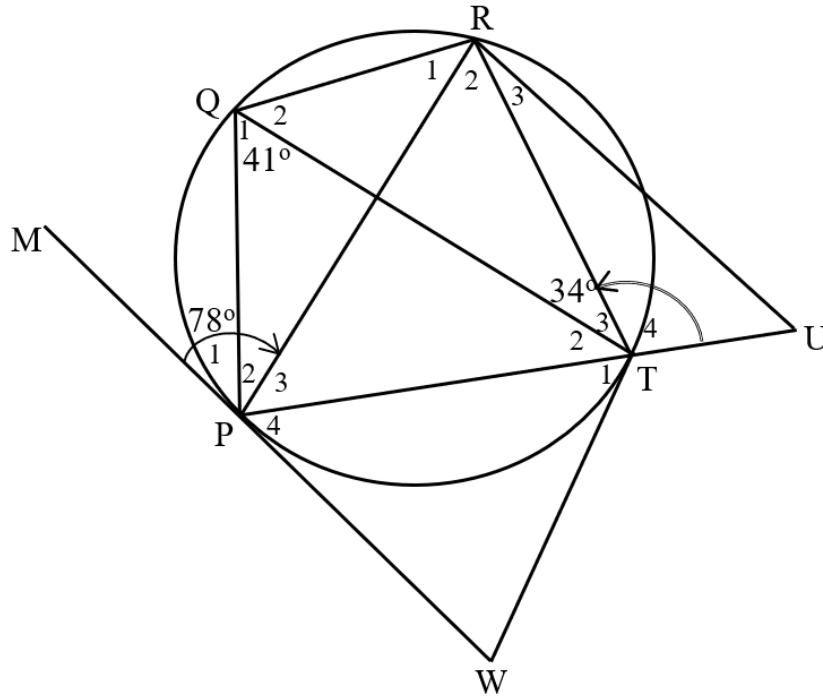
QUESTION 8/VRAAG 8		
8.1		(1)
8.2		
8.2.1		(2)
8.2.2		(4)
		[7]

QUESTION 9/VRAAG 9

9.1

(1)

9.2



9.2.1

(6)

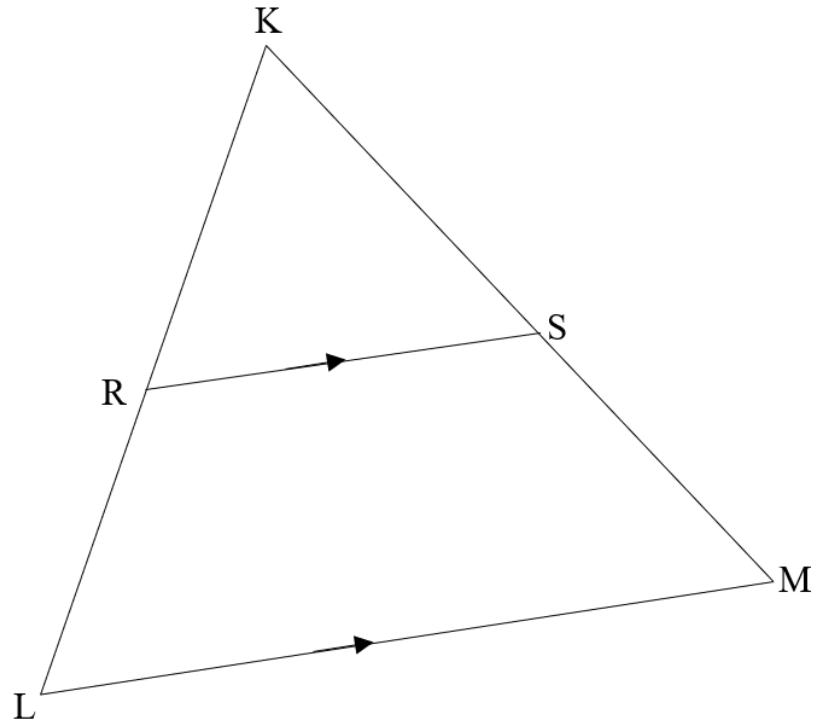
9.2.2(a)

(2)

9.2.2(b)		(2)
9.2.2(c)		(2)
9.2.2(d)		(2)
9.2.3(a)		(2)
9.2.3(b)		(2)
9.2.3(c)		(2)
		[21]

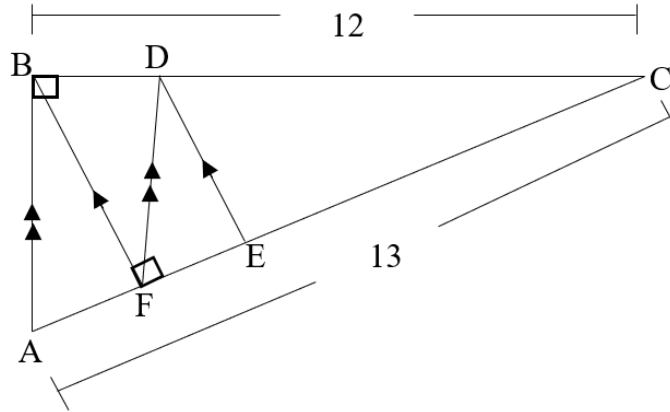
QUESTION 10/VRAAG 10

10.1



(5)

10.2



10.2.1		(1)
10.2.2(a)		(3)
10.2.2(b)		(3)
10.2.3		(2)

10.2.4		(1)
10.2.5		(5)
		[20]

TOTAL/TOTAAL: 150



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL
SENIOR CERTIFICATE/
*NASIONALE
SENIOR SERTIFIKAAT***

GRADE/*GRAAD* 12

JUNE/*JUNIE* 2019

**MATHEMATICS P2/*WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN***

MARKS/*PUNTE*: 150

This marking guideline consists of 17 pages./
Hierdie nasienriglyn bestaan uit 17 bladsye.

