basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2018

MARKS: 150

TIME: 3 hours

This question paper consists of 15 pages, 1 information sheet and an answer book of 31 pages.

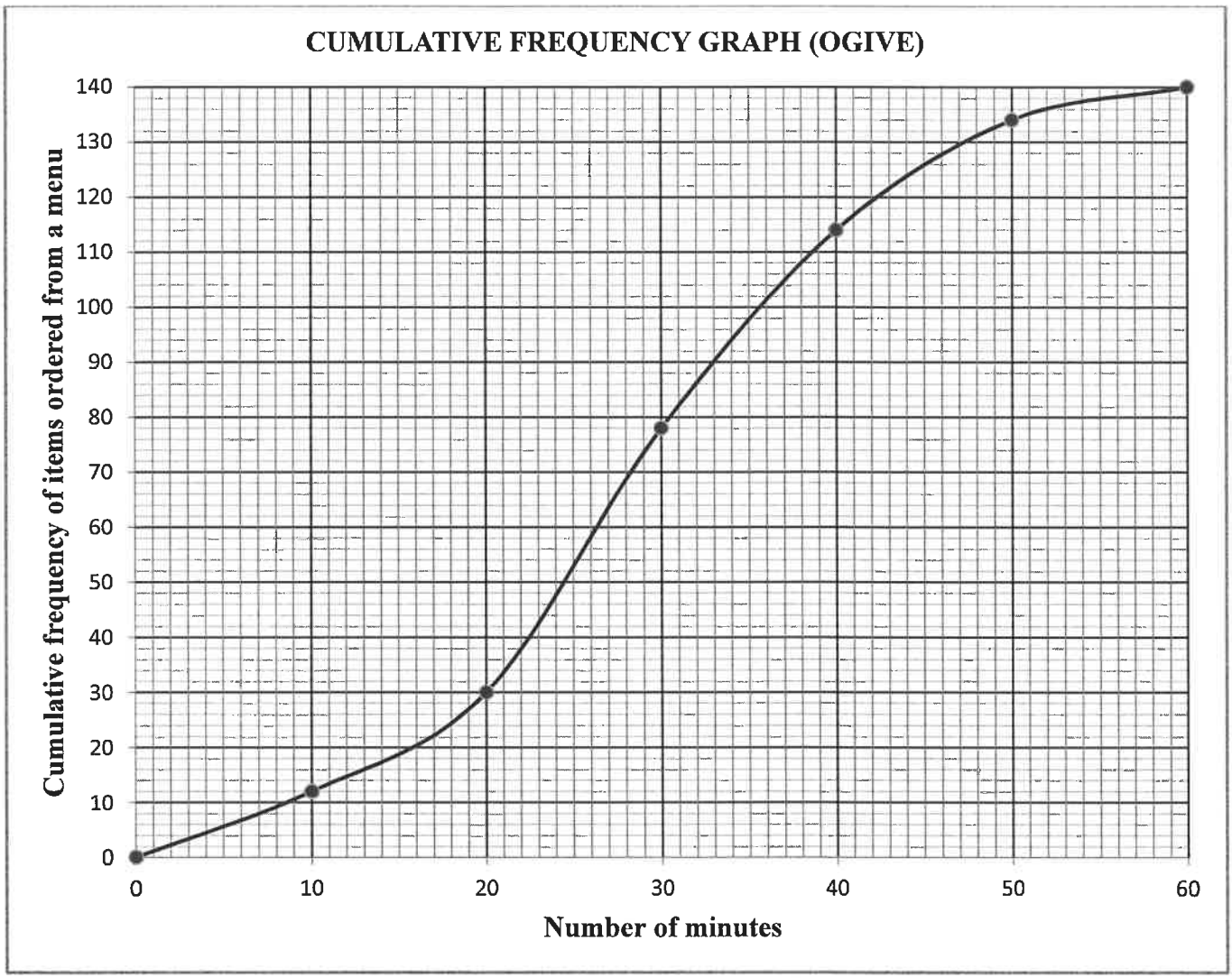
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers correct to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 The cumulative frequency graph (ogive) drawn below shows the total number of food items ordered from a menu over a period of 1 hour.



1.1.1 Write down the total number of food items ordered from the menu during this hour. (1)

1.1.2 Write down the modal class of the data. (1)

1.1.3 How long did it take to order the first 30 food items? (1)

1.1.4 How many food items were ordered in the last 15 minutes? (2)

1.1.5 Determine the 75^{th} percentile for the data. (2)

1.1.6 Calculate the interquartile range of the data. (2)

1.2 Reggie works part-time as a waiter at a local restaurant. The amount of money (in rands) he made in tips over a 15-day period is given below.

35	70	75	80	80
90	100	100	105	105
110	110	115	120	125

1.2.1 Calculate:

(a) The mean of the data (2)

(b) The standard deviation of the data (2)

1.2.2 Mary also works part-time as a waitress at the same restaurant. Over the same 15-day period Mary collected the same mean amount in tips as Reggie, but her standard deviation was R14.

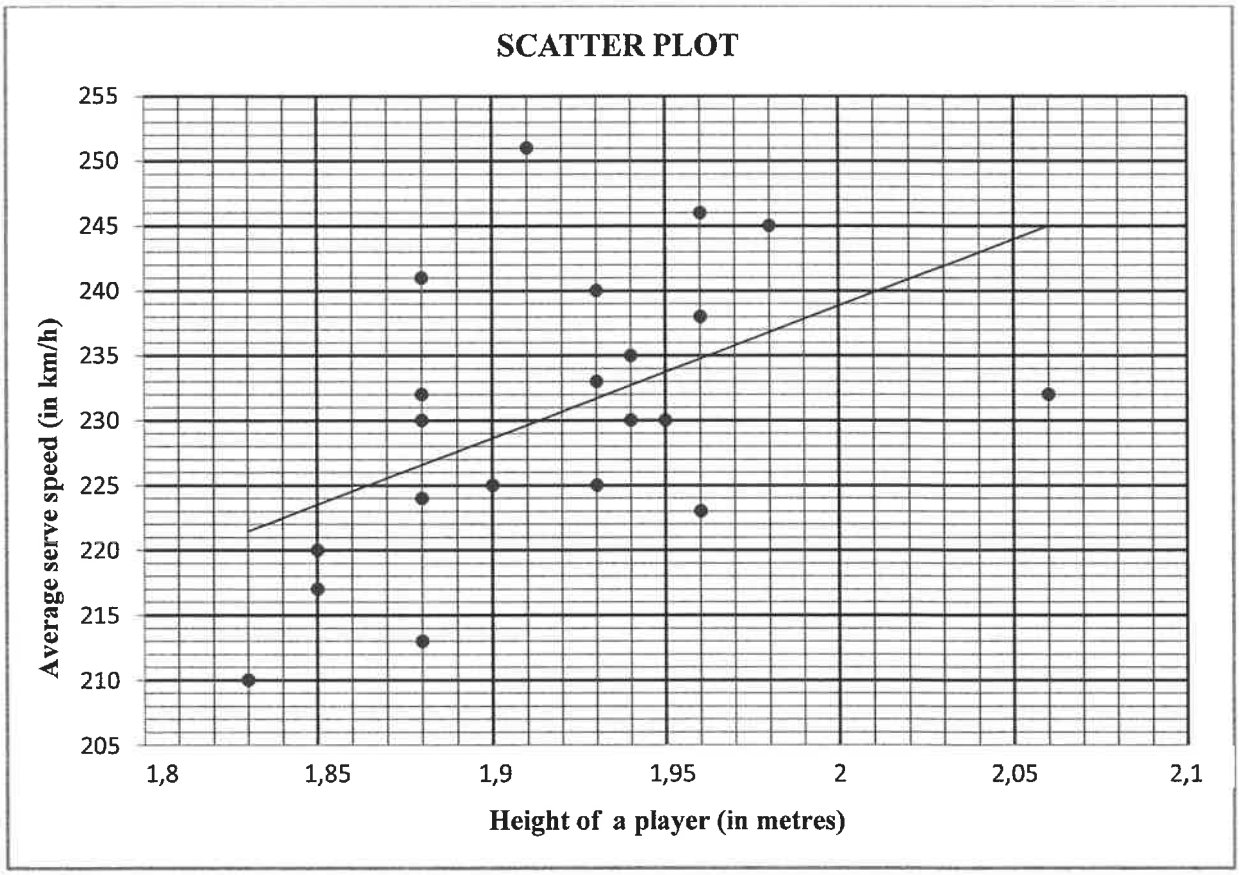
Using the available information, comment on the:

(a) Total amount in tips that they EACH collected over the 15-day period (1)

(b) Variation that EACH of them received in daily tips over this period (1)
[15]

QUESTION 2

A familiar question among professional tennis players is whether the speed of a tennis serve (in km/h) depends on the height of a player (in metres). The heights of 21 tennis players and the average speed of their serves were recorded during a tournament. The data is represented in the scatter plot below. The least squares regression line is also drawn.



