Province of the
EASTERN CAPE
EDUCATION

NASIONALE SENIOR SERTIFIKAAT

GRAAD 12

JUNIE 2019

WISKUNDE V2

PUNTE: **150**

TYD: **3 uur**

* J M A T H A 2 *

Hierdie vraestel bestaan uit 14 bladsye, insluitend ’n inligtingsblad en ’n spesiale antwoordeboek van 19 bladsye.

INSTRUKSIES EN INLIGTING

Lees die volgende instruksies sorgvuldig deur voordat jy die vrae beantwoord.

1. Hierdie vraestel bestaan uit TIEN (10) vrae.

2. Beantwoord AL die vrae in die ANTWOORDEBOEK wat voorsien is.

3. Toon duidelik ALLE berekeninge, diagramme, grafieke, ensovoorts wat jy gebruik het om jou antwoorde te bepaal.

4. Antwoorde alleen sal NIE noodwendig volpunte verdien NIE.

5. Jy mag ’n goedgekeurde wetenskaplike sakrekenaar (nieprogrammeerbaar en niegrafies) gebruik, tensy anders vermeld.

6. Indien nodig, rond antwoorde af tot TWEE desimale plekke, tensy anders vermeld.

7. Diagramme is NIE noodwendig volgens skaal geteken NIE.

8. ’n Inligtingsblad met formules is aan die einde van die vraestel ingesluit.

9. Skryf netjies en leesbaar.

VRAAG 1

Eastern Hoërskool het die Kwartaal 1 persentasies van 20 graad 12 leerlinge, wat uit 10 seuns en 10 meisies bestaan, vergelyk. Die volgende data is aangeteken:

Seuns se punte	41	30	24	65	72	15	83	52	60	38

Meisies se gemiddelde punt = 51 Meisies se standaardafwyking = 15,95

1.1 Bereken die gemiddelde punt vir die seuns. (1)

1.2 Bereken die standaardafwyking vir die seuns se punte. (2)

1.3 Het die seuns of die meisies beter presteer? Gee ’n rede vir jou antwoord. (2)

1.4 Met watter persentasie moet elk van die seuns se punte aangepas word sodat die gemiddelde van die seuns dieselfde as die van die meisies kan wees? (1)

1.5 Sal die standaardafwyking van die seuns se punte vermeerder, verminder of dieselfde bly na die aanpassing in VRAAG 1.4 hierbo? (1)
[7]

VRAAG 2

Die ouderdomme van mense, wat by een stemlokaal geregistreer het om te stem, is in die frekwensietabel hieronder aangeteken.

Ouderdomme (in jare)	Frekwensie	Kumulatiewe Frekwensie
$18 \leq x < 28$		4
$28 \leq x < 38$		14
$38 \leq x < 48$		28
$48 \leq x < 58$	17	
$58 \leq x < 68$	12	
$68 \leq x < 78$	3	

2.1 Voltooi die frekwensietabel. (2)

2.2 Teken die kumulatiewefrekwensie-grafiek (ogief). (3)

2.3 Skryf die modale-klas neer. (1)

2.4 Mense wat 60 jaar en ouer is, word as senior burgers beskou, en hoef nie in ’n ry te staan nie maar word na voor in die ry geneem. Beraam die aantal senior burgers. (2)

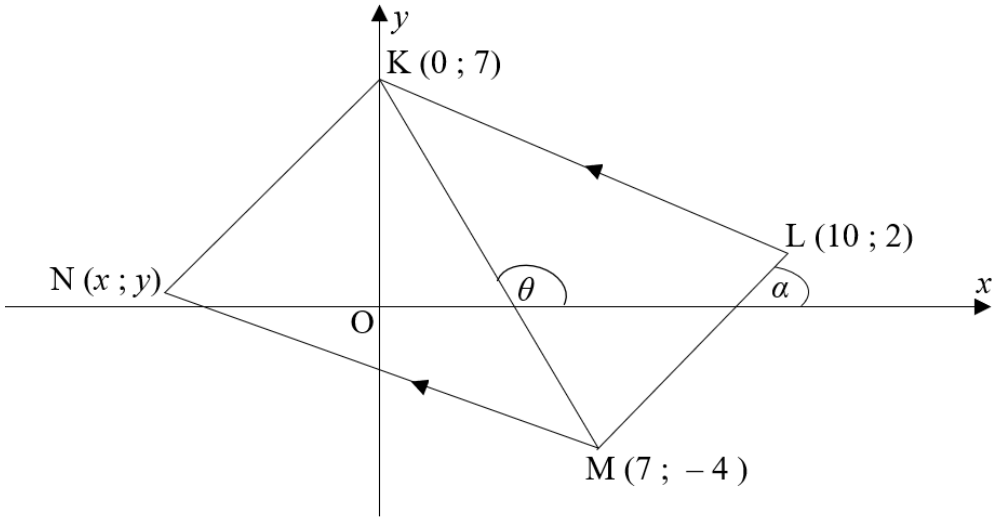
2.5 Skryf die onderste (Q_1), middel (Q_2) en boonste (Q_3) kwartiele neer. (3)

2.6 Teken ’n mond-en-snor-diagram om die ouderdomme van die kiesers voor te stel. (2)

[13]

VRAAG 3

In die diagram hieronder is K(0 ; 7), L(10 ; 2), M(7 ; -4) en N(x ; y) die hoekpunte van vierhoek KLMN, met MN || KL. θ en α is die hoeke wat deur KM en ML onderskeidelik met die x-as gevorm word.



3.1 Bepaal:

3.1.1 Die lengte van KL. Laat jou antwoord in eenvoudigste wortelvorm (2)

3.1.2 Die gradiënt van KM (2)

3.1.3 Die grootte van α, die inklinasiehoek van LM (3)

3.1.4 Die grootte van $L\hat{M}K$ (4)

3.2 Bepaal die koördinate van N as KLMN 'n parallelogram is. Toon ALLE berekeninge. (4)

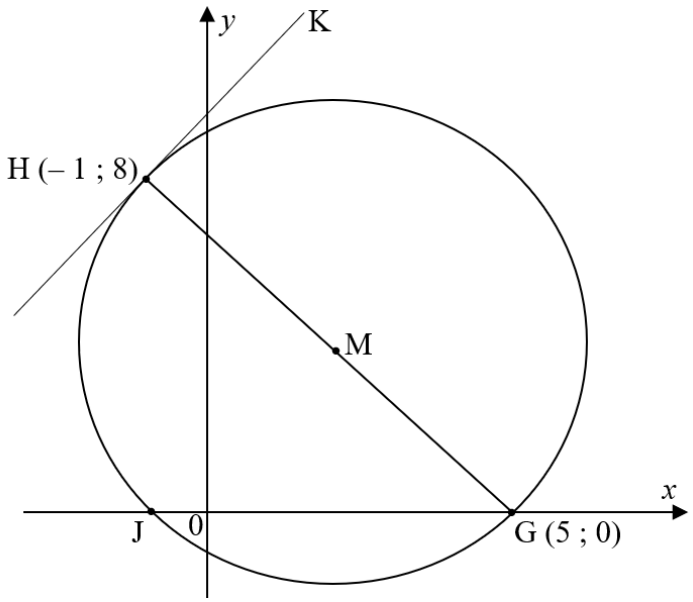
3.3 Is $L\hat{M}N$ 'n regtehoek of nie? Ondersteun jou antwoord met berekeninge. (2)

3.4 Bepaal die oppervlakte van Δ KNM. (5)

[22]

VRAAG 4

In die diagram hieronder is sirkel met middelpunt M, middellyn GH met G(5 ; 0) en raaklyn HK met raakpunt by H(– 1 ; 8) gegee.



4.1 Skryf die koördinate van M neer. (2)

4.2 Bepaal die vergelyking van die sirkel in die vorm $(x-a)^2+(y-b)^2=r^2$ (3)

4.3 Bepaal die vergelyking van die raaklyn HK. (4)

4.4 Bepaal die koördinate van J. (3)

4.5 Vind die nuwe koördinate van J as die sirkel 180° om die middelpunt M roteer word. (2)

4.6 Die vergelyking van 'n ander sirkel word gegee as $x^2+y^2-12x-2y+17=0$.
Lê die middelpunt van die nuwe sirkel op, binne of buite die omtrek van die oorspronklike sirkel? Ondersteun jou antwoord met toepaslike berekeninge. (5)
[19]

VRAAG 5

5.1 As $\sin 42° = k$, bepaal die volgende in terme van k.

5.1.1 $\tan 42°$ (2)

5.1.2 $\sin 84°$ (3)

5.1.3 $\sin 3°$ (4)

5.2 Vereenvoudig tot 'n enkel trigonometriese verhouding:

$$\frac{\sin(x-450°).\tan(180°+x).\sin(90°-x)}{\cos(-x)}$$ (6)