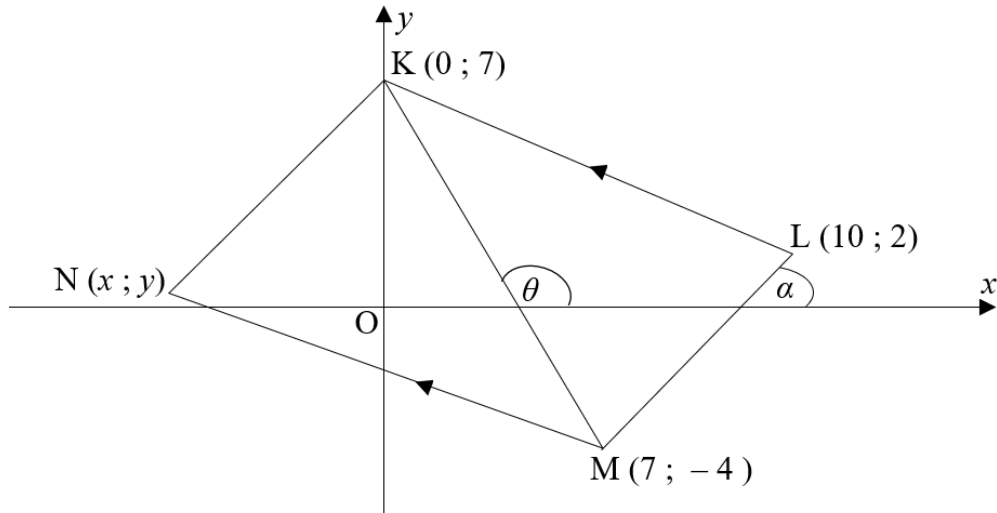
QUESTION 1/VRAAG 1

1.1	Mean/ <i>Gemiddelde</i> = 48	✓ 48 (1)
1.2	SD/SA = 22,08 Penalty of 1 mark for incorrect rounding <i>Penaliseer 1 punt vir verkeerde afronding</i>	✓✓ 22,08 (2)
1.3	Girls performed better. The girls' mean percentage is bigger than that of boys and the girls standard deviation is smaller than that of boys <i>Meisies het beter gevaar. Die meisies se gemiddelde persentasie is groter as dié van die seuns en die standaardafwyking is kleiner as dié van die seuns.</i>	✓ Girls / <i>Meisies</i> ✓ Reason / <i>Rede</i> (2)
1.4	$51 - 48 = 3$ ∴ each boy's percentage must be increased by 3. <i>∴ elke seun se persentasie moet met 3 vermeerder word.</i>	✓ 3 (1)
1.5	Boys' standard deviation will remain the same <i>Die seuns se standaardafwyking sal dieselfde bly.</i>	✓ remain the same <i>dieselfde bly</i> (1)
		[7]

QUESTION 2/VRAAG 2

2.1	<table border="1"> <thead> <tr> <th>Ages (in years) <i>Ouderdom (in jare)</i></th> <th>Frequency <i>Frekwensie</i></th> <th>Cumulative Frequency <i>Kumulatiewe Frekwensie</i></th> </tr> </thead> <tbody> <tr> <td>$18 \leq x < 28$</td> <td>4</td> <td>4</td> </tr> <tr> <td>$28 \leq x < 38$</td> <td>10</td> <td>14</td> </tr> <tr> <td>$38 \leq x < 48$</td> <td>14</td> <td>28</td> </tr> <tr> <td>$48 \leq x < 58$</td> <td>17</td> <td>45</td> </tr> <tr> <td>$58 \leq x < 68$</td> <td>12</td> <td>57</td> </tr> <tr> <td>$68 \leq x < 78$</td> <td>3</td> <td>60</td> </tr> </tbody> </table>	Ages (in years) <i>Ouderdom (in jare)</i>	Frequency <i>Frekwensie</i>	Cumulative Frequency <i>Kumulatiewe Frekwensie</i>	$18 \leq x < 28$	4	4	$28 \leq x < 38$	10	14	$38 \leq x < 48$	14	28	$48 \leq x < 58$	17	45	$58 \leq x < 68$	12	57	$68 \leq x < 78$	3	60	<p>✓ frequency <i>frekwensie</i></p> <p>✓ cumulative frequency <i>kumulatiewe frekwensie</i></p> <p>(2)</p>
Ages (in years) <i>Ouderdom (in jare)</i>	Frequency <i>Frekwensie</i>	Cumulative Frequency <i>Kumulatiewe Frekwensie</i>																					
$18 \leq x < 28$	4	4																					
$28 \leq x < 38$	10	14																					
$38 \leq x < 48$	14	28																					
$48 \leq x < 58$	17	45																					
$58 \leq x < 68$	12	57																					
$68 \leq x < 78$	3	60																					
2.2	<p style="text-align: center;">Ogive - Ages of people registering to vote Ogief - Ouderdomme van mense wat registreer om te stem</p>	<p>✓ grounding at / <i>anker by (0 ; 18)</i></p> <p>✓ upper limits <i>boonste limiete</i></p> <p>✓ shape / <i>vorm</i></p> <p>(3)</p>																					
2.3	$48 \leq x < 58$	<p>✓ answer / <i>antwoord</i></p> <p>(1)</p>																					
2.4	$60 - 49 = 11$ senior citizens / <i>senior burgers</i>	<p>✓ 49</p> <p>✓ answer / <i>antwoord</i></p> <p>(2)</p>																					
2.5	<p>$Q_1 = 39$</p> <p>$Q_2 = 50$</p> <p>$Q_3 = 58$</p>	<p>✓ value of / <i>waarde van</i> Q_1</p> <p>✓ value of / <i>waarde van</i> Q_2</p> <p>✓ value of / <i>waarde van</i> Q_3</p> <p>(3)</p>																					
2.6		<p>✓ minimum and maximum <i>minimum en maksimum</i></p> <p>✓ box / <i>boks</i></p> <p>(2)</p>																					
		[13]																					