2.1 Write down the fastest average serve speed (in km/h) achieved in this tournament. (1)

2.2 Consider the following correlation coefficients:

A. $r = 0,93$ B. $r = -0,42$ C. $r = 0,52$

2.2.1 Which ONE of the given correlation coefficients best fits the plotted data? (1)

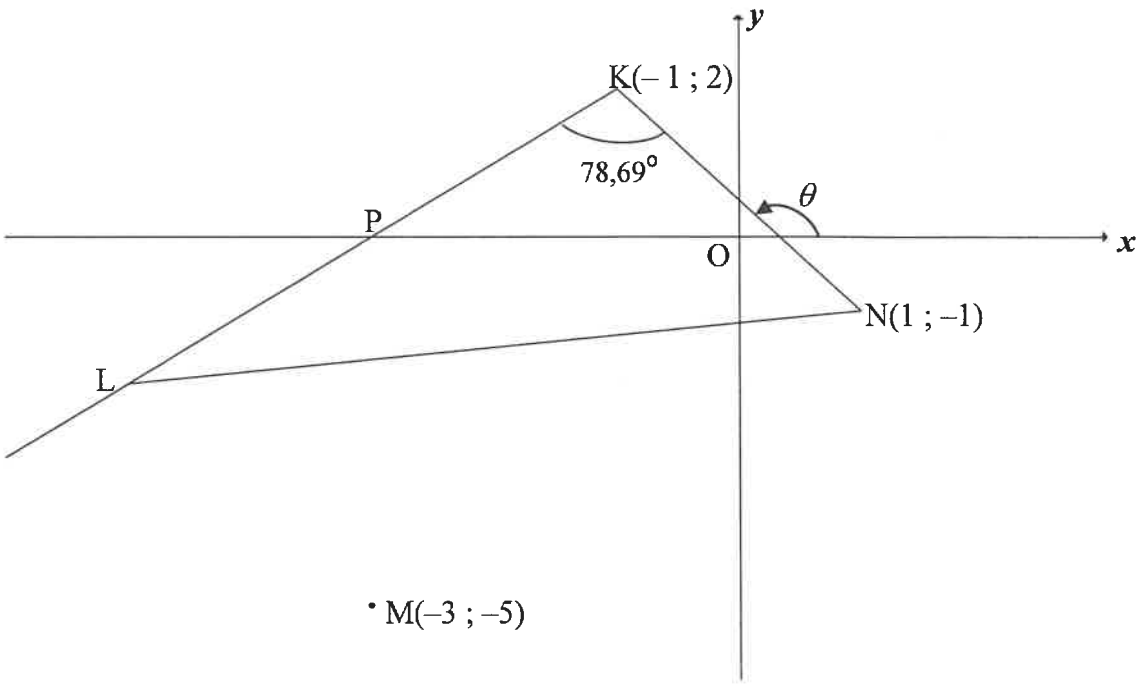
2.2.2 Use the scatter plot and least squares regression line to motivate your answer to QUESTION 2.2.1. (1)

2.3 What does the data suggest about the speed of a tennis serve (in km/h) and the height of a player (in metres)? (1)

2.4 The equation of the regression line is given as $\hat{y} = 27,07 + bx$.
Explain why, in this context, the least squares regression line CANNOT intersect the y-axis at (0 ; 27,07). (1)
[5]

QUESTION 3

In the diagram, K(– 1 ; 2), L and N(1 ; – 1) are vertices of ΔKLN such that $L\hat{K}N = 78{,}69°$. KL intersects the x-axis at P. KL is produced. The inclination of KN is θ. The coordinates of M are (–3 ; –5).



3.1 Calculate:

3.1.1 The gradient of KN (2)

3.1.2 The size of θ, the inclination of KN (2)

3.2 Show that the gradient of KL is equal to 1. (2)

3.3 Determine the equation of the straight line KL in the form $y = mx + c$. (2)

3.4 Calculate the length of KN. (2)

3.5 It is further given that KN = LM.

3.5.1 Calculate the possible coordinates of L. (5)

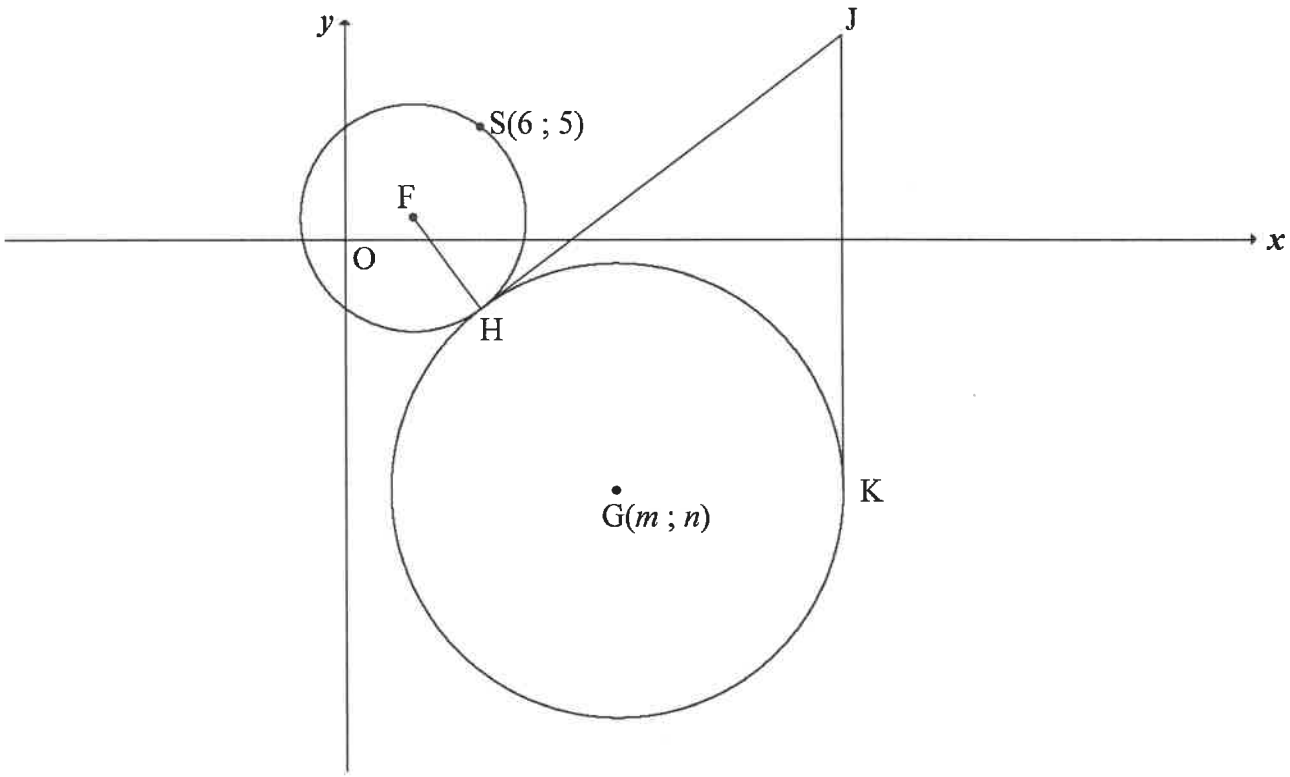
3.5.2 Determine the coordinates of L if it is given that KLMN is a parallelogram. (3)

3.6 T is a point on KL produced. TM is drawn such that TM = LM. Calculate the area of ΔKTN. (4)

[22]

QUESTION 4

In the diagram, the equation of the circle with centre F is $(x-3)^2+(y-1)^2=r^2$. S(6 ; 5) is a point on the circle with centre F. Another circle with centre G$(m\,;\,n)$ in the 4th quadrant touches the circle with centre F, at H such that FH : HG = 1 : 2. The point J lies in the first quadrant such that HJ is a common tangent to both these circles. JK is a tangent to the larger circle at K.