5.3 Beskou die identiteit: $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

5.3.1 Voltooi: cos (A + B) = … (1)

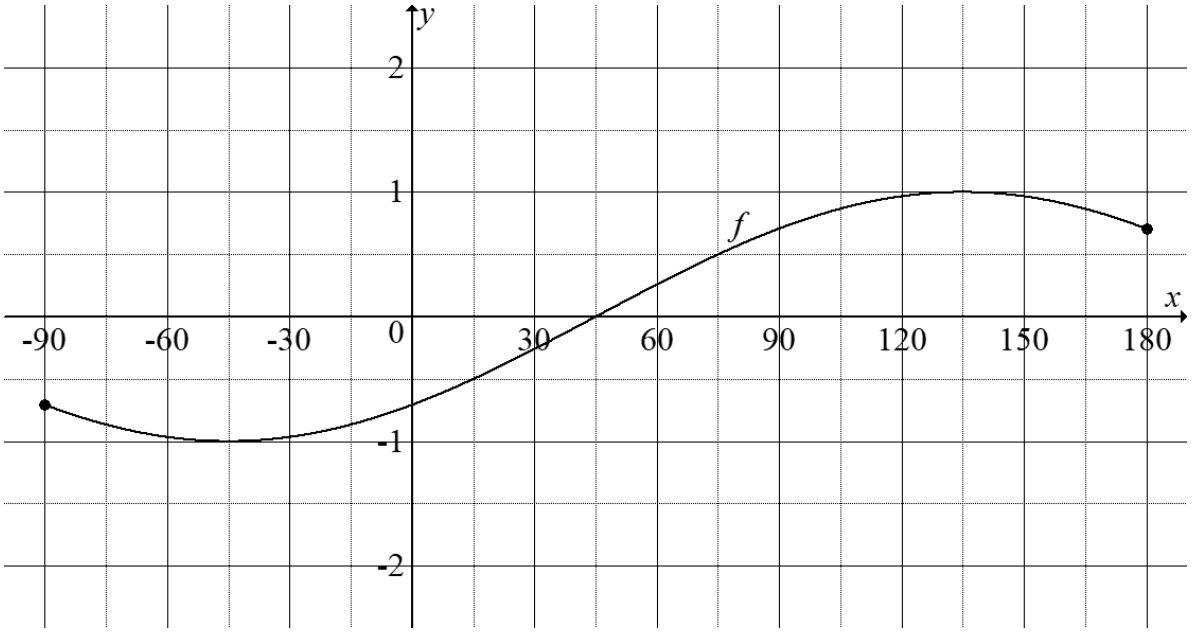
5.3.2 Bewys die identiteit: $cos\,3\theta = 4\,cos^3\theta - 3\,cos\,\theta$ (4)

5.4 As $\cos\theta = 2p$ en $\cos 2\theta = 7p$, bepaal die moontlike waarde(s) van p. (5)

[25]

VRAAG 6

Gegee hieronder is die grafiek van $f(x) = \sin(x - 45°)$, vir $x \in [-90° ; 180°]$.



6.1 Skryf die waardeversameling/terrein van f neer. (1)

6.2 Skets, op dieselfde assestelsel, die grafiek van $g(x) = \tan x$, vir $x \in [-90° ; 180°]$ in die SPESIALE ANTWOORDEBOEK. Toon ALLE afsnitte met die asse sowel as die asimptote en die eindpunte. (3)

6.3 Skryf die periode van g neer. (1)

6.4 Skryf die waarde(s) van x neer waarvoor $f(x) = g(x)$, vir $x \in [-90° ; 180°]$. (1)

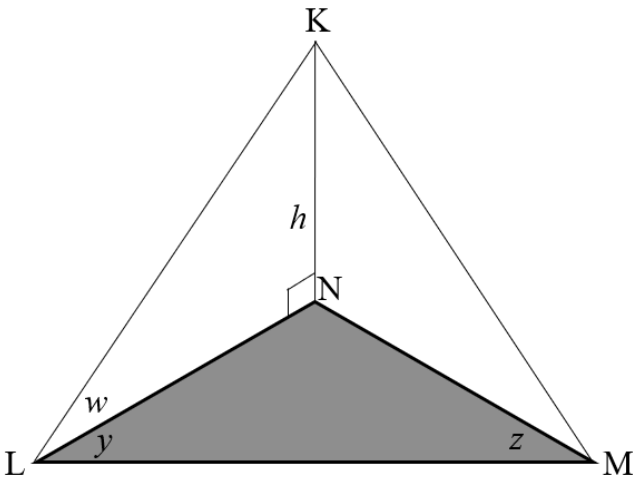
6.5 Vir watter waarde(s) van x is $f(x).g(x) \geq 0$, vir $x \in [0° ; 180°]$? (2)

6.6 Skryf die vergelyking van $h(x)$ neer, as $h(x)$ die gevolg is wanneer $f(x)$ geskuif word sodat dit 'n minimumwaarde van nul het. (1)

[**9**]

VRAAG 7

In die diagram stel KN 'n vertikale toring, met hoogte *h* meter, voor wat op 'n horisontale vlak LMN staan. Die hoogtehoek van K, soos gesien van L, is w. $\hat{NLM} = y$ en $\hat{NML} = z$. (**LET WEL**: alle hoeke word in grade gemeet).



7.1 Toon aan dat $\text{LN} = \dfrac{h}{\tan w}$ (1)

7.2 Bewys vervolgens dat $\text{LM} = \dfrac{h\sin(y+z)}{\tan w \sin z}$ (4)

7.3 Bereken LM as $h = 38\,\text{m}$, $w = 21^\circ$, $y = 52^\circ$ en $z = 59^\circ$ is. (2)

[7]

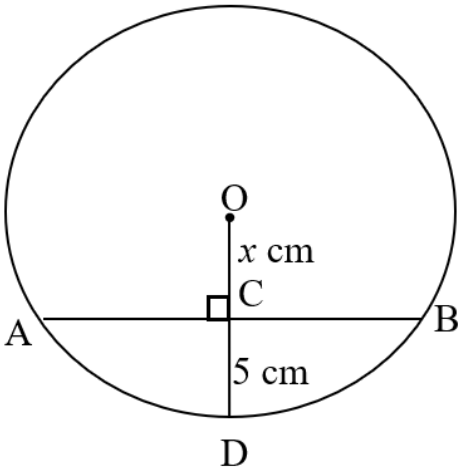
Gee redes vir jou bewerings in VRAE 8, 9 en 10.

VRAAG 8

8.1 Voltooi:
Die middelloodlyn van 'n koord gaan deur … (1)

8.2 In die diagram hieronder is O die middelpunt van die sirkel, AB 'n koord en OC ⊥ AB.

OC verleng, ontmoet die sirkel by D. AB = 20 cm, CD = 5 cm en OC = x cm.



Bepaal, meld redes:

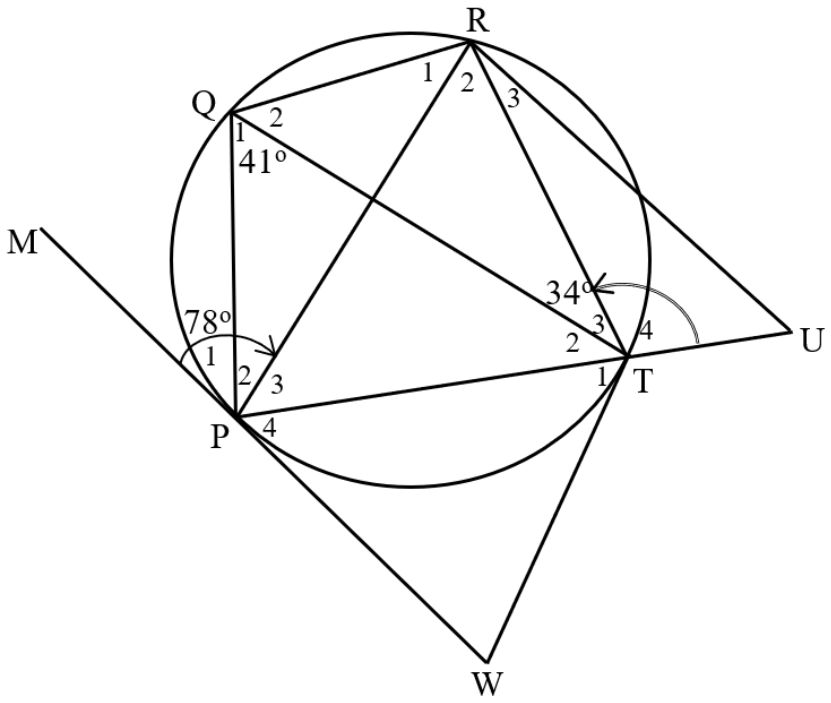
8.2.1 Die lengte van AC (2)

8.2.2 Die radius van die sirkel (4)
[7]

VRAAG 9

9.1 Voltooi:
Buitehoek van 'n koordevierhoek is gelyk aan … (1)

9.2 In die diagram hieronder lê punte P, Q, R en T op die omtrek van die sirkel. MW en TW is raaklyne aan die sirkel by P en T onderskeidelik. PT is verleng om RU by U te ontmoet. Verder is $\hat{MPR} = 78°$, $\hat{PQT} = 41°$ en $\hat{QTR} = 34°$.