QUESTION 3/VRAAG 3

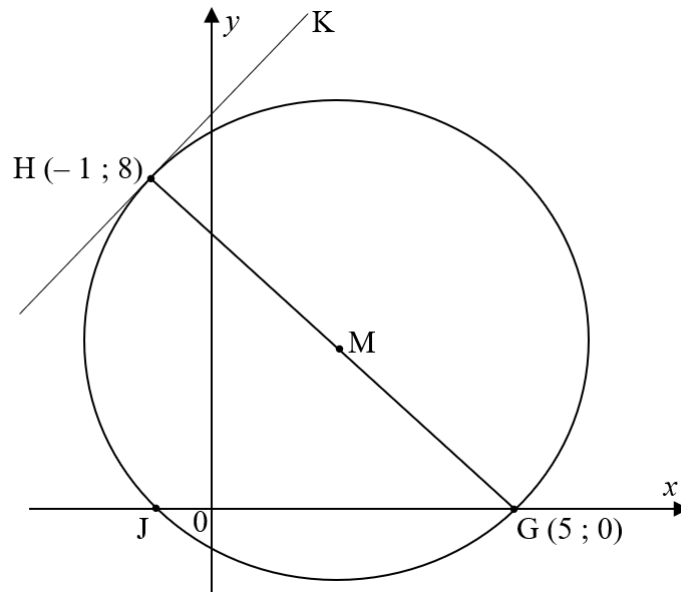


3.1.1	$KL = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(10 - 0)^2 + (2 - 7)^2}$ $= \sqrt{125} = 5\sqrt{5}$	✓ substitution / <i>vervanging</i> ✓ answer / <i>antwoord</i>	(2)
3.1.2	$m_{KM} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-4 - 7}{7 - 0}$ $= -\frac{11}{7}$	✓ substitution / <i>vervanging</i> ✓ gradient of KM / <i>gradiënt van KM</i>	(2)
3.1.3	$m_{LM} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-4 - 2}{7 - 10}$ $= 2$ $\tan \alpha = 2$ $\therefore \alpha = 63,43^\circ$	✓ gradient of LM / <i>gradiënt van LM</i> ✓ $\tan \alpha = 2$ ✓ value of α / <i>waarde van α</i>	(3)
3.1.4	$\tan \theta = -\frac{11}{7}$ $\text{Ref } \angle = 57,53^\circ$ $\therefore \theta = 122,47^\circ$ $\hat{\text{LMK}} = 122,47^\circ - 63,43^\circ$ $= 59,04^\circ$	✓ $\tan \theta = -\frac{11}{7}$ ✓ reference angle / <i>verwysingshoek</i> ✓ value of θ / <i>waarde van θ</i> ✓ value of $\hat{\text{LMK}}$ / <i>waarde van $\hat{\text{LMK}}$</i>	(4)

<p>3.2</p> $m_{KN} = m_{LM}$ $\frac{y-7}{x-0} = 2$ $y = 2x+7$ $m_{MN} = m_{KL}$ $\frac{y+4}{x-7} = -\frac{1}{2}$ $y = -\frac{x}{2} - \frac{1}{2}$ $2x+7 = -\frac{x}{2} - \frac{1}{2}$ $\therefore y = -\frac{x}{2} + \frac{13}{2}$ $4x+14 = -x-1$ $x = -3$ $y = 1$ <p style="text-align: center;">OR / OF</p> $m_{KL} = \frac{-5}{10}$ $m_{NM} = \frac{-5}{10}$ <p>Hence the coordinates of/ <i>Vervolgens die koördinate van N(-3;1)</i></p> <p style="text-align: center;">OR/OF</p> <p>Midpoint of KM / <i>Middelpunt van KM</i> = $\left(\frac{7}{2}, \frac{3}{2}\right)$</p> <p>Midpoint of LN / <i>Middelpunt van LN</i> = $\left(\frac{7}{2}, \frac{3}{2}\right)$</p> $\therefore \frac{x+10}{2} = \frac{7}{2} \text{ and/en } \frac{y+2}{2} = \frac{3}{2}$ $\therefore N(-3;1)$	<p>✓ $y = 2x+7$</p> <p>✓ $y = -\frac{x}{2} - \frac{1}{2}$</p> <p>✓ value of x / <i>waarde van x</i></p> <p>✓ value of y / <i>waarde van y</i></p> <p style="text-align: center;">OR/OF</p> <p>✓ $m_{KL} = \frac{-5}{10}$</p> <p>✓ $m_{NM} = \frac{-5}{10}$</p> <p>✓ value of x / <i>waarde van x</i></p> <p>✓ value of y / <i>waarde van y</i></p> <p style="text-align: center;">OR/OF</p> <p>✓ Midpoint of KM / <i>Middelpunt van KM</i> = $\left(\frac{7}{2}, \frac{3}{2}\right)$</p> <p>✓ Midpoint of LN / <i>Middelpunt van LN</i> = $\left(\frac{7}{2}, \frac{3}{2}\right)$</p> <p>✓ value of x / <i>waarde van x</i></p> <p>✓ value of y / <i>waarde van y</i></p> <p style="text-align: right;">(4)</p>
---	---