4.1 Write down the coordinates of F. (2)

4.2 Calculate the length of FS. (2)

4.3 Write down the length of HG. (1)

4.4 Give a reason why JH = JK. (1)

4.5 Determine:

4.5.1 The distance FJ, with reasons, if it is given that JK = 20 (4)

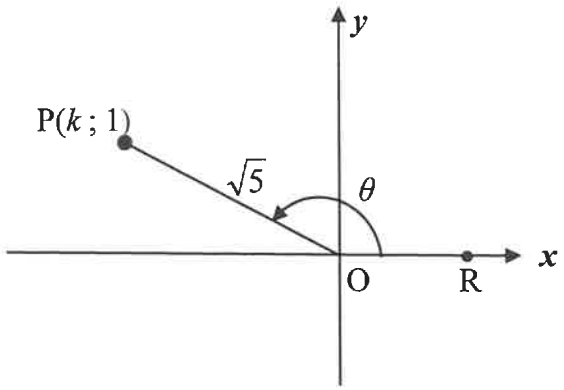
4.5.2 The equation of the circle with centre G in terms of m and n in the form $(x-a)^2+(y-b)^2=r^2$ (1)

4.5.3 The coordinates of G, if it is further given that the equation of tangent JK is $x=22$ (7)

[18]

QUESTION 5

5.1 In the diagram, P(k ; 1) is a point in the 2nd quadrant and is $\sqrt{5}$ units from the origin. R is a point on the positive x-axis and obtuse $\hat{ROP} = \theta$.



5.1.1 Calculate the value of k. (2)

5.1.2 **Without using a calculator**, calculate the value of:

(a) $\tan\theta$ (1)

(b) $\cos(180° + \theta)$ (2)

(c) $\sin(\theta + 60°)$ in the form $\frac{a+b}{\sqrt{20}}$ (5)

5.1.3 **Use a calculator** to calculate the value of $\tan(2\theta - 40°)$ correct to ONE decimal place. (3)

5.2 Prove the following identity: $\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$ (5)

5.3 Evaluate, **without using a calculator**: $\sum_{A=38°}^{52°} \cos^2 A$ (5)

[23]

QUESTION 6

Consider: $f(x) = -2\tan\frac{3}{2}x$

6.1 Write down the period of f. (1)

6.2 The point A(t ; 2) lies on the graph. Determine the general solution of t. (3)

6.3 On the grid provided in the ANSWER BOOK, draw the graph of f for the interval $x \in [-120° ; 180°]$. Clearly show ALL asymptotes, intercepts with the axes and endpoint(s) of the graph. (4)

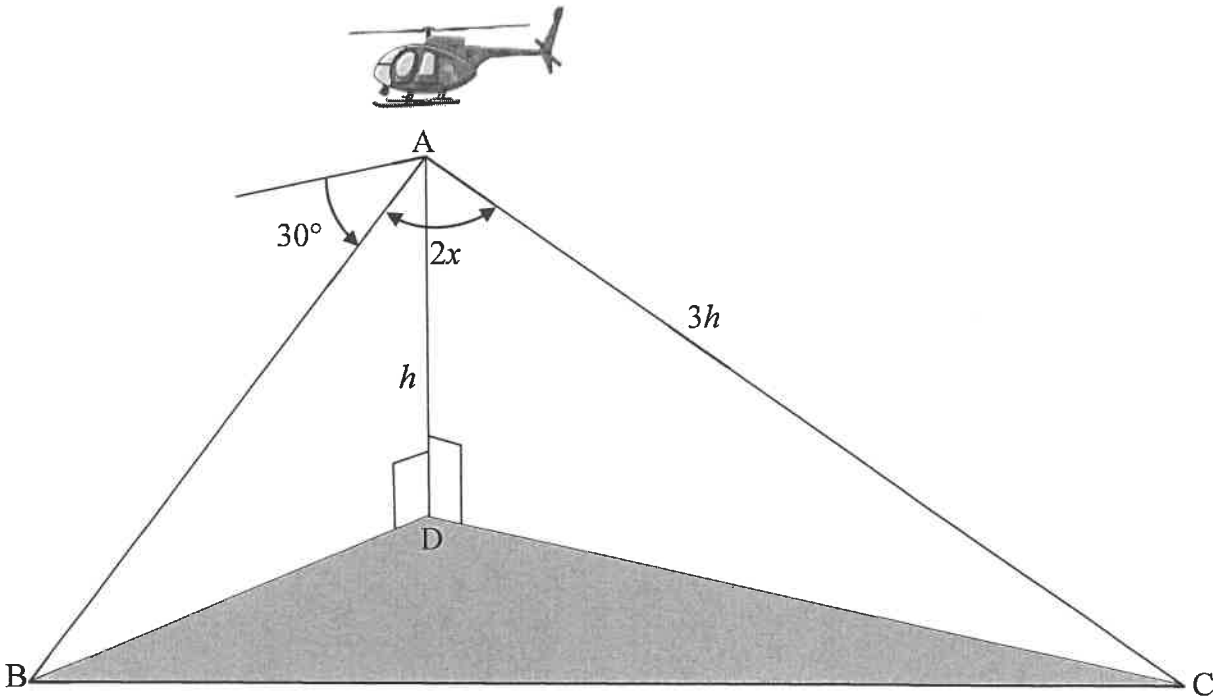
6.4 Use the graph to determine for which value(s) of x will $f(x) \geq 2$ for $x \in [-120° ; 180°]$. (3)

6.5 Describe the transformation of graph f to form the graph of $g(x) = -2\tan\left(\frac{3}{2}x + 60°\right)$. (2)

[13]

QUESTION 7

A pilot is flying in a helicopter. At point A, which is h metres directly above point D on the ground, he notices a strange object at point B. The pilot determines that the angle of depression from A to B is 30°. He also determines that the control room at point C is $3h$ metres from A and $B\hat{A}C = 2x$. Points B, C and D are in the same horizontal plane. This scenario is shown in the diagram below.