9.2.1 Skryf neer, met redes, DRIE ander hoeke wat elk gelyk is aan 41°. (6)

9.2.2 Bepaal, met melding van redes, die volgende:

(a) $\hat{T}_2$ (2)

(b) $\hat{Q}_2$ (2)

(c) $\hat{T}_4$ (2)

(d) $\hat{W}$ (2)

9.2.3 Bepaal, met redes, of:

(a) QR || PT is of nie (2)

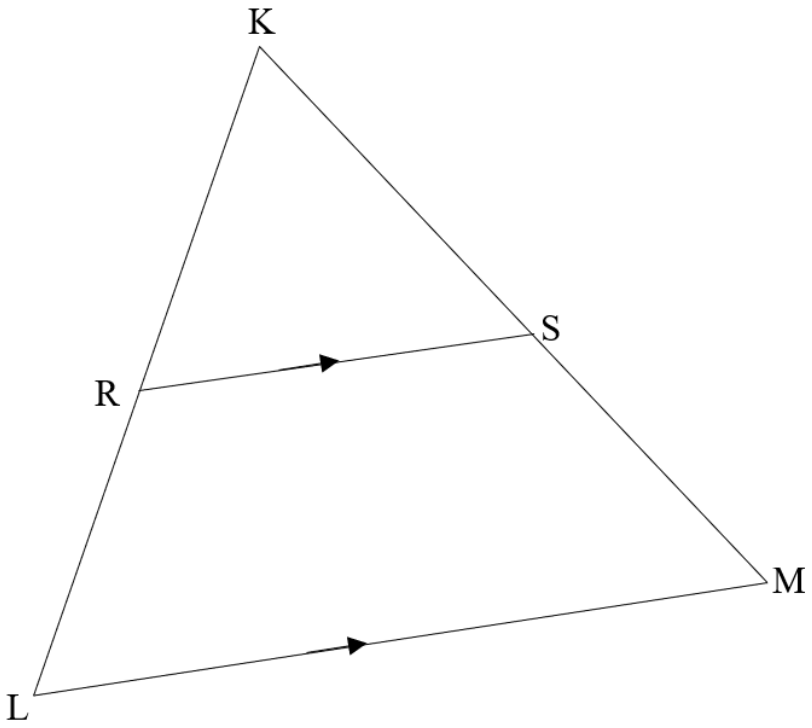
(b) PRTW 'n koordevierhoek is of nie (2)

(c) TQ 'n middellyn is of nie (2)

[21]

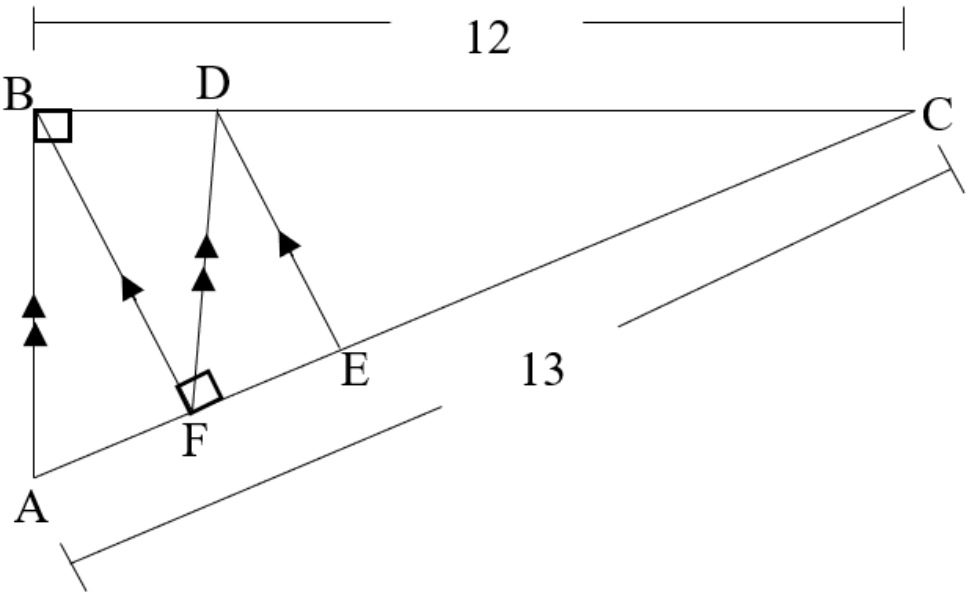
VRAAG 10

10.1 In die diagram hieronder is ΔKLM gegee met R en S op KL en KM onderskeidelik sodat RS || LM.



Bewys die stelling wat beweer dat $\frac{KR}{RL}=\frac{KS}{SM}$ (5)

10.2 In die diagram hieronder is ΔABC geteken met D op BC en F en E op AC sodat AB || FD, BF || DE, AB ⊥ BC en BF ⊥ CA. Verder is CA = 13 eenhede en CB = 12 eenhede.



10.2.1 Skryf die lengte van AB neer. (1)

10.2.2 Bewys, met redes, dat:

(a) $\Delta CBA \;|||\; \Delta CFB$ (3)

(b) $CF = \dfrac{CB^2}{CA}$ (3)

10.2.3 Bepaal vervolgens die lengte van CF, korrek tot die naaste eenheid. (2)

10.2.4 Gee die lengte van AF. (1)

10.2.5 Bepaal die lengte van FE.
Laat jou antwoord in die vorm $\dfrac{a}{b}$. (5)

[20]

TOTAAL: 150

INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin \mathrm{A}} = \dfrac{b}{\sin \mathrm{B}} = \dfrac{c}{\sin \mathrm{C}}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\,\Delta\mathrm{ABC} = \frac{1}{2}ab.\sin \mathrm{C}$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})} \qquad \mathrm{P(A\ of\ B) = P(A) + P(B) - P(A\ en\ B)}$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

Province of the
EASTERN CAPE
EDUCATION

LEARNER'S NAME: *LEERDERNAAM:*	

GRADE 12 *GRAAD 12*	

NATIONAL SENIOR CERTIFICATE/*NASIONALE* SENIOR *SERTIFIKAAT*

GRADE 12/*GRAAD 12*

JUNE/*JUNIE* 2019

MATHEMATICS P2/*WISKUNDE V2*
SPECIAL ANSWER BOOK/*SPESIALE ANTWOORDEBOEK*

Marker/*Merker*				**Moderator's Initials / *Moderator se paraaf***															
Question *Vraag*	**Mark** *Punt*			**Initial** **Parafeer**	**Marks** *Punte*			**S M**	**Marks** *Punte*			**D M**	**Marks** *Punte*			**P M**	**Marks** *Punte*			**N M**
1																				
2																				
3																				
4																				
5																				
6																				
7																				
8																				
9																				
10																				
TOTAL *TOTAAL*																				

This special answer book consists of 19 pages./
Hierdie spesiale antwoordeboek bestaan uit 19 bladsye.

QUESTION 1/*VRAAG 1*		
1.1		(1)
1.2		(2)
1.3		(2)
1.4		(1)
1.5		(1)
		[7]

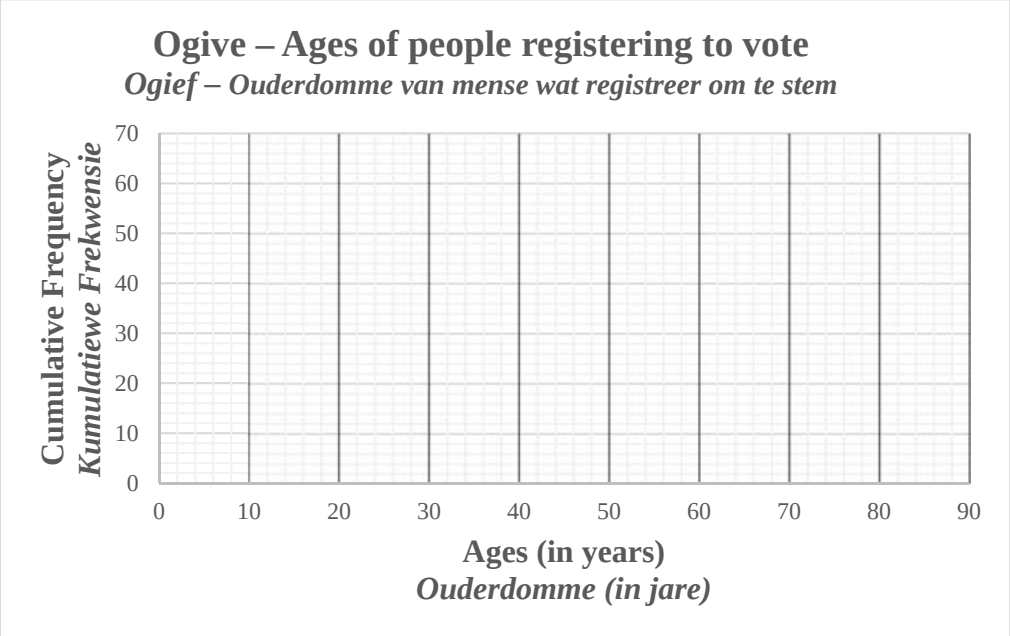
QUESTION 2/*VRAAG 2*

2.1

Ages (in years) ***Ouderdom (in jare)***	**Frequency** ***Frekwensie***	**Cumulative Frequency** ***Kumulatiewe Frekwensie***
$18 \leq x < 28$		4
$28 \leq x < 38$		14
$38 \leq x < 48$		28
$48 \leq x < 58$	17	
$58 \leq x < 68$	12	
$68 \leq x < 78$	3	

(2)

2.2



(3)

2.3

(1)

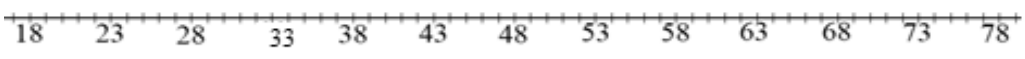
2.4

(2)

2.5

(3)

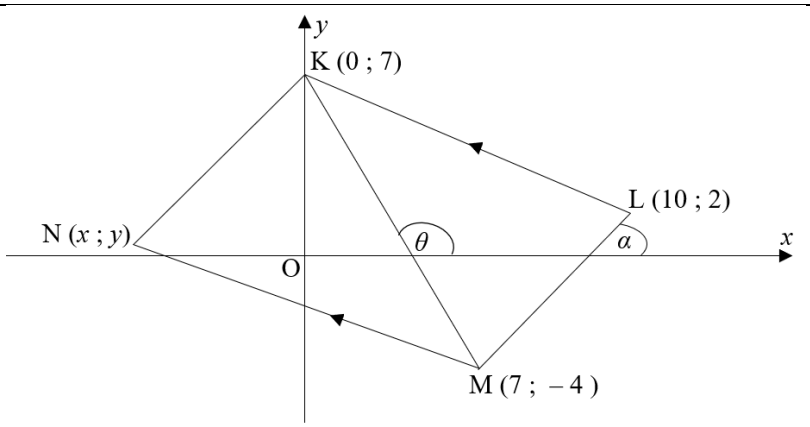
2.6



(2)