3.3	$m_{LM} \times m_{MN} = 2 \times \left(-\frac{1}{2}\right)$ $= -1$ $\therefore \hat{LMN} = 90^\circ$ <p>OR/OF \hat{LMN} is a right angle / is 'n regtehoek</p>	<p>✓ product of gradients / produk van gradiënte</p> <p>✓ conclusion / gevolgtrekking</p> <p style="text-align: right;">(2)</p>
3.4	$KL = NM = 5\sqrt{5}$ $KM = \sqrt{7^2 + 11^2}$ $= \sqrt{170}$ $\hat{LMN} = 90^\circ \text{ and / en } \hat{LMK} = 59,04^\circ$ $\therefore \hat{KMN} = 90^\circ - 59,04^\circ = 30,96^\circ$ $\text{Area of } \Delta KMN = \frac{1}{2} \times \sqrt{170} \times 5\sqrt{5} \times \sin 30,96^\circ$ $= 37,50 \text{ square units / vierkante eenhede}$	<p>✓ $KL = NM = 5\sqrt{5}$</p> <p>✓ $KM = \sqrt{170}$</p> <p>✓ $\hat{LMN} = 90^\circ$ and / en $\hat{LMK} = 59,04^\circ$</p> <p>✓ $\hat{KMN} = 30,96^\circ$</p> <p>✓ Area of ΔKMN</p> <p style="text-align: right;">(5)</p>
		[22]

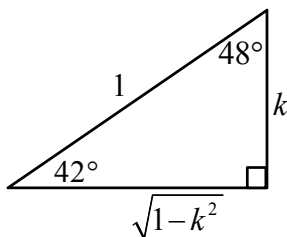
QUESTION 4/VRAAG 4

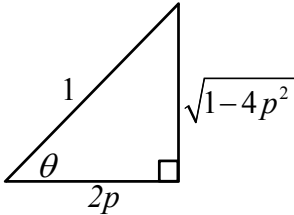


4.1	$M(2; 4)$	✓ value of x / waarde van x ✓ value of y / waarde van y	(2)
4.2	$r^2 = (5-2)^2 + (0-4)^2$ $= 25$ $\therefore (x-2)^2 + (y-4)^2 = 25$	✓ $(x-2)^2$ ✓ $(y-4)^2$ ✓ 25	(3)
4.3	$m_{GH} = \frac{8-0}{-1-5}$ $= -\frac{8}{6} = -\frac{4}{3}$ $m_{\text{tan}} = \frac{3}{4}$ $y-8 = \frac{3}{4}(x+1)$ $\therefore y = \frac{3}{4}x + \frac{35}{4}$	✓ m_{GH} ✓ m_{tan} ✓ substitution / vervanging ✓ equation / vergelyking	(4)
4.4	At/By J, $y=0$ $(x-2)^2 + (0-4)^2 = 25$ $(x-2)^2 = 9$ $x-2 = \pm 3$ $x=5$ or/of $x=-1$ $\therefore J(-1; 0)$	✓ $y=0$ ✓ substitution / vervanging ✓ $x=-1$	(3)

4.5	<p>HJG is a right angled triangle (8, 6 and 10). So the rotation of J around M will complete a rectangle Hence, $j((-1 + 6; 0 + 8)) = J^l(5; 8)$</p> <p><i>HJG is 'n reghoekige driehoek (8, 6 en 10)</i> <i>Dus sal die rotasie van J om M die reghoek voltooi.</i> <i>Vervolgens is $J^l((-1 + 6; 0 + 8)) = J^l(5; 8)$</i></p>	<p>✓ value of x / <i>waarde van x</i> ✓ value of y / <i>waarde van y</i></p> <p style="text-align: right;">(2)</p>
4.6	<p>$x^2 + y^2 - 12x - 2y + 17 = 0$ $x^2 - 12x + y^2 - 2y = -17$ $x^2 - 12x + 36 + y^2 - 2y + 1 = -17 + 36 + 1$ $(x - 6)^2 + (y - 1)^2 = 20$</p> <p>Distance between the centres: <i>Afstand tussen die middelpunte:</i></p> <p>$\sqrt{(2 - 6)^2 + (4 - 1)^2} = 5$</p> <p>$\therefore$ the centre lies on the original circle / <i>die middelpunt lê op die omtrek van die oorspronklike sirkel</i></p>	<p>✓ completing the square / <i>voltooiing van die vierkant</i></p> <p>✓ factorisation/<i>faktorisering: x</i> ✓ factorisation/<i>faktorisering: y</i></p> <p>✓ distance formula / <i>afstand formule</i></p> <p>✓ conclusion / <i>gevolgtrekking</i></p> <p style="text-align: right;">(5)</p>
		[19]