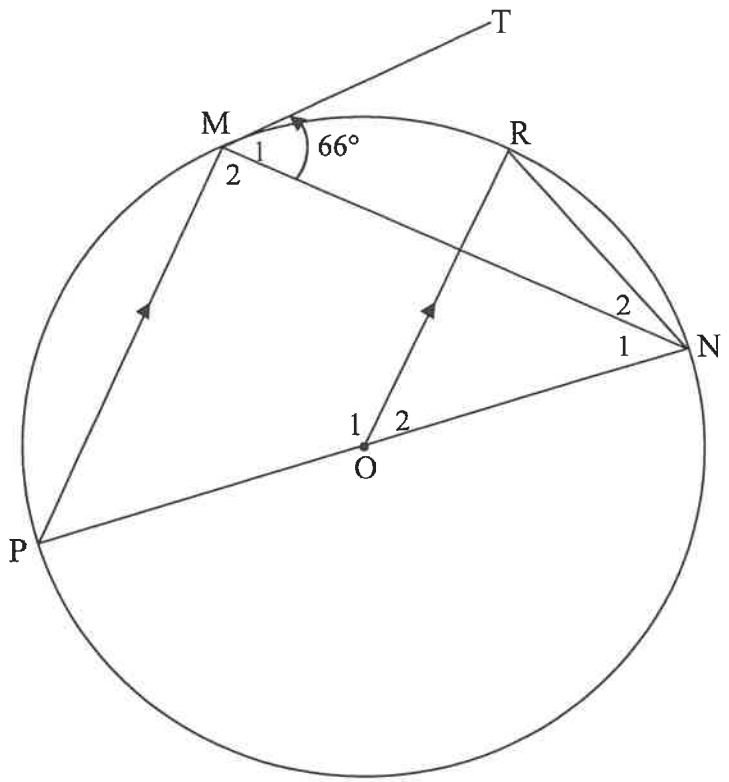


7.1 Determine the distance AB in terms of h. (2)

7.2 Show that the distance between the strange object at point B and the control room at point C is given by $BC = h\sqrt{25 - 24\cos^2 x}$. (4)
[6]

QUESTION 8

8.1 PON is a diameter of the circle centred at O. TM is a tangent to the circle at M, a point on the circle. R is another point on the circle such that OR || PM. NR and MN are drawn. Let $\hat{M}_1 = 66°$.



Calculate, with reasons, the size of EACH of the following angles:

8.1.1 $\hat{P}$ (2)

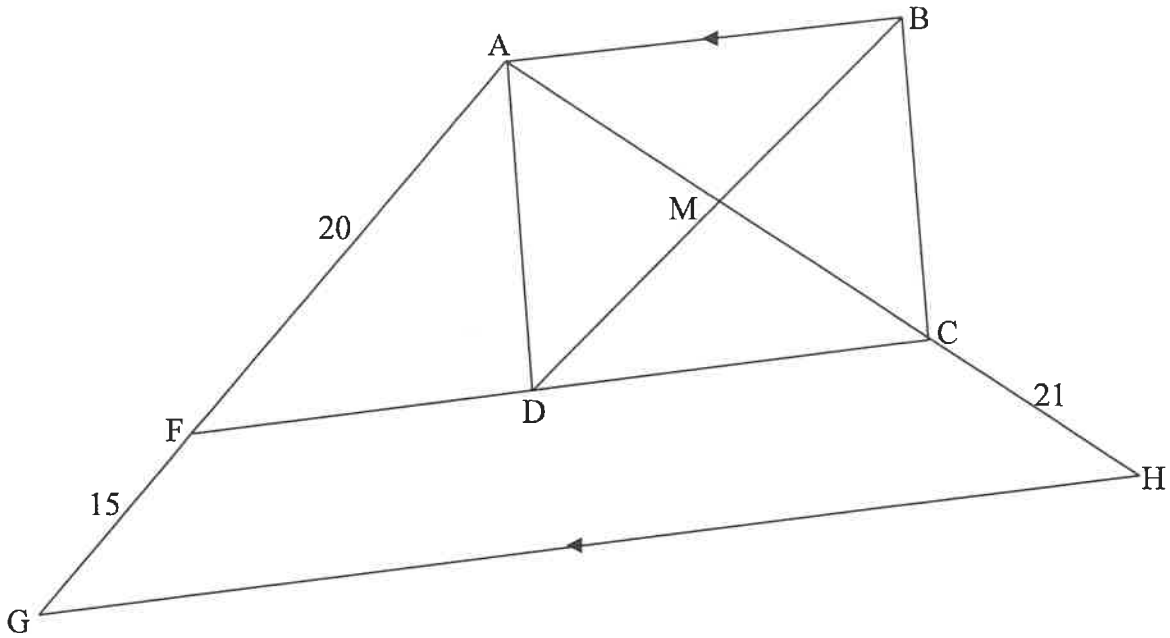
8.1.2 $\hat{M}_2$ (2)

8.1.3 $\hat{N}_1$ (1)

8.1.4 $\hat{O}_2$ (2)

8.1.5 $\hat{N}_2$ (3)

8.2 In the diagram, ΔAGH is drawn. F and C are points on AG and AH respectively such that AF = 20 units, FG = 15 units and CH = 21 units. D is a point on FC such that ABCD is a rectangle with AB also parallel to GH. The diagonals of ABCD intersect at M, a point on AH.

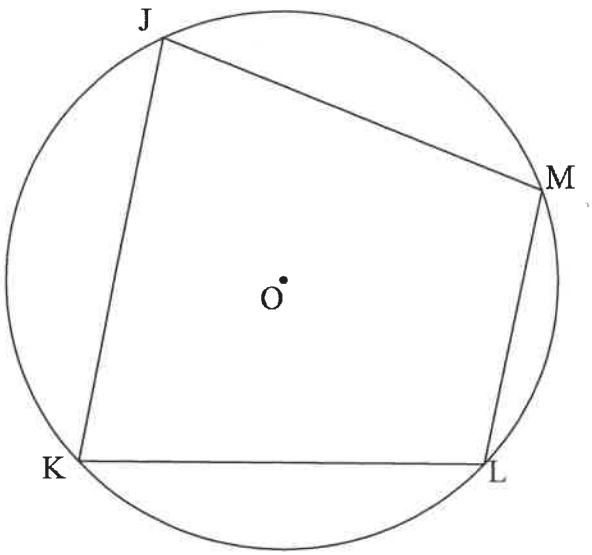


8.2.1 Explain why FC || GH. (1)

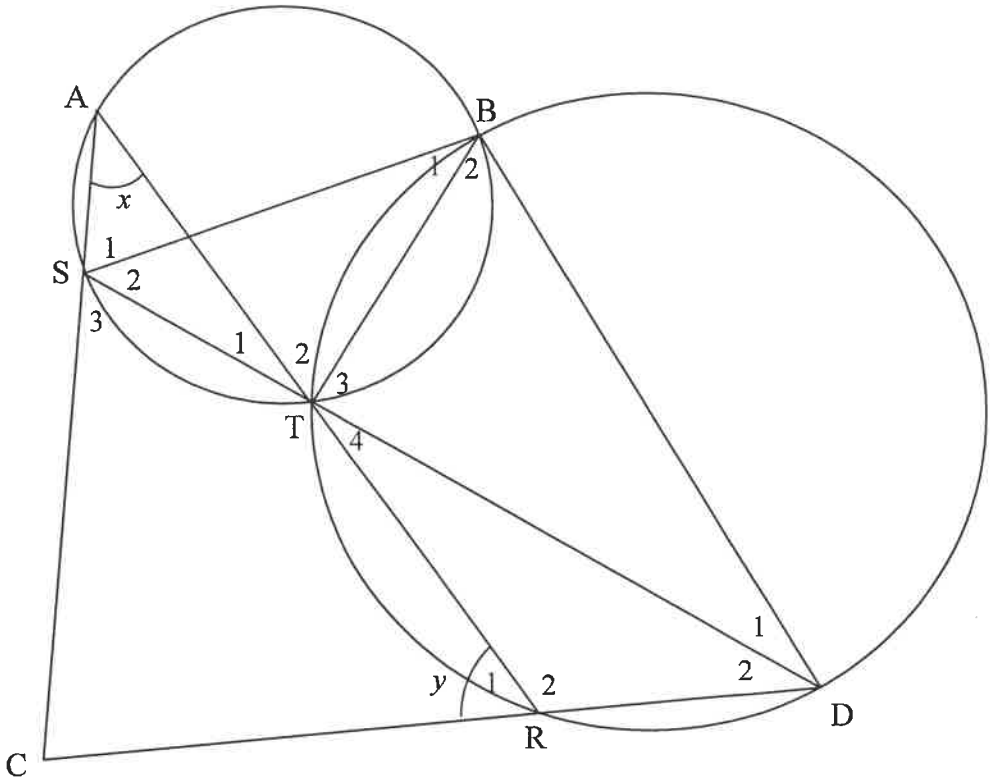
8.2.2 Calculate, with reasons, the length of DM. (5)
[16]

QUESTION 9

9.1 In the diagram, JKLM is a cyclic quadrilateral and the circle has centre O.
Prove the theorem which states that $\hat{J} + \hat{L} = 180°$. (5)



9.2 In the diagram, a smaller circle ABTS and a bigger circle BDRT are given. BT is a common chord. Straight lines STD and ATR are drawn. Chords AS and DR are produced to meet in C, a point outside the two circles. BS and BD are drawn. $\hat{A} = x$ and $\hat{R}_1 = y$.