[13]

QUESTION 3/*VRAAG 3*



3.1.1 (2)

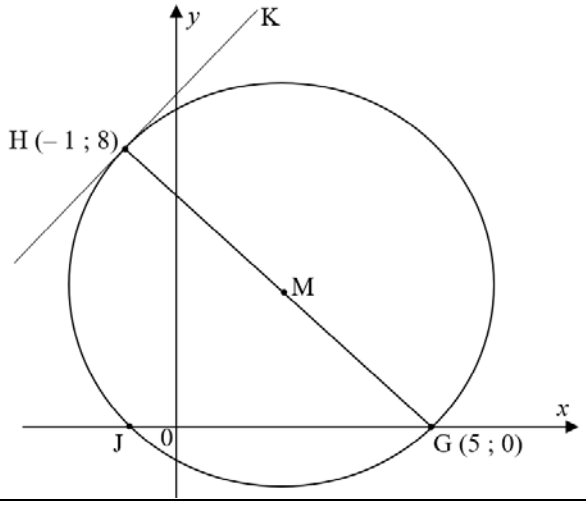
3.1.2 (2)

3.1.3 (3)

3.1.4 (4)

3.2		(4)
3.3		(2)
3.4		(5)
		[22]

QUESTION 4/*VRAAG 4*



4.1 (2)

4.2 (3)

4.3 (4)

4.4		(3)
4.5		(2)
4.6		(5)
		[19]

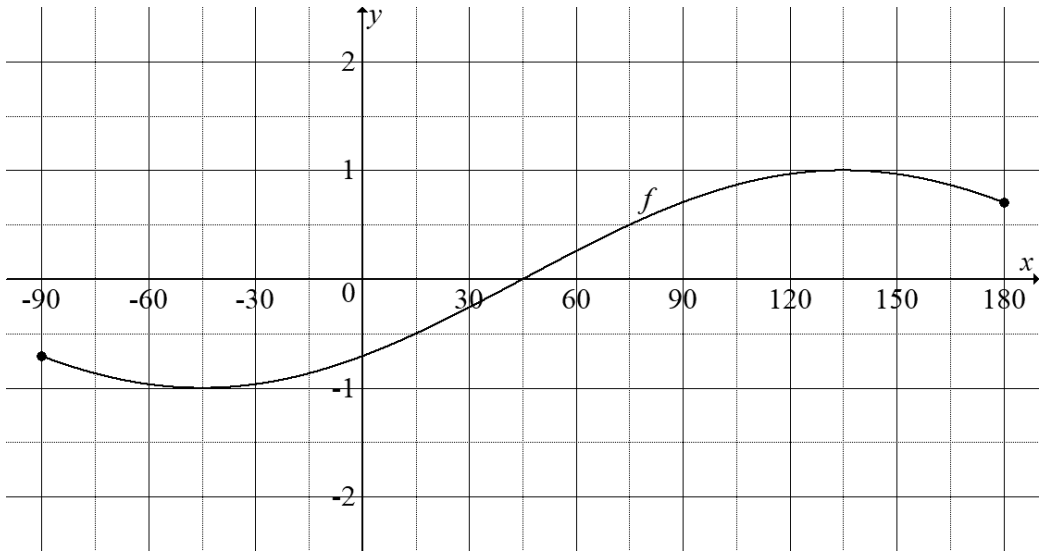
QUESTION 5/*VRAAG 5*		
5.1.1		(2)
5.1.2		(3)
5.1.3		(4)
5.2		(6)

5.3.1		
		(1)
5.3.2		
		(4)
5.4		
		(5)
		[25]

QUESTION 6/*VRAAG 6*

6.1 (1)

6.2



(3)

6.3 (1)

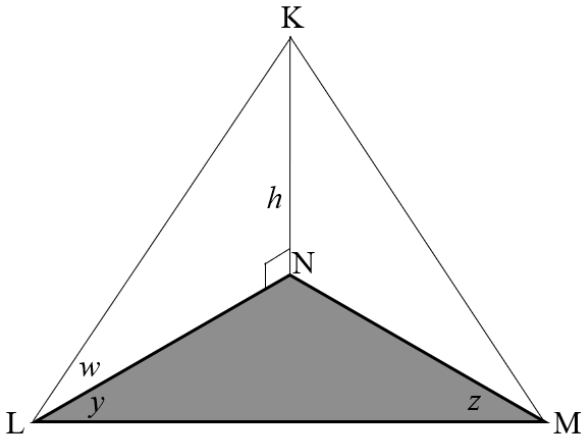
6.4 (1)

6.5 (2)

6.6 (1)

[9]

QUESTION 7/*VRAAG 7*



7.1 (1)

7.2 (4)

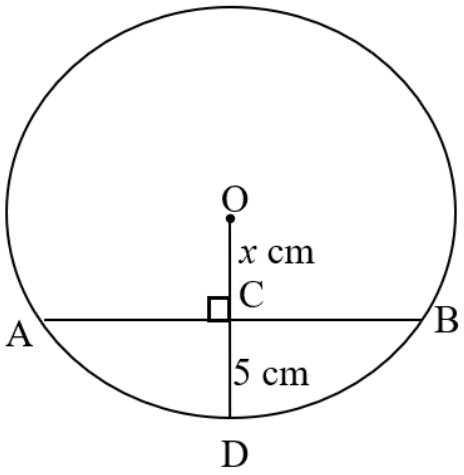
7.3 (2)

[7]

QUESTION 8/*VRAAG 8*

8.1 (1)

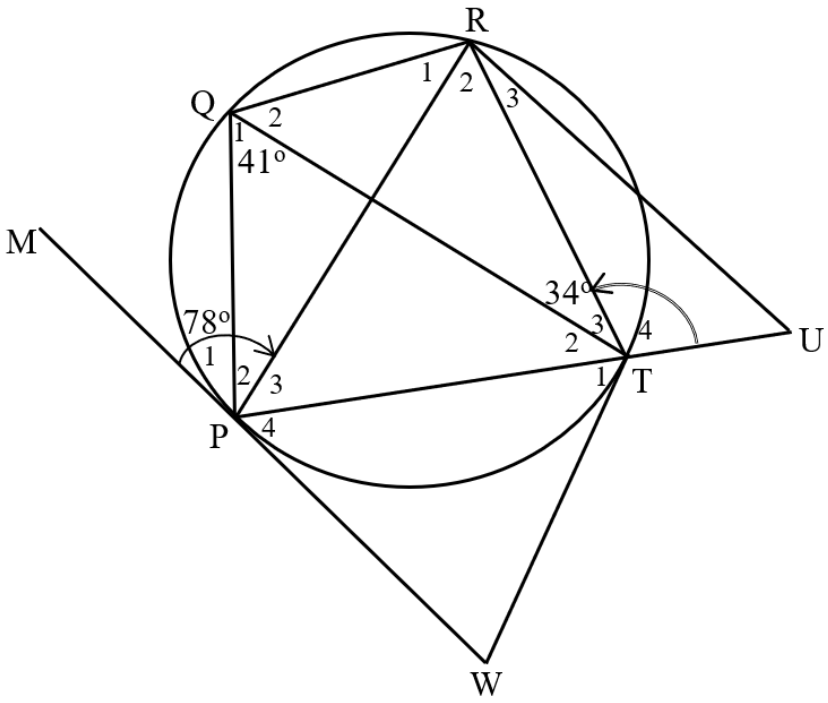
8.2



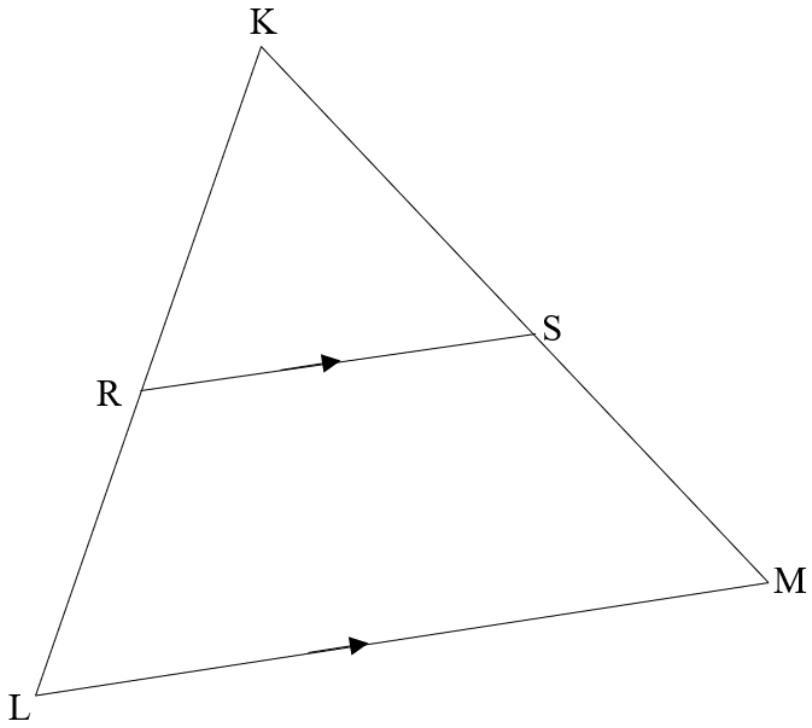
8.2.1 (2)