QUESTION 5/VRAAG 5

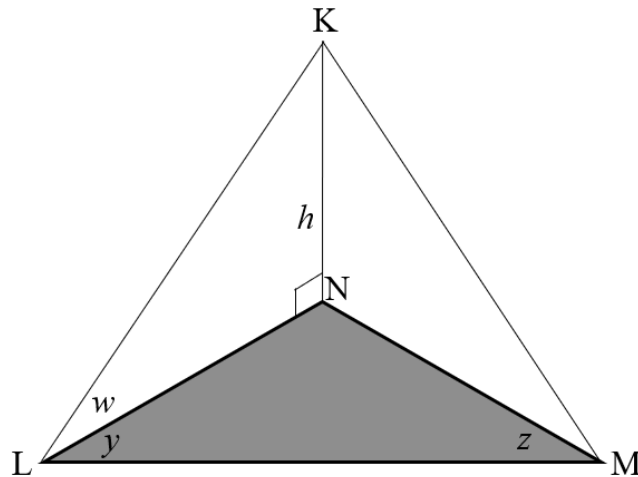
<p>5.1.1</p>	$\sin 42^\circ = \frac{k}{1}$ $\tan 42^\circ = \frac{k}{\sqrt{1-k^2}}$ 	$\checkmark \sqrt{1-k^2}$ $\checkmark \tan 42^\circ = \frac{k}{\sqrt{1-k^2}}$ <p style="text-align: right;">(2)</p>
<p>5.1.2</p>	$\sin 84^\circ = \sin 2 \times 42^\circ$ $= 2 \sin 42^\circ \cos 42^\circ$ $= 2 \cdot k \cdot \sqrt{1-k^2} = 2k \sqrt{1-k^2}$	$\checkmark \text{double angle/dubbelhoek}$ $\checkmark \text{expansion/identiteit/ontwikkeling}$ $\checkmark \text{substitution/vervang}$ <p style="text-align: right;">(3)</p>
<p>5.1.3</p>	$\sin 3^\circ = \sin(45^\circ - 42^\circ)$ $= \sin 45^\circ \cos 42^\circ - \cos 45^\circ \sin 42^\circ$ $= \frac{\sqrt{2}}{2} \cdot \sqrt{1-k^2} - \frac{\sqrt{2}}{2} \cdot k$	$\checkmark 3^\circ = 45^\circ - 42^\circ$ $\checkmark \text{expansion/identiteit/ontwikkeling}$ $\checkmark \text{substitution/vervang}$ $\checkmark \text{substitution/vervang}$ <p style="text-align: right;">(4)</p>
<p>5.2</p>	$\frac{\sin(x-45^\circ) \cdot \tan(180^\circ+x) \cdot \sin(90^\circ-x)}{\cos(-x)}$ $\frac{-\cos x \cdot \tan x \cdot \cos x}{\cos x}$ $-\cos x \cdot \frac{\sin x}{\cos x}$ $-\sin x$	$\checkmark -\cos x$ $\checkmark \tan x$ $\checkmark \cos x$ $\checkmark \cos x$ $\checkmark \frac{\sin x}{\cos x}$ $\checkmark \text{answer / antwoord}$ <p style="text-align: right;">(6)</p>
<p>5.3.1</p>	$\cos(A+B) = \cos A \cos B - \sin A \sin B$	$\checkmark \text{expansion/identiteit/ontwikkeling}$ <p style="text-align: right;">(1)</p>
<p>5.3.2</p>	$\text{LHS/LK} = \cos 3\alpha$ $= \cos(2\alpha + \alpha)$ $= \cos 2\alpha \cos \alpha - \sin 2\alpha \sin \alpha$ $= (2 \cos^2 \alpha - 1) \cdot \cos \alpha - 2 \sin \alpha \cos \alpha \cdot \sin \alpha$ $= 2 \cos^3 \alpha - \cos \alpha - 2 \sin^2 \alpha \cos \alpha$ $= 2 \cos^3 \alpha - \cos \alpha - 2(1 - \cos^2 \alpha) \cos \alpha$ $= 2 \cos^3 \alpha - \cos \alpha - 2 \cos \alpha + 2 \cos^3 \alpha$ $= 4 \cos^3 \alpha - 3 \cos \alpha$ $= \text{RHS/RK}$	$\checkmark \text{compound angle identity}$ $\text{saamgesteldehoek-identiteit}$ $\checkmark \text{cos double angle identity}$ $\text{cos dubbelhoek-identiteit}$ $\checkmark \text{sin double angle identity}$ $\text{sin dubbelhoek-identiteit}$ $\checkmark (1 - \cos^2 \alpha)$ <p style="text-align: right;">(4)</p>

5.4	$\cos \theta = \frac{2p}{1}$  $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $7p = (2p)^2 - (\sqrt{1-4p^2})^2$ $7p = 4p^2 - 1 + 4p^2$ $8p^2 - 7p - 1 = 0$ $(8p+1)(p-1) = 0$ $\therefore p = -\frac{1}{8} \text{ or / of } p = 1$ <p style="text-align: center;">OR/OF</p> $\cos 2\theta = 2\cos^2 \theta - 1$ $7p = 2 \cdot (2p)^2 - 1$ $7p = 8p^2 - 1$ $8p^2 - 7p - 1 = 0$ $(8p+1)(p-1) = 0$ $\therefore p = -\frac{1}{8} \text{ or / of } p = 1$ <p style="text-align: center;">OR/OF</p> $\cos 2\theta = 1 - 2\sin^2 \theta$ $7p = 1 - 2 \cdot (\sqrt{1-4p^2})^2$ $7p = 1 - 2(1 - 4p^2)$ $7p = 1 - 2 + 8p^2$ $8p^2 - 7p - 1 = 0$ $(8p+1)(p-1) = 0$ $\therefore p = -\frac{1}{8} \text{ or / of } p = 1$	$\checkmark \sqrt{1-4p^2}$ $\checkmark \cos 2\theta = \cos^2 \theta - \sin^2 \theta$ \checkmark substitution/ <i>vervanging</i> \checkmark standard form <i>standaardvorm</i> \checkmark values of p / <i>waardes van p</i> <p style="text-align: center;">OR/OF</p> $\checkmark 2\cos^2 \theta - 1$ $\checkmark \checkmark$ substitution/ <i>vervanging</i> \checkmark standard form / <i>standaardvorm</i> \checkmark values of p / <i>waardes van p</i> <p style="text-align: center;">OR/OF</p> $\checkmark \sqrt{1-4p^2}$ $\checkmark 1 - 2\sin^2 \theta$ \checkmark substitution / <i>vervanging</i> \checkmark standard form <i>standaardvorm</i> \checkmark values of p / <i>waardes van p</i> <p style="text-align: right;">(5)</p> <p style="text-align: right;">[25]</p>
-----	--	--