9.2.1 Name, giving a reason, another angle equal to:

(a) x (2)

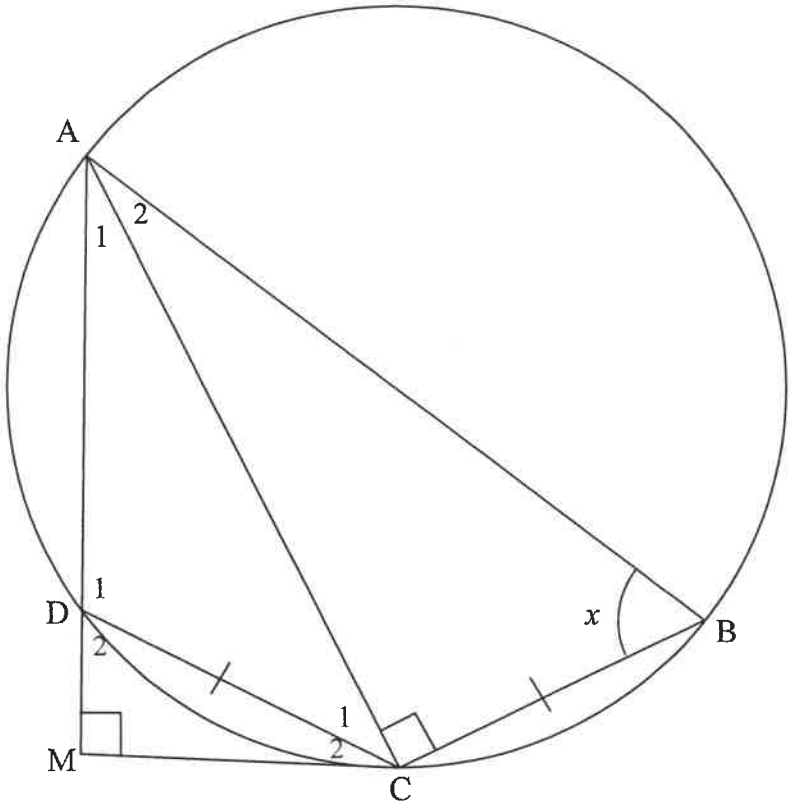
(b) y (2)

9.2.2 Prove that SCDB is a cyclic quadrilateral. (3)

9.2.3 It is further given that $\hat{D}_2 = 30°$ and $A\hat{S}T = 100°$.
Prove that SD is not a diameter of circle BDS. (4)
[16]

QUESTION 10

In the diagram, ABCD is a cyclic quadrilateral such that $AC \perp CB$ and $DC = CB$. AD is produced to M such that $AM \perp MC$. Let $\hat{B} = x$.



10.1 Prove that:

10.1.1 MC is a tangent to the circle at C (5)

10.1.2 $\Delta ACB \,|||\, \Delta CMD$ (3)

10.2 Hence, or otherwise, prove that:

10.2.1 $\dfrac{CM^2}{DC^2} = \dfrac{AM}{AB}$ (6)

10.2.2 $\dfrac{AM}{AB} = \sin^2 x$ (2)

[16]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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Candidate Barcode label/Stafieskodeplakker

NSC Answer Book
NSS-antwoordeboek

National Senior Certificate/*Nasionale Senior Sertifikaat* (Grade 12/*Graad 12*)

CENTRE NUMBER *SENTRUMNOMMER*	
EXAMINATION NUMBER *EKSAMENNOMMER*	
DATE *DATUM*	
BOOK NUMBER *BOEKNOMMER*	OF *VAN* BOOKS *BOEKE*
SUBJECT CODE *VAKKODE*	
PAPER NUMBER *VRAESTELNOMMER*	2
SUBJECT NAME *VAKNAAM*	**MATHEMATICS/*WISKUNDE***

MARKER/*NASIENER*				MODERATOR'S INITIALS IN RELEVANT BLOCK *MODERATOR SE VOORLETTERS IN RELEVANTE BLOKKIE*											
Question *Vraag*	Marks *Punte*	Marker's Code & Initials *Nasiener se kode & Voorletters*	Marks *Punte*	SM	Marks *Punte*	DCM *AHN*	Marks *Punte*	CM *HN*	Marks *Punte*	IM	Marks *Punte*	EM
1												
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10												
		TOTAL *TOTAAL*										

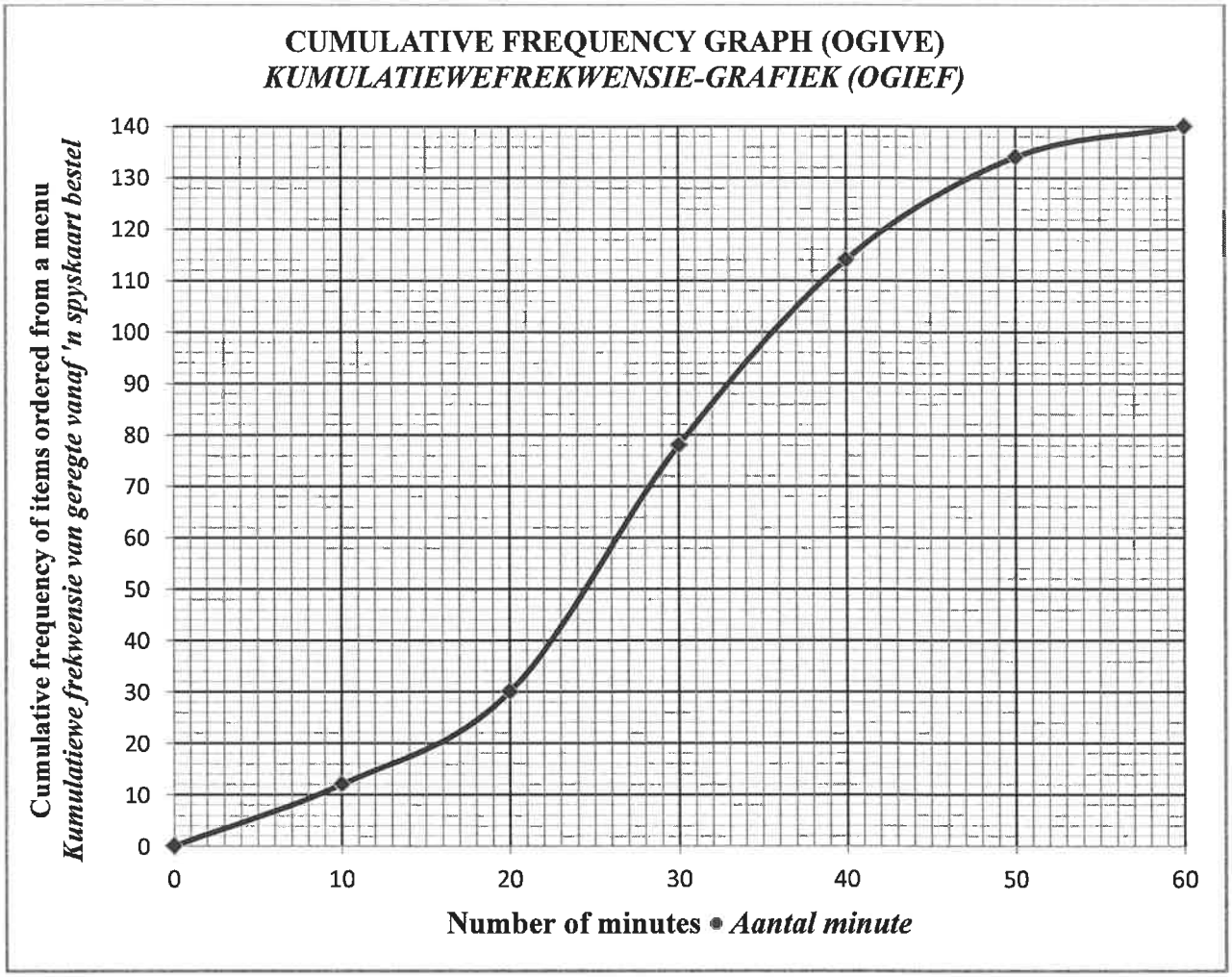
CONTROLLED AND CERTIFIED CORRECT (SURNAME AND INITIALS OF EA)
GEKONTROLEER EN AS KORREK GESERTIFISEER (VAN EN VOORLETTERS VAN EA)

READ INSTRUCTIONS ON THE NEXT PAGE.
LEES INSTRUKSIES OP VOLGENDE BLADSY.