8.2.2 (4)

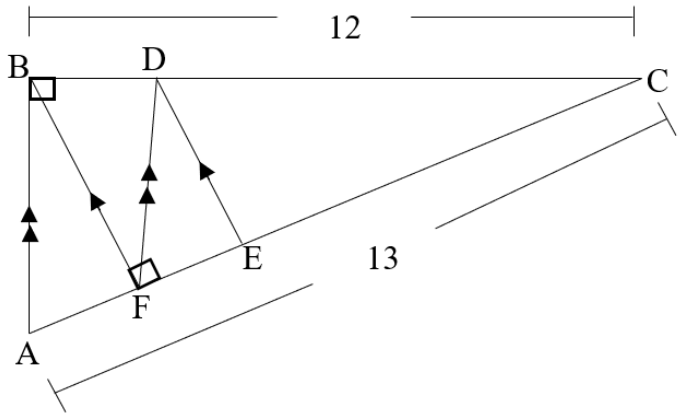
[7]

QUESTION 9/*VRAAG 9*		
9.1		(1)
9.2		
9.2.1		(6)
9.2.2(a)		(2)

9.2.2(b)		(2)
9.2.2(c)		(2)
9.2.2(d)		(2)
9.2.3(a)		(2)
9.2.3(b)		(2)
9.2.3(c)		(2)
		[21]

QUESTION 10/*VRAAG 10*		
10.1		(5)

10.2



10.2.1		(1)
10.2.2(a)		(3)
10.2.2(b)		(3)
10.2.3		(2)

10.2.4		
		(1)
10.2.5		
		(5)
		[20]

TOTAL/TOTAAL: 150

	Additional Space/*Addisionele ruimte*	

	Additional Space/*Addisionele ruimte*	

Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE/*GRAAD* 12

JUNE/JUNIE 2019

MATHEMATICS P2/*WISKUNDE V2* MARKING GUIDELINE/*NASIENRIGLYN*

MARKS/*PUNTE*: 150

This marking guideline consists of 17 pages./
Hierdie nasienriglyn bestaan uit 17 bladsye.

QUESTION 1/*VRAAG 1*

1.1	Mean/*Gemiddelde* = 48	✓ 48 (1)
1.2	SD/SA = 22,08 Penalty of 1 mark for incorrect rounding *Penaliseer 1 punt vir verkeerde afronding*	✓✓22,08 (2)
1.3	Girls performed better. The girls' mean percentage is bigger than that of boys and the girls standard deviation is smaller than that of boys *Meisies het beter gevaar.* *Die meisies se gemiddelde persentasie is groter as dié van die seuns en die standaardafwyking is kleiner as dié van die seuns.*	✓Girls / *Meisies* ✓Reason / *Rede* (2)
1.4	51 – 48 = 3 ∴ each boy's percentage must be increased by 3. *∴elke seun se persentasie moet met 3 vermeerder word.*	✓3 (1)
1.5	Boys' standard deviation will remain the same *Die seuns se standaardawyking sal dieselfde bly.*	✓remain the same *dieselfde bly* (1)
		[7]

QUESTION 2/*VRAAG 2*

2.1

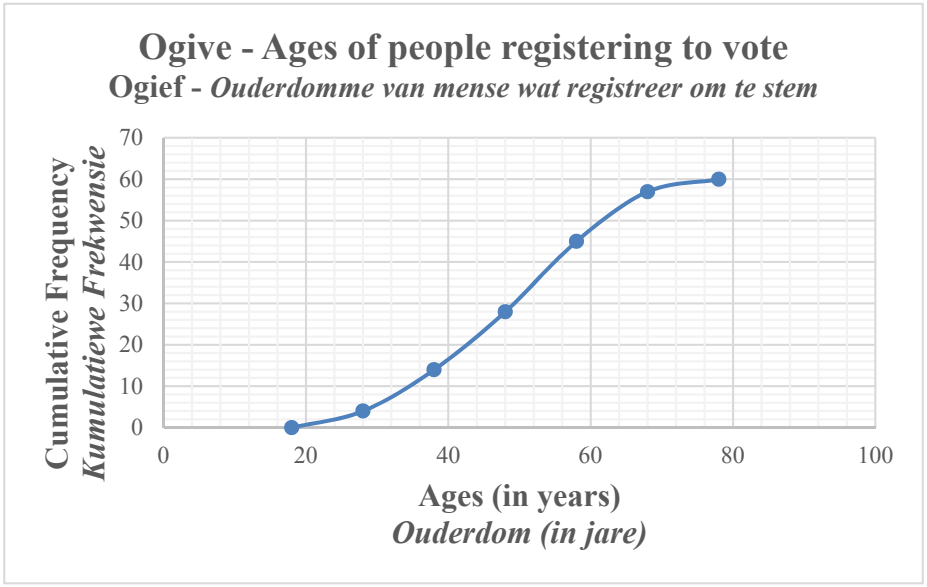
Ages (in years) *Ouderdom (in jare)*	**Frequency** *Frekwensie*	**Cumulative Frequency** *Kumulatiewe Frekwensie*
$18 \le x < 28$	4	4
$28 \le x < 38$	10	14
$38 \le x < 48$	14	28
$48 \le x < 58$	17	45
$58 \le x < 68$	12	57
$68 \le x < 78$	3	60

✓frequency *frekwensie*

✓cumulative frequency *kumulatiewe frekwensie*

(2)

2.2



✓grounding at / *anker by* (0 ; 18)

✓upper limits *boonste limiete*

✓shape / *vorm*

(3)

2.3 $48 \le x < 58$

✓answer / *antwoord* (1)

2.4 60 – 49 = 11 senior citizens / *senior burgers*

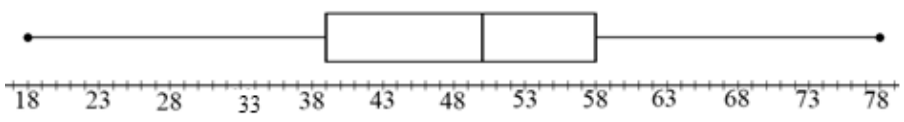
✓49
✓answer / *antwoord* (2)

2.5 $Q_1 = 39$
$Q_2 = 50$
$Q_3 = 58$

✓value of / *waarde van* Q_1
✓value of / *waarde van* Q_2
✓value of / *waarde van* Q_3

(3)

2.6



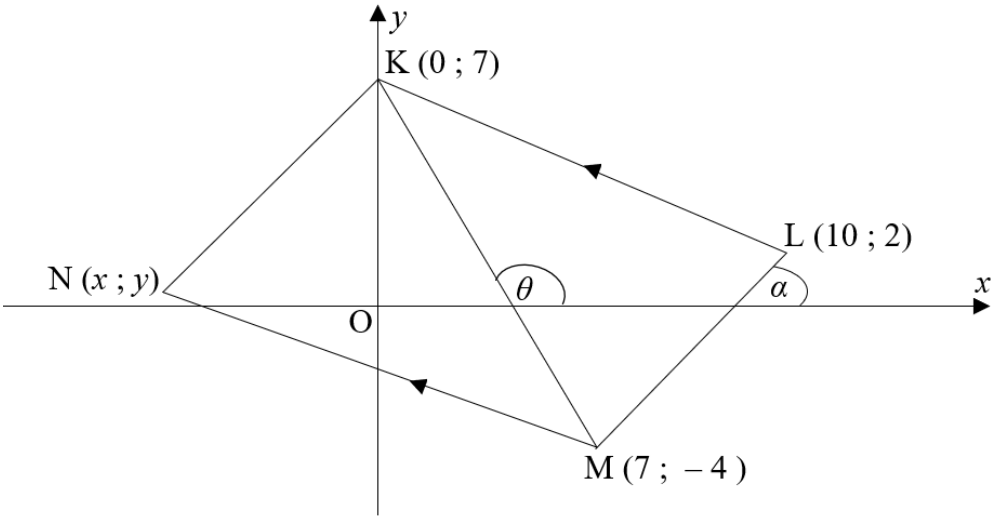
✓minimum and maximum *minimum en maksimum*

✓ box / *boks*

(2)

[13]