QUESTION 6/VRAAG 6

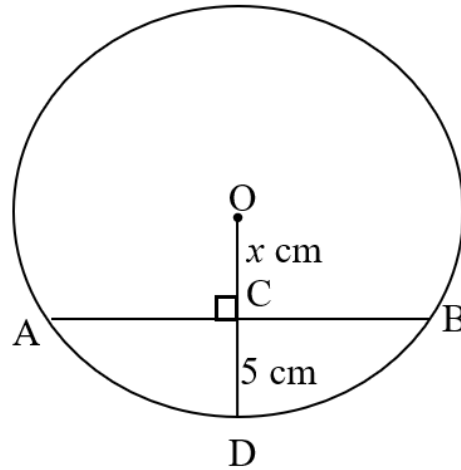
6.1	$y \in [-1; 1]$ OR/OF $-1 \leq y \leq 1$	✓ answer / <i>antwoord</i> (1)
6.2		<i>g</i> : ✓ asymptotes at -90° and 90° <i>asymptote vir -90° en 90°</i> ✓ <i>x</i> -intercepts <i>x-afsnitte</i> ✓ shape / <i>vorm</i> (3)
6.3	180°	✓ 180° (1)
6.4	$x = -45^\circ$	✓ -45° (1)
6.5	$x \in [45^\circ; 90^\circ)$ OR/OF $45^\circ \leq x < 90^\circ$	✓ critical values <i>kritiese waardes</i> ✓ notation / <i>notasie</i> (2)
6.6	$h(x) = \sin(x - 45^\circ) + 1$	✓ $h(x) = \sin(x - 45^\circ) + 1$ (1)
		[9]

QUESTION 7/VRAAG 7



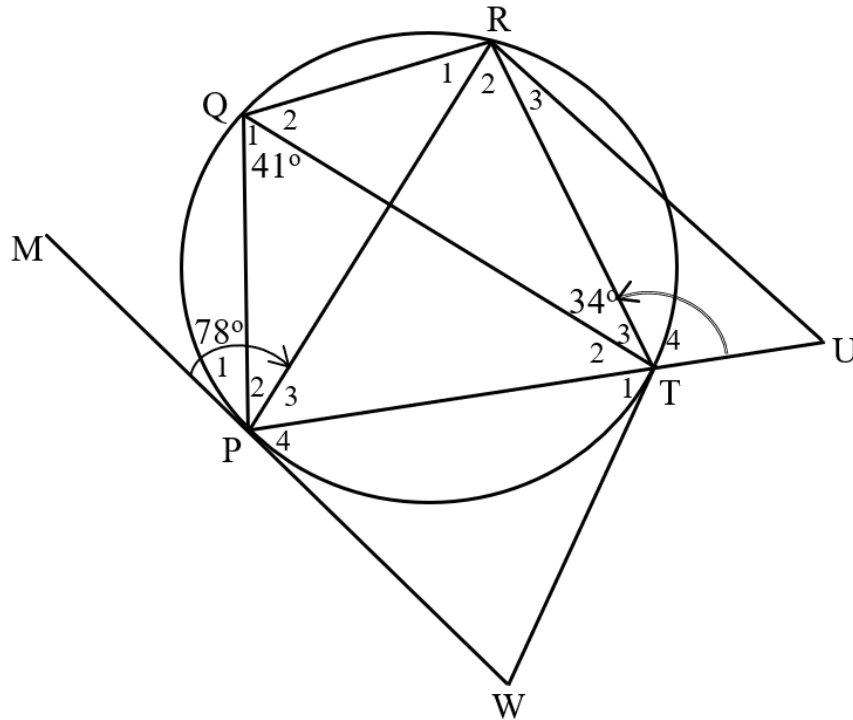
7.1	In $\triangle KLN$: $\tan w = \frac{h}{LN}$ $LN = \frac{h}{\tan w}$	$\checkmark LN = \frac{h}{\tan w}$ (1)
7.2	In $\triangle NLM$ $\frac{LM}{\sin \hat{N}} = \frac{LN}{\sin \hat{M}}$ $\frac{LM}{\sin (180^\circ - (y+z))} = \frac{LN}{\sin z}$ $\therefore LM = \frac{LN \cdot \sin (y+z)}{\sin z}$ But $LN = \frac{h}{\tan w}$ $\therefore LM = \frac{h \sin (y+z)}{\tan w \sin z}$	\checkmark correct sine rule <i>korrekte sinusreël</i> \checkmark substitution <i>vervanging</i> \checkmark isolating LM <i>isoleer LM</i> \checkmark answer / <i>antwoord</i> (4)
7.3	$LM = \frac{h \sin (y+z)}{\tan w \sin z} \text{ and/en}$ $h = 38 \text{ m}, w = 21^\circ, y = 52^\circ \text{ and/en } z = 59^\circ$ $\therefore LM = \frac{38 \cdot \sin (52^\circ + 59^\circ)}{\tan 21^\circ \sin 59^\circ}$ $= 107,82 \text{ m}$	\checkmark substitution <i>vervanging</i> \checkmark answer / <i>antwoord</i> (2)
		[7]