This answer book consists of 31 pages./*Hierdie antwoordeboek bestaan uit 31 bladsye.*

PLEASE FOLLOW THESE INSTRUCTIONS CAREFULLY	VOLG ASSEBLIEF HIERDIE INSTRUKSIES NOUKEURIG
1. Clearly write your examination number and centre number in the space provided and attach your barcode label in the space provided.	1. Skryf jou eksamennommer en sentrumnommer duidelik in die ruimtes verskaf en plak jou stafieskodeplakker in die ruimte verskaf.
2. Remember that your own name (or the name of your school) may NOT appear anywhere on or in this answer book.	2. Onthou dat jou eie naam (of die naam van jou skool) NIE op of in hierdie antwoordeboek mag voorkom NIE.
3. Answer ALL questions in the spaces provided.	3. Beantwoord ALLE vrae in die ruimtes wat verskaf is.
4. NO pages may be torn from this answer book.	4. GEEN bladsye mag uit hierdie antwoordeboek geskeur word NIE.
5. Read the instructions printed on your timetable carefully as well as any other instructions which may be given in each question paper.	5. Lees die instruksies, wat op jou eksamenrooster gedruk is, sorgvuldig deur, asook enige ander instruksies wat op elke vraestel gegee word.
6. Candidates may NOT retain an answer book or remove it from the examination room.	6. GEEN antwoordeboek mag deur die kandidaat behou of uit die eksamenlokaal verwyder word NIE.
7. Answers must be written in black/blue ink as distinctly as possible. Do NOT write in the margins.	7. Skryf die antwoorde so duidelik moontlik met swart/blou ink. Laat die kantlyne oop.
8. Write the numbers of the questions you have answered on the front cover of the answer book where marks are to be recorded.	8. Skryf die nommers van die vrae wat jy beantwoord het op die voorblad van die antwoordeboek waar die punte aangebring word.
9. If you require additional space for your answers: 9.1 Use the additional space provided at the end of the answer book. 9.2 When answering a question in the additional space, clearly indicate the question number in the column on the LHS. 9.3 Rule off after each answer.	9. In geval jy addisionele ruimte benodig vir jou antwoorde: 9.1 Gebruik die addisionele ruimte wat aan die einde van die antwoordeboek verskaf word. 9.2 As 'n vraag in die addisionele ruimte beantwoord word, dui duidelik die vraagnommer in die kolom aan die LK aan. 9.3 Trek 'n lyn na elke antwoord.
10. Draw a neat line through any work/rough work that must not be marked.	10. Trek 'n netjiese lyn deur enige werk/rofwerk wat nie nagesien moet word nie.

QUESTION/*VRAAG* 1



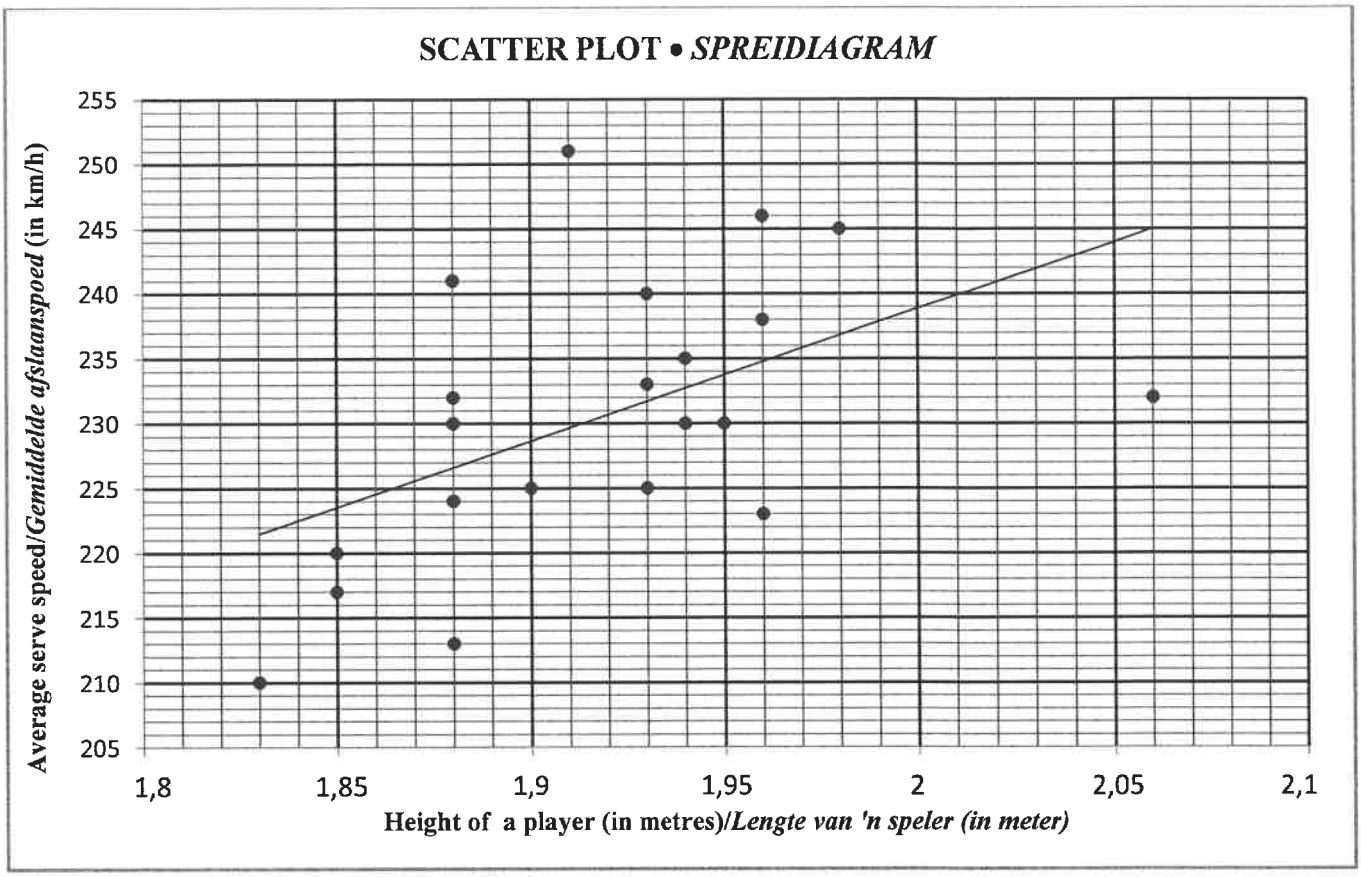
	Solution/*Oplossing*	Marks *Punte*
1.1.1		(1)
1.1.2		(1)
1.1.3		(1)

	Solution/*Oplossing*	**Marks** ***Punte***
1.1.4		(2)
1.1.5		(2)
1.1.6		(2)

35	70	75	80	80
90	100	100	105	105
110	110	115	120	125

	Solution/*Oplossing*	**Marks** ***Punte***
1.2.1(a)		(2)
1.2.1(b)		(2)
1.2.2(a)		(1)
1.2.2(b)		(1)
		[15]