QUESTION 3/*VRAAG 3*

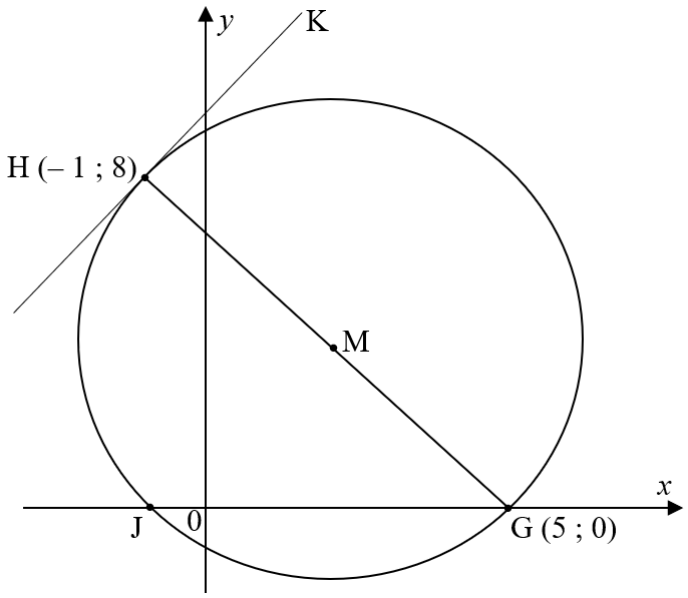


3.1.1	$KL = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(10-0)^2 + (2-7)^2}$ $= \sqrt{125} = 5\sqrt{5}$	✓substitution / *vervanging* ✓answer / *antwoord* (2)
3.1.2	$m_{KM} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-4-7}{7-0}$ $= -\frac{11}{7}$	✓substitution / *vervanging* ✓gradient of KM / *gradiënt van* KM (2)
3.1.3	$m_{LM} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-4-2}{7-10}$ $= 2$ $\tan\alpha = 2$ $\therefore \alpha = 63,43°$	✓gradient of LM / *gradiënt van* LM ✓ $\tan\alpha = 2$ ✓value of α / *waarde van* α (3)
3.1.4	$\tan\theta = -\frac{11}{7}$ $\text{Ref}\angle = 57,53°$ $\therefore \theta = 122,47°$ $\hat{LMK} = 122,47° - 63,43°$ $= 59,04°$	✓ $\tan\theta = -\frac{11}{7}$ ✓reference angle / *verwysingshoek* ✓value of θ / waarde van θ ✓value of $\hat{LMK}$ / *waarde van* $\hat{LMK}$ (4)

3.2	$m_{KN} = m_{LM}$ $\frac{y-7}{x-0} = 2$ $y = 2x+7$ $m_{MN} = m_{KL}$ $\frac{y+4}{x-7} = -\frac{1}{2}$ $y = -\frac{x}{2}-\frac{1}{2}$ $2x+7 = -\frac{x}{2}-\frac{1}{2}$ $\therefore y = -\frac{x}{2}+\frac{13}{2}$ $4x+14 = -x-1$ $x = -3$ $y = 1$ OR / *OF* $m_{KL} = \frac{-5}{10}$ $m_{NM} = \frac{-5}{10}$ Hence the coordinates of/ *Vervolgens die koördinate van* N(-3;1) OR/*OF* Midpoint of KM/ *Middelpunt van KM* $= \left(\frac{7}{2};\frac{3}{2}\right)$ Midpoint of LN/ *Middelpunt van LN* $= \left(\frac{7}{2};\frac{3}{2}\right)$ $\therefore \frac{x+10}{2} = \frac{7}{2}$ and/*en* $\frac{y+2}{2} = \frac{3}{2}$ $\therefore$ N$(-3;1)$	✓ $y = 2x+7$ ✓ $y = -\frac{x}{2}-\frac{1}{2}$ ✓value of x / *waarde van x* ✓value of y / *waarde van y* OR/*OF* ✓ $m_{KL} = \frac{-5}{10}$ ✓ $m_{NM} = \frac{-5}{10}$ ✓value of x / *waarde van x* ✓value of y / *waarde van y* OR/*OF* ✓ Midpoint of KM / *Middelpunt van KM* $= \left(\frac{7}{2};\frac{3}{2}\right)$ ✓ Midpoint of LN / *Middelpunt van LN* $= \left(\frac{7}{2};\frac{3}{2}\right)$ ✓value of x / *waarde van x* ✓value of y / *waarde van y* (4)

3.3	$m_{LM} \times m_{MN} = 2 \times \left(-\frac{1}{2}\right)$ $= -1$ $\therefore \hat{LMN} = 90°$ OR/OF $\hat{LMN}$ is a right angle / *is 'n regtehoek*	✓product of gradients / *produk van gradiënte* ✓conclusion / *gevolgtrekking* (2)
3.4	KL = NM = $5\sqrt{5}$ $KM = \sqrt{7^2 + 11^2}$ $= \sqrt{170}$ $\hat{LMN} = 90°$ and / *en* $\hat{LMK} = 59,04°$ $\therefore \hat{KMN} = 90° - 56,04° = 30,96°$ Area of $\Delta KMN = \frac{1}{2} \times \sqrt{170} \times 5\sqrt{5} \times \sin 30,96°$ $= 37,50$ square units / *vierkante eenhede*	✓ KL = NM = $5\sqrt{5}$ ✓ KM = $\sqrt{170}$ ✓ $\hat{LMN} = 90°$ and/*en* $\hat{LMK} = 59,04°$ ✓ $\hat{KMN} = 30,96°$ ✓ Area of ΔKMN (5)
		[22]

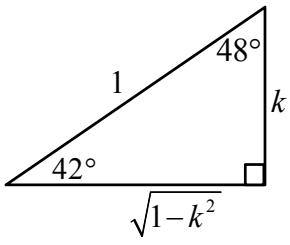
QUESTION 4/*VRAAG 4*

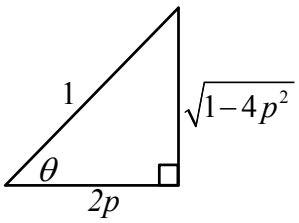


4.1	M (2 ; 4)	✓value of x / *waarde van x* ✓value of y / *waarde van y* (2)
4.2	$r^2 = (5-2)^2 + (0-4)^2$ $= 25$ $\therefore (x-2)^2 + (y-4)^2 = 25$	✓$(x-2)^2$ ✓$(y-4)^2$ ✓25 (3)
4.3	$m_{GH} = \frac{8-0}{-1-5}$ $= -\frac{8}{6} = -\frac{4}{3}$ $m_{\tan} = \frac{3}{4}$ $y - 8 = \frac{3}{4}(x+1)$ $\therefore y = \frac{3}{4}x + \frac{35}{4}$	✓m_{GH} ✓$m_{\tan}$ ✓substitution / *vervanging* ✓equation / *vergelyking* (4)
4.4	At/By J, $y = 0$ $(x-2)^2 + (0-4)^2 = 25$ $(x-2)^2 = 9$ $x-2 = \pm 3$ $x = 5$ or/*of* $x = -1$ $\therefore J(-1;0)$	✓$y = 0$ ✓substitution / *vervanging* ✓$x = -1$ (3)

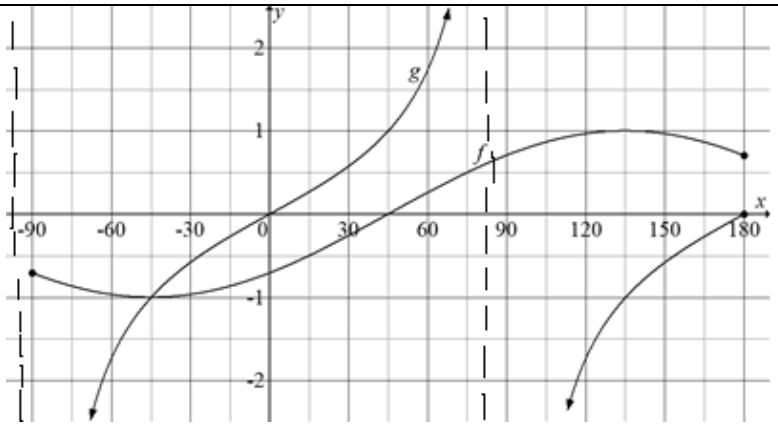
4.5	HJG is a right angled triangle (8, 6 and 10). So the rotation of J around M will complete a rectangle Hence, j $((-1+6;0+8)) = J^{|}(5;8)$ *HJG is 'n reghoekige driehoek (8, 6 en 10)* *Dus sal die rotasie van J om M die reghoek voltooi.* *Vervolgens is* $J^{|}((-1+6;0+8)) = J^{|}(5;8)$	✓value of *x / waarde van x* ✓value of *y / waarde van y* (2)
4.6	$x^2 + y^2 - 12x - 2y + 17 = 0$ $x^2 - 12x + y^2 - 2y = -17$ $x^2 - 12x + 36 + y^2 - 2y + 1 = -17 + 36 + 1$ $(x-6)^2 + (y-1)^2 = 20$ Distance between the centres: *Afstand tussen die middelpunte*: $\sqrt{(2-6)^2 + (4-1)^2} = 5$ ∴ the centre lies on the original circle */ die middelpunt lê op die omtrek van die oorspronklike sirkel*	✓completing the square / *voltooiing van die vierkant* ✓factorisation/*faktorisering*: *x* ✓factorisation/*faktorisering*: *y* ✓distance formula / *afstand formule* ✓conclusion / *gevolgtrekking* (5)
		[19]