QUESTION 8/VRAAG 8



8.1	the centre of a circle / <i>die middelpunt van die sirkel</i>	✓ answer / <i>antwoord</i> (1)
8.2.1	AC = 10 cm (line from centre \perp chord) (<i>lyn vanaf die middelpunt \perp op koord</i>)	✓ length of AC <i>lengte van AC</i> ✓ Reason/ <i>rede</i> (2)
8.2.2	$(x+5)^2 = 10^2 + x^2$ $x^2 + 10x + 25 = 100 + x^2$ $10x = 75$ $\therefore x = 7,5 \text{ cm}$ $\therefore \text{radius} = 7,5 \text{ cm} + 5 \text{ cm}$ $= 12,5 \text{ cm}$	✓ radius = $(x + 5)$ ✓ applying Pythagoras theorem / <i>toepassing van Pythagoras se stelling</i> ✓ value of x / <i>waarde van x</i> ✓ length of radius / <i>lengte van radius</i> (4)
		[7]

QUESTION 9/VRAAG 9

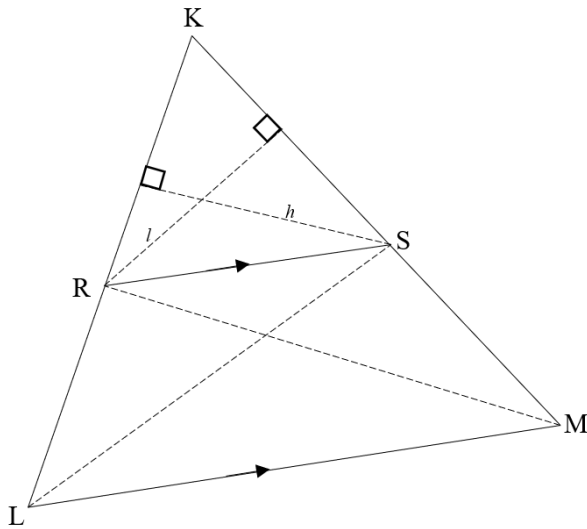


9.1	interior opposite angle / teenoorstaande binnehoek	✓ answer / antwoord (1)
9.2.1	$\hat{R}_2 = \hat{Q}_2 = 41^\circ$ (\angle s in the same seg)/(\angle e in dieselfde seg.) $\hat{P}_4 = \hat{Q}_1 = 41^\circ$ (tan-chord theorem)/(raaklyn-koord stelling) $\hat{T}_1 = \hat{P}_4 = 41^\circ$ (\angle s opp. = sides)/(\angle e teenoor gelyke sye) OR/OF $\hat{T}_1 = \hat{R}_2 = 41^\circ$ (tan – chord theorem)/(raaklyn-koord stelling)	✓ Statement / bewering ✓ Reason / rede ✓ Statement / bewering ✓ Reason / rede ✓ Statement / bewering ✓ Reason / rede (6)
9.2.2(a)	$\hat{T}_2 + 34^\circ = 78^\circ$ (tan – chord theorem)/(raaklyn-koord stelling) $\therefore \hat{T}_2 = 44^\circ$	✓ Statement / bewering ✓ Reason / rede (2)
9.2.2(b)	$41^\circ + \hat{Q}_2 + 44^\circ + 34^\circ = 180^\circ$ (opp. \angle s of a cyclic quad.) $\therefore \hat{Q}_2 = 61^\circ$ (teenorst. \angle evan'n koordevierhoek)	✓ Statement / bewering ✓ Reason / rede (2)
9.2.2(c)	$\hat{T}_4 = 41^\circ + 61^\circ$ (ext. \angle s of a cyclic quad.) $\therefore \hat{T}_4 = 102^\circ$ (buite \angle van koordevierhoek) OR/OF $\hat{T}_4 + 44^\circ + 34^\circ = 180^\circ$ (int. \angle s of a Δ) $\therefore \hat{T}_4 = 102^\circ$ (binne \angle evan'n Δ)	✓ Statement / bewering ✓ Reason / rede OR/OF ✓ Statement / bewering ✓ Reason / rede (2)