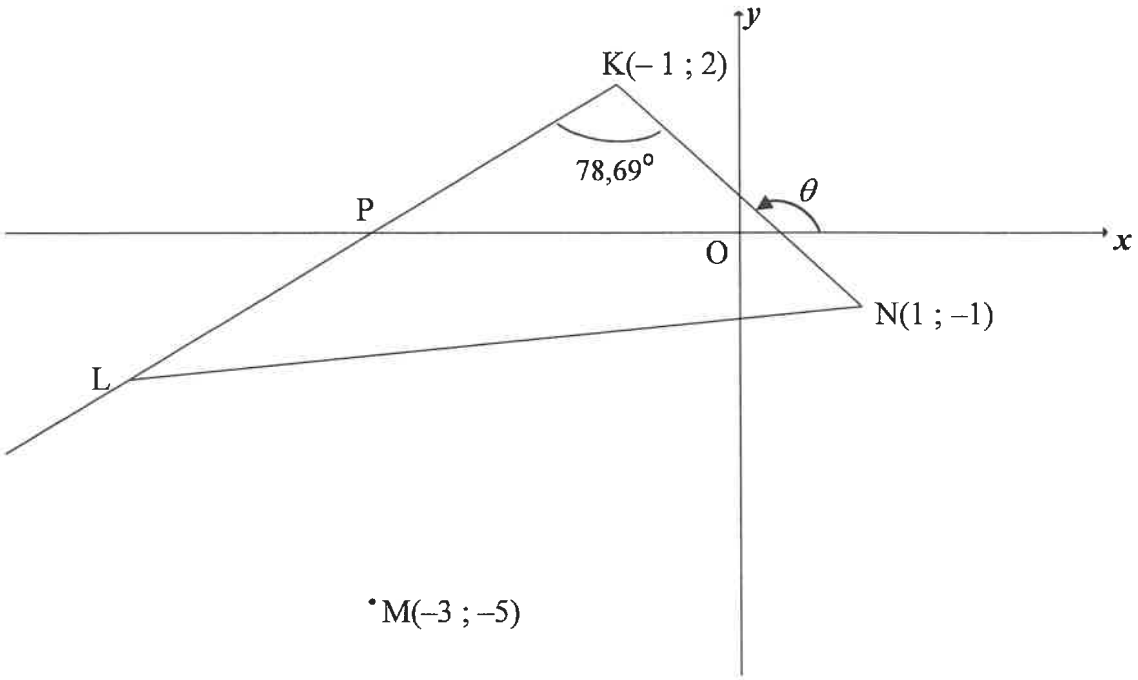
QUESTION/*VRAAG* 2



	Solution/*Oplossing*	Marks *Punte*
2.1		(1)
2.2.1		(1)
2.2.2		(1)

	Solution/*Oplossing*	**Marks** ***Punte***
2.3		(1)
2.4		(1)
		[5]

QUESTION/*VRAAG* 3

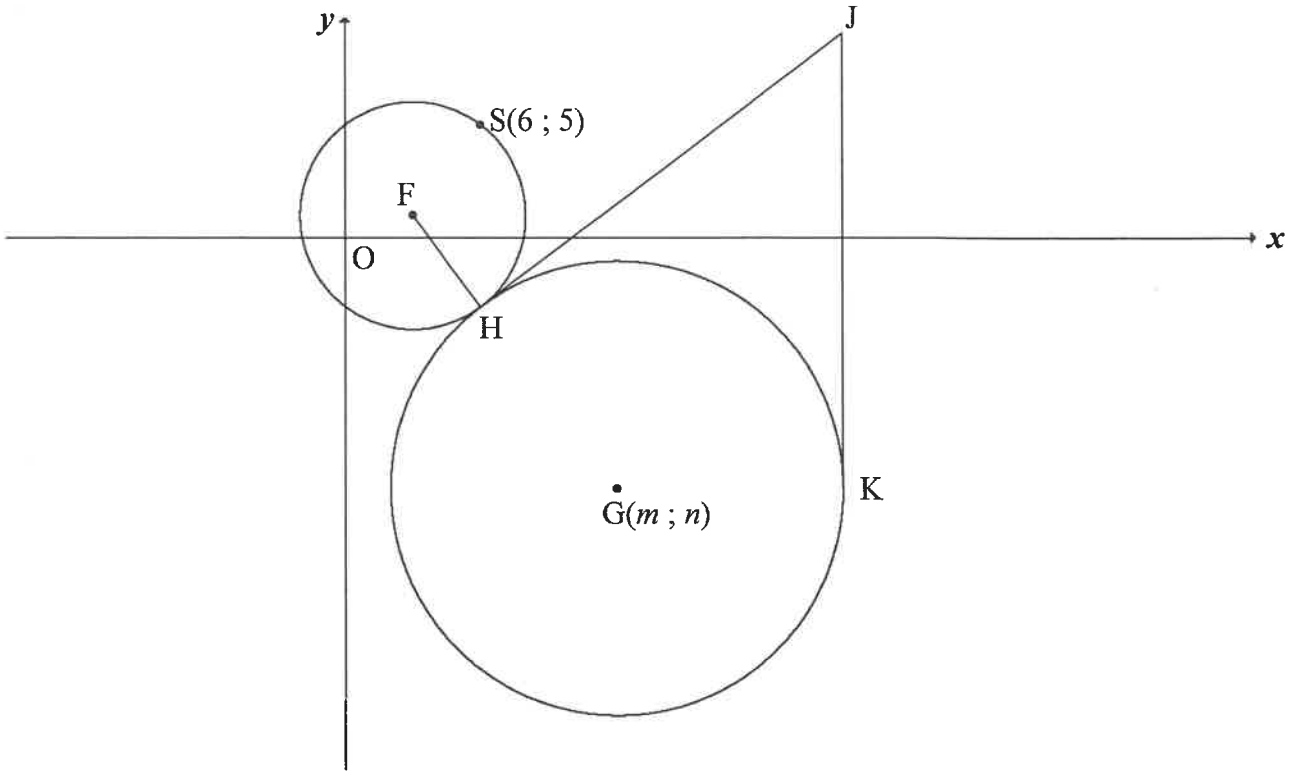


	Solution/*Oplossing*	**Marks** ***Punte***
3.1.1		(2)
3.1.2		(2)

	Solution/*Oplossing*	**Marks** ***Punte***
3.2		(2)
3.3		(2)
3.4		(2)
3.5.1		(5)

	Solution/*Oplossing*	**Marks** ***Punte***
3.5.2		(3)
3.6		(4)
		[22]

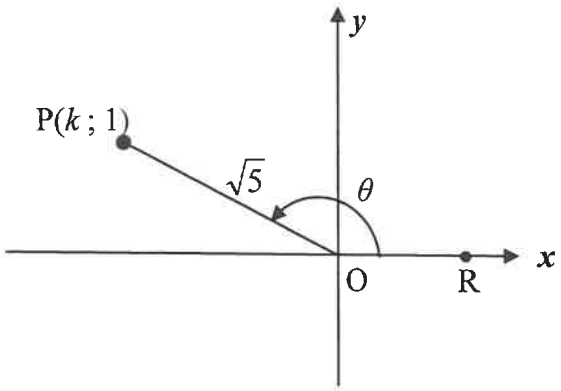
QUESTION/*VRAAG* 4



	Solution/*Oplossing*	Marks *Punte*
4.1		(2)
4.2		(2)
4.3		(1)
4.4		(1)

	Solution/*Oplossing*	**Marks** ***Punte***
4.5.1		(4)
4.5.2		(1)
4.5.3		(7)
		[18]