QUESTION 5/*VRAAG 5*

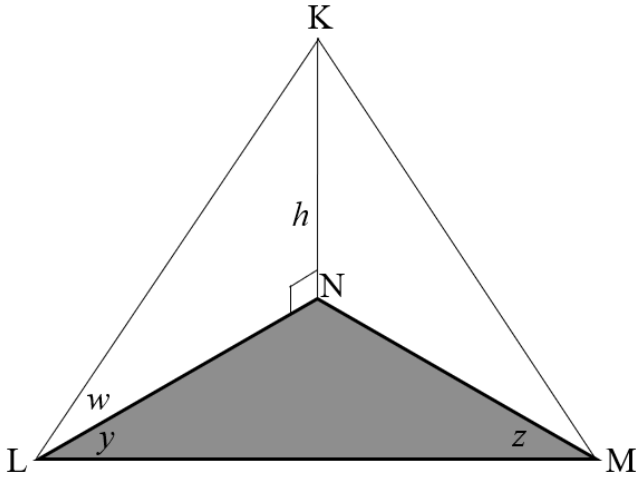
5.1.1	$\sin 42° = \frac{k}{1}$ $\tan 42° = \frac{k}{\sqrt{1-k^2}}$  	✓ $\sqrt{1-k^2}$ ✓ $\tan 42° = \frac{k}{\sqrt{1-k^2}}$ (2)
5.1.2	$\sin 84° = \sin 2 \times 42°$ $= 2\sin 42° \cos 42°$ $= 2.k.\sqrt{1-k^2} \quad = 2k\sqrt{1-k^2}$	✓double angle/*dubbelhoek* ✓expansion/*identiteit/ontwikkeling* ✓substitution/*vervanging* (3)
5.1.3	$\sin 3° = sin(45° - 42°)$ $= sin\,45° cos\,42° - cos\,45° sin\,42°$ $= \frac{\sqrt{2}}{2}.\sqrt{1-k^2} - \frac{\sqrt{2}}{2}.k$	✓$3° = 45° - 42°$ ✓expansion/*identiteit/ontwikkeling* ✓substitution/*vervanging* ✓substitution/*vervanging* (4)
5.2	$\frac{\sin(x-450°).\tan(180°+x).\sin(90°-x)}{\cos(-x)}$ $\frac{-\cos x.\tan x.\cos x}{\cos x}$ $-\cos x.\frac{\sin x}{\cos x}$ $-\sin x$	✓ $-\cos x$ ✓ $\tan x$ ✓ $\cos x$ ✓ $\cos x$ ✓ $\frac{\sin x}{\cos x}$ ✓answer / *antwoord* (6)
5.3.1	$\cos(A+B) = \cos A \cos B - \sin A \sin B$	✓expansion/*identiteit/ontwikkeling* (1)
5.3.2	LHS/LK $= cos\,3\alpha$ $= \cos(2\alpha + \alpha)$ $= \cos 2\alpha \cos\alpha - \sin 2\alpha \sin\alpha$ $= (2\cos^2\alpha - 1).\cos\alpha - 2\sin\alpha\cos\alpha.\sin\alpha$ $= 2\cos^3\alpha - \cos\alpha - 2\sin^2\alpha\cos\alpha$ $= 2\cos^3\alpha - \cos\alpha - 2(1-\cos^2\alpha)\cos\alpha$ $= 2\cos^3\alpha - \cos\alpha - 2\cos\alpha + 2\cos^3\alpha$ $= 4\cos^3\alpha - 3\cos\alpha$ = RHS/*RK*	✓compound angle identity *saamgesteldehoek-identiteit* ✓cos double angle identity cos *dubbelhoek-identiteit* ✓sin double angle identity sin *dubbelhoek-identiteit* ✓$(1-\cos^2\alpha)$ (4)

5.4	$\cos\theta = \frac{2p}{1}$  $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $7p = (2p)^2 - \left(\sqrt{1-4p^2}\right)^2$ $7p = 4p^2 - 1 + 4p^2$ $8p^2 - 7p - 1 = 0$ $(8p+1)(p-1) = 0$ $\therefore p = -\frac{1}{8}$ *or / of* $p = 1$ OR/*OF* $\cos 2\theta = 2\cos^2\theta - 1$ $7p = 2.(2p)^2 - 1$ $7p = 8p^2 - 1$ $8p^2 - 7p - 1 = 0$ $(8p+1)(p-1) = 0$ $\therefore p = -\frac{1}{8}$ *or / of* $p = 1$ OR/*OF* $\cos 2\theta = 1 - 2\sin^2\theta$ $7p = 1 - 2.\left(\sqrt{1-4p^2}\right)^2$ $7p = 1 - 2(1-4p^2)$ $7p = 1 - 2 + 8p^2$ $8p^2 - 7p - 1 = 0$ $(8p+1)(p-1) = 0$ $\therefore p = -\frac{1}{8}$ *or / of* $p = 1$	✓ $\sqrt{1-4p^2}$ ✓ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ ✓substitution/*vervanging* ✓standard form *standaardvorm* ✓values of p / *waardes van p* OR/*OF* ✓ $2\cos^2\theta - 1$ ✓✓substitution/*vervanging* ✓standard form / *standaardvorm* ✓values of p / *waardes van p* OR/OF ✓ $\sqrt{1-4p^2}$ ✓ $1 - 2\sin^2\theta$ ✓substitution / *vervanging* ✓standard form *standaardvorm* ✓values of p / *waardes van p* (5)
		[25]

QUESTION 6/*VRAAG 6*

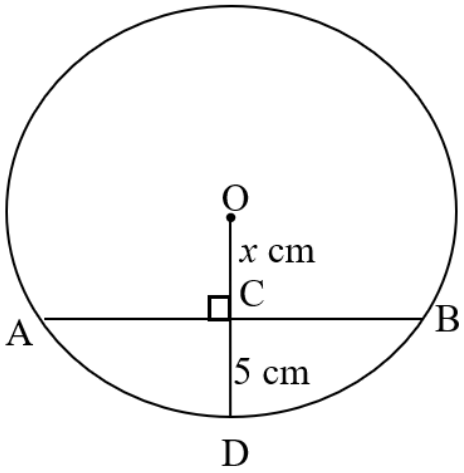
6.1	$y \in [-1;1]$ OR/*OF* $-1 \le y \le 1$	✓answer / *antwoord* (1)
6.2		*g:* ✓asymptotes at -90° and 90° *asymptote vir* -90° en -90° ✓*x*-intercepts *x-afsnitte* ✓shape / *vorm* (3)
6.3	180°	✓180° (1)
6.4	$x = -45°$	✓$-45°$ (1)
6.5	$x \in [45°; 90°)$ OR/*OF* $45° \le x < 90°$	✓critical values *kritiese waardes* ✓notation / *notasie* (2)
6.6	$h(x) = \sin(x - 45°) + 1$	✓$h(x) = \sin(x - 45°) + 1$ (1)
		[9]

QUESTION 7/*VRAAG 7*



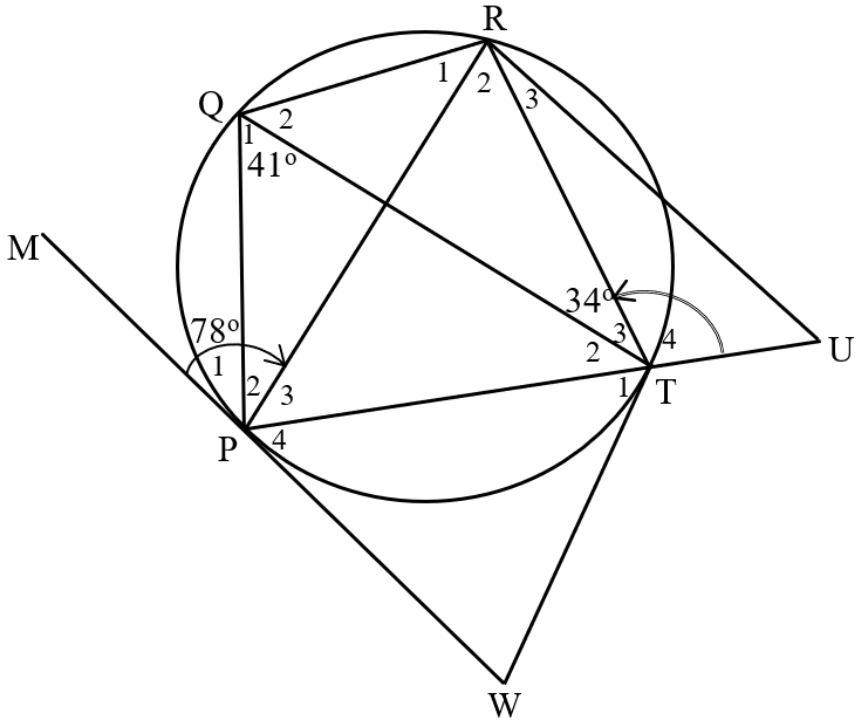
7.1	In Δ KLN : $\tan w = \frac{h}{\text{LN}}$ $\text{LN} = \frac{h}{\tan w}$	✓ $\text{LN} = \frac{h}{\tan w}$ (1)
7.2	In ΔNLM $\frac{\text{LM}}{\sin \hat{\text{N}}} = \frac{\text{LN}}{\sin \hat{\text{M}}}$ $\frac{\text{LM}}{\sin(180° - (y+z))} = \frac{\text{LN}}{\sin z}$ $\therefore \text{LM} = \frac{\text{LN}.\sin(y+z)}{\sin z}$ But $\text{LN} = \frac{h}{\tan w}$ $\therefore \text{LM} = \frac{h \sin(y+z)}{\tan w \sin z}$	✓correct sine rule *korrekte sinusreël* ✓substitution *vervanging* ✓isolating LM *isoleer* LM ✓answer / *antwoord* (4)
7.3	$\text{LM} = \frac{h \sin(y+z)}{\tan w \sin z}$ and/*en* $h = 38\,\text{m},\ w = 21^\circ,\ y = 52^\circ$ and/*en* $z = 59$ $\therefore\ \text{LM} = \frac{38.\sin(52° + 59°)}{\tan 21° \sin 59°}$ $= 107,82\ \text{m}$	✓substitution *vervanging* ✓answer / *antwoord* (2)
		[7]