9.2.2(d)	$\hat{W} + 41^\circ + 41^\circ = 180^\circ$ (int. \angle s of a Δ)/(binne \angle e van Δ) $\therefore \hat{W} = 98^\circ$	✓ Statement /bewering ✓ Reason / rede (2)
9.2.3(a)	$\hat{Q}_2 = 61^\circ$ and/en $\hat{T}_2 = 44^\circ$ $\therefore \hat{Q}_2 \neq \hat{T}_2$ \therefore QR is not parallel to PT (alt. \angle s are not equal) <i>QR is nie ewewydig aan PT nie (verw. \anglee is nie gelyk nie)</i>	✓ $\hat{Q}_2 \neq \hat{T}_2$ ✓ alt. \angle s are not equal <i>verw. \anglee is nie gelyk nie</i> (2)
9.2.3(b)	$\hat{R}_2 + \hat{W} = 41^\circ + 98^\circ$ $= 139^\circ \neq 180^\circ$ \therefore PRTW is not a cyclic quad. (Opp. \angle s are not supp.) <i>PRTW is nie 'n koordevierhoek nie</i> <i>(teenoorst. \anglee is nie supplementêr nie)</i>	✓ $\hat{R}_2 + \hat{W} \neq 180^\circ$ ✓ PRTW is not a cyclic quad. <i>PRTW is nie 'n koordevierhoek nie</i> (2)
9.2.3(c)	$\hat{R}_1 = \hat{T}_2 = 44^\circ$ (\angle s in same seg)/(\angle e in dieselfde segment) $\hat{R}_1 + \hat{R}_2 = 44^\circ + 41^\circ$ $= 95^\circ \neq 90^\circ$ \therefore TQ is not a diameter (\angle subt. by TQ is not a right angle) \therefore <i>TQ is nie 'n middellyn nie (Zonder span deur TQ is nie 'n reghoek nie)</i>	✓ $\hat{R}_1 + \hat{R}_2 \neq 90^\circ$ ✓ TQ is not a diameter <i>TQ is nie 'n middellyn nie</i> (2)
		[21]

QUESTION 10/VRAAG 10

10.1



Construction: Draw heights h and l on KR and KS respectively. Join LS and MR

Konstruksie: Teken hoogtes h en l op KR en KS onderskeidelik. Verbind LS en MR

Proof/Bewys:

$$\frac{\text{Area of } \triangle KRS}{\text{Area of } \triangle LRS} = \frac{\frac{1}{2} \cdot KR \cdot h}{\frac{1}{2} \cdot RL \cdot h} = \frac{KR}{RL}$$

$$\frac{\text{Area of } \triangle KRS}{\text{Area of } \triangle MSR} = \frac{\frac{1}{2} \cdot KS \cdot l}{\frac{1}{2} \cdot SM \cdot l} = \frac{KS}{SM}$$

But/*Maar*: area of/*van* $\triangle LRS$ = area of/*van* $\triangle MSR$

$$\frac{\text{Area of } \triangle KRS}{\text{Area of } \triangle LRS} = \frac{\text{Area of } \triangle KRS}{\text{Area of } \triangle MSR}$$

$$\therefore \frac{KR}{RL} = \frac{KS}{SM}$$

✓ construction / *konstruksie*

✓ ratio of areas /
verhouding van oppervlaktes

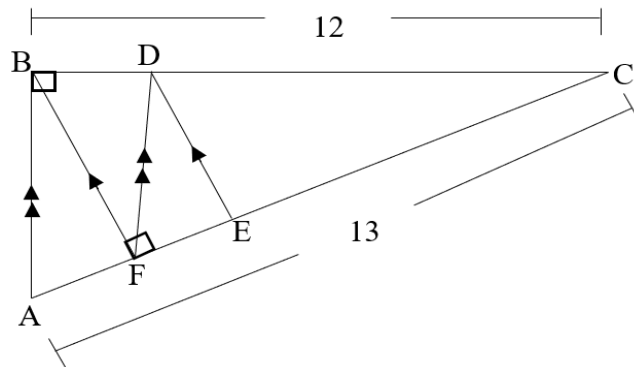
✓ ratio of areas
verhouding van oppervlaktes

✓ same base and same height
dieselfde basis en hoogte

✓ $\frac{\text{Area of } \triangle KRS}{\text{Area of } \triangle LRS} = \frac{\text{Area of } \triangle KRS}{\text{Area of } \triangle MSR}$

(5)

10.2



10.2.1	AB = 5 units	✓5 (1)
10.2.2(a)	\hat{C} is common / <i>is gemeen</i> $\hat{CBA} = \hat{CFB}$ (both/beide = 90°) $\hat{CAB} = \hat{CBF}$ (sum of \angle s of Δ)/(som van die \angle e van Δ) $\therefore \Delta CBA \parallel \Delta CFB$ (\angle, \angle, \angle)	✓ Statement/bewering/ Reason / rede ✓ Statement/bewering/ Reason / rede ✓ Reason/Rede (3)
10.2.2(b)	$\frac{CB}{CF} = \frac{CA}{CB}$ ($\parallel \Delta$ s) $CB^2 = CF \cdot CA$ $\therefore CF = \frac{CB^2}{CA}$	✓ proportion / verhouding ✓ reason / rede ✓ $CB^2 = CF \cdot CA$ (3)
10.2.3	$CF = \frac{CB^2}{CA}$ $CF = \frac{(12)^2}{13}$ ≈ 11 units/eenhede	✓ substitution / vervanging ✓ length of CF / lengte van CF (2)
10.2.4	AF = 13 – 11 = 2 units/eenhede	✓ length of AF / lengte van AF (1)
10.2.5	$\frac{CB}{BD} = \frac{CA}{AF}$ (prop. theorem/verh. stelling; $DF \parallel BA$) $\frac{12}{BD} = \frac{13}{2}$ $\therefore BD = \frac{24}{13}$ $\frac{CF}{FE} = \frac{CB}{BD}$ (prop. theorem/verh. stelling; $DF \parallel BA$) $\frac{11}{FE} = \frac{12}{\frac{24}{13}}$ $\therefore FE = \frac{22}{13}$ units/eenhede	✓ proportion / verhouding ✓ reason / rede ✓ length of BD / lengte van BD ✓ proportion / verhouding ✓ length of FE / lengte van FE (5)
		[20]
		TOTAL/TOTAAL: 150