QUESTION 8/*VRAAG 8*



8.1	the centre of a circle / *die middelpunt van die sirkel*	✓answer / *antwoord* (1)
8.2.1	AC = 10 cm (line from centre ⊥ chord) (*lyn vanaf die middelpunt ⊥ op koord*)	✓length of AC *lengte van* AC ✓Reason/*rede* (2)
8.2.2	$(x+5)^2 = 10^2 + x^2$ $x^2 + 10x + 25 = 100 + x^2$ $10x = 75$ $\therefore \quad x = 7,5\,\text{cm}$ $\therefore$ radius $= 7,5\,\text{cm} + 5\,\text{cm}$ $= 12,5\,\text{cm}$	✓radius = $(x + 5)$ ✓applying Pythagoras theorem / *toepassing van Pythagoras se stelling* ✓value of x / *waarde van x* ✓length of radius / *lengte van radius* (4)
		[7]

QUESTION 9/*VRAAG 9*

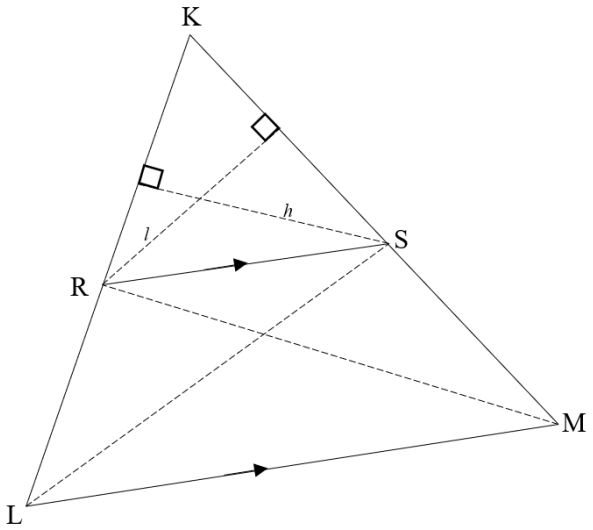


9.1	interior opposite angle / *teenoorstaande binnehoek*	✓answer / *antwoord* (1)
9.2.1	$\hat{R}_2 = \hat{Q}_2 = 41°$ (∠s in the same seg)/(*∠e in dieselfde seg.*) $\hat{P}_4 = \hat{Q}_1 = 41°$ (tan-chord theorem)/(*raaklyn-koord stelling*) $\hat{T}_1 = \hat{P}_4 = 41°$ (∠s opp. = sides)/(*∠e teenoor gelyke sye*) OR/*OF* $\hat{T}_1 = \hat{R}_2 = 41°$ (tan – chord theorem)/(*raaklyn-koord stelling*)	✓Statement / *bewering* ✓Reason / *rede* ✓Statement / *bewering* ✓Reason / *rede* ✓Statement / *bewering* ✓Reason / *rede* (6)
9.2.2(a)	$\hat{T}_2 + 34° = 78°$ (tan – chord theorem)/(*raaklyn - koord stelling*) $\therefore \hat{T}_2 = 44°$	✓Statement / *bewering* ✓Reason / *rede* (2)
9.2.2(b)	$41° + \hat{Q}_2 + 44° + 34° = 180°$ (opp. ∠s of a cyclic quad.) $\therefore \hat{Q}_2 = 61°$ (*teenoorst.∠e van 'n koordevierhoek*)	✓Statement / *bewering* ✓Reason / *rede* (2)
9.2.2(c)	$\hat{T}_4 = 41° + 61°$ (ext. ∠s of a cyclic quad.) $\therefore \hat{T}_4 = 102°$ (*buite ∠ van koordevierhoek*) OR/*OF* $\hat{T}_4 + 44° + 34° = 180°$ (int. ∠s of a Δ) $\therefore \hat{T}_4 = 102°$ (*binne ∠e van 'n Δ*)	✓Statement / *bewering* ✓Reason / *rede* OR/*OF* ✓Statement / *bewering* ✓Reason / *rede* (2)

9.2.2(d)	$\hat{W} + 41° + 41° = 180°$ (int . ∠s of a Δ)/*(binne ∠e van Δ*) $\therefore \hat{W} = 98°$	✓Statement /*bewering* ✓ Reason / *rede* (2)
9.2.3(a)	$\hat{Q}_2 = 61°$ and/*en* $\hat{T}_2 = 44°$ $\therefore \hat{Q}_2 \neq \hat{T}_2$ ∴ QR is not parallel to PT (alt. ∠s are not equal) *QR is nie ewewydig aan PT nie* (*verw. ∠e is nie gelyk nie*)	✓$\hat{Q}_2 \neq \hat{T}_2$ ✓ alt. ∠s are not equal *verw. ∠e is nie gelyk nie* (2)
9.2.3(b)	$\hat{R}_2 + \hat{W} = 41° + 98°$ $= 139° \neq 180°$ ∴ PRTW is not a cyclic quad. (Opp. ∠s are not supp.) *PRTW is nie 'n koordevierhoek nie* (*teenoorst. ∠e is nie* supplementêr *nie*)	✓$\hat{R}_2 + \hat{W} \neq 180°$ ✓ PRTW is not a cyclic quad. *PRTW is nie 'n koordevierhoek nie* (2)
9.2.3(c)	$\hat{R}_1 = \hat{T}_2 = 44°$(∠s in same seg)/*(∠e in dieselfde segment*) $\hat{R}_1 + \hat{R}_2 = 44° + 41°$ $= 95° \neq 90°$ ∴ TQ is not a diameter (∠subt. by TQ is not a right angle) *∴TQ is nie 'n middellyn nie (∠onder span deur TQ* is *nie 'n reghoek nie)*	✓$\hat{R}_1 + \hat{R}_2 \neq 90°$ ✓ TQ is not a diameter *TQ is nie 'n middellyn nie* (2)
		[21]

QUESTION 10/*VRAAG 10*

10.1



Construction: Draw heights h and l on KR and KS respectively. Join LS and MR
Konstruksie: Teken hoogtes h en l op KR en KS onderskeidelik. Verbind LS en MR

✓construction / *konstruksie*

Proof/*Bewys*:

$$\frac{\text{Area of } \Delta KRS}{\text{Area of } \Delta LRS} = \frac{\frac{1}{2}.KR.h}{\frac{1}{2}.RL.h} = \frac{KR}{RL}$$

✓ratio of areas / *verhouding van oppervlaktes*

$$\frac{\text{Area of } \Delta KRS}{\text{Area of } \Delta MSR} = \frac{\frac{1}{2}.KS.l}{\frac{1}{2}.SM.l} = \frac{KS}{SM}$$

✓ratio of areas *verhouding van oppervlaktes*

But/*Maar*: area of/*van* ΔLRS = area of/*van* ΔMSR

✓same base and same height *dieselfde basis en hoogte*

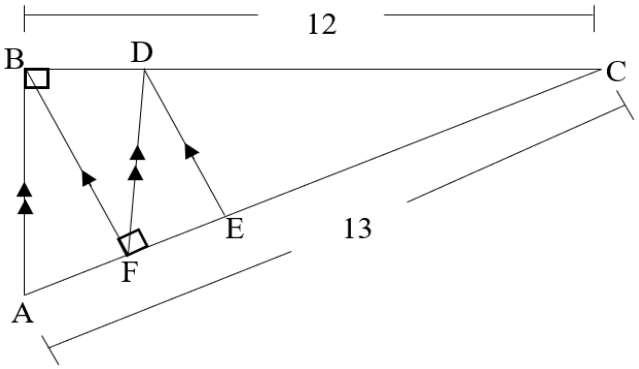
$$\frac{\text{Area of } \Delta KRS}{\text{Area of } \Delta LRS} = \frac{\text{Area of } \Delta KRS}{\text{Area of } \Delta MSR}$$

✓ $\frac{\text{Area of } \Delta KRS}{\text{Area of } \Delta LRS} = \frac{\text{Area of } \Delta KRS}{\text{Area of } \Delta MSR}$

$$\therefore \frac{KR}{RL} = \frac{KS}{SM}$$

(5)

10.2



10.2.1	AB = 5 units	✓5 (1)
10.2.2(a)	$\hat{C}$ is common / *is gemeen* $C\hat{B}A = C\hat{F}B$ (both/*beide* = 90°) $C\hat{A}B = C\hat{B}F$ (sum of ∠s of Δ)/(*som van die ∠e van* Δ) ∴ ΔCBA ||| ΔCFB (∠,∠,∠)	✓Statement/*bewering*/ Reason / *rede* ✓Statement/*bewering*/ Reason / *rede* ✓Reason/*Rede* (3)
10.2.2(b)	$\frac{CB}{CF} = \frac{CA}{CB}$ (|||s) $CB^2 = CF.CA$ $\therefore CF = \frac{CB^2}{CA}$	✓proportion / *verhouding* ✓reason / *rede* ✓ $CB^2 = CF.CA$ (3)
10.2.3	$CF = \frac{CB^2}{CA}$ $CF = \frac{(12)^2}{13}$ ≈ 11 units/*eenhede*	✓substitution / *vervanging* ✓length of CF / *lengte van* CF (2)
10.2.4	AF = 13 – 11 = 2 units/*eenhede*	✓length of AF / *lengte van* AF (1)
10.2.5	$\frac{CB}{BD} = \frac{CA}{AF}$ (prop. theorem/*verh. stelling*; DF || BA) $\frac{12}{BD} = \frac{13}{2}$ $\therefore BD = \frac{24}{13}$ $\frac{CF}{FE} = \frac{CB}{BD}$ (prop. theorem/*verh. stelling*; DF || BA) $\frac{11}{FE} = \frac{12}{\frac{24}{13}}$ $\therefore FE = \frac{22}{13}$ units/*eenhede*	✓proportion / *verhouding* ✓reason / *rede* ✓length of BD / *lengte van* BD ✓proportion / *verhouding* ✓length of FE / *lengte van* FE (5)
		[20]
		TOTAL/*TOTAAL*: 150