QUESTION/*VRAAG* 5

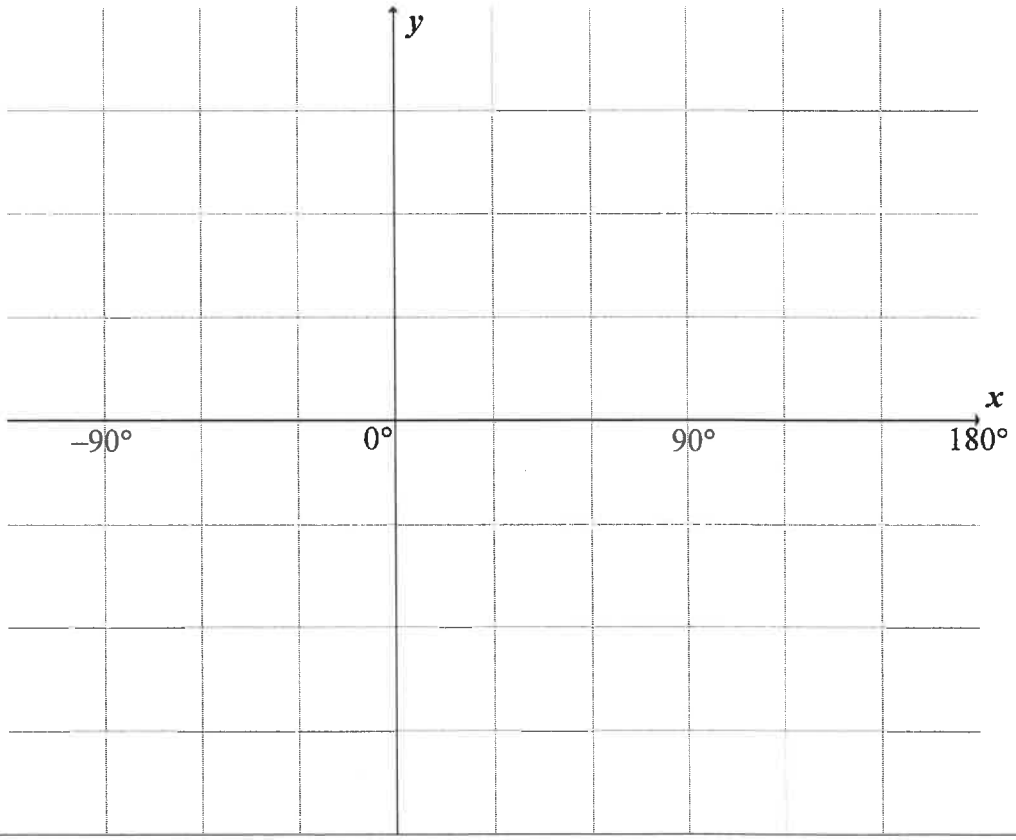


	Solution/*Oplossing*	Marks *Punte*
5.1.1		(2)
5.1.2(a)		(1)
5.1.2(b)		(2)

	Solution/*Oplossing*	**Marks** ***Punte***
5.1.2(c)		(5)
5.1.3		(3)

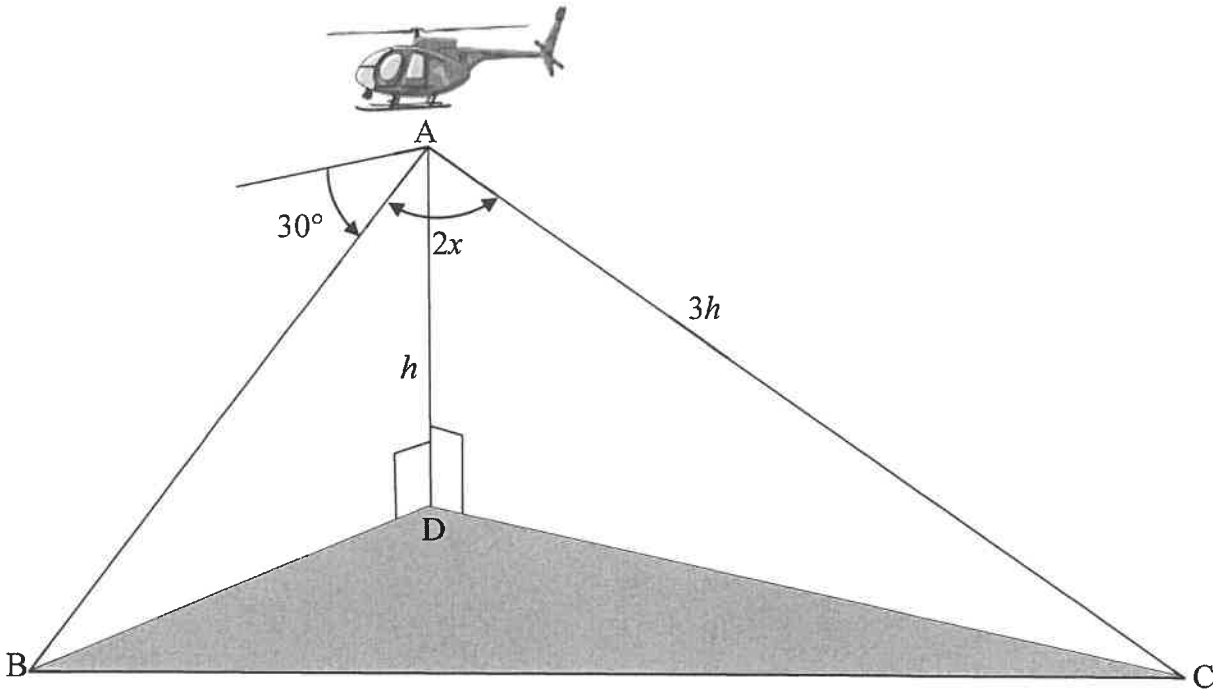
	Solution/*Oplossing*	**Marks** ***Punte***
5.2		(5)
5.3		(5)
		[23]

QUESTION/*VRAAG* 6

	Solution/*Oplossing*	**Marks** ***Punte***
6.1		(1)
6.2		(3)
6.3		(4)

	Solution/*Oplossing*	**Marks** ***Punte***
6.4		(3)
6.5		(2)
		[13]

QUESTION/*VRAAG* 7

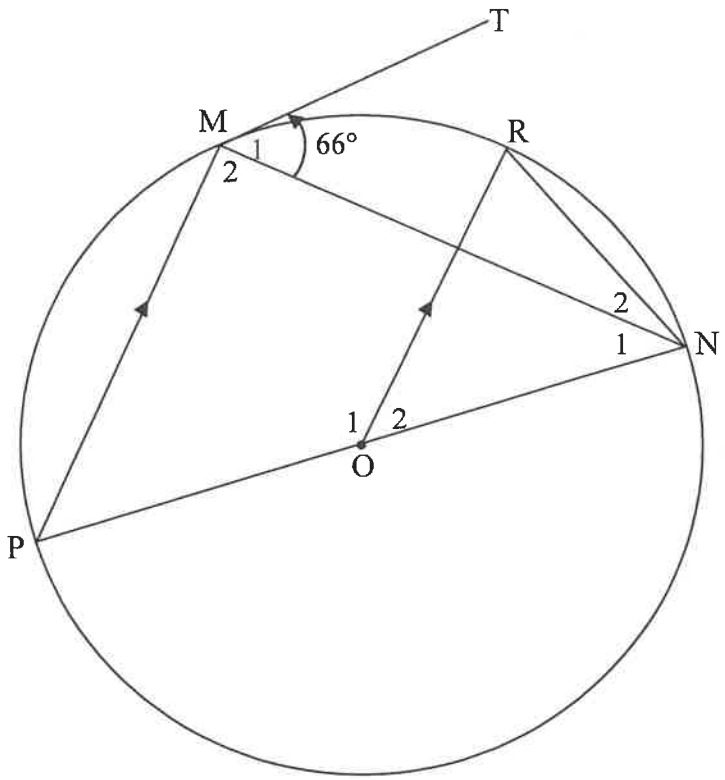


	Solution/*Oplossing*	**Marks** ***Punte***
7.1		(2)

	Solution/*Oplossing*	**Marks** ***Punte***
7.2		(4)
		[6]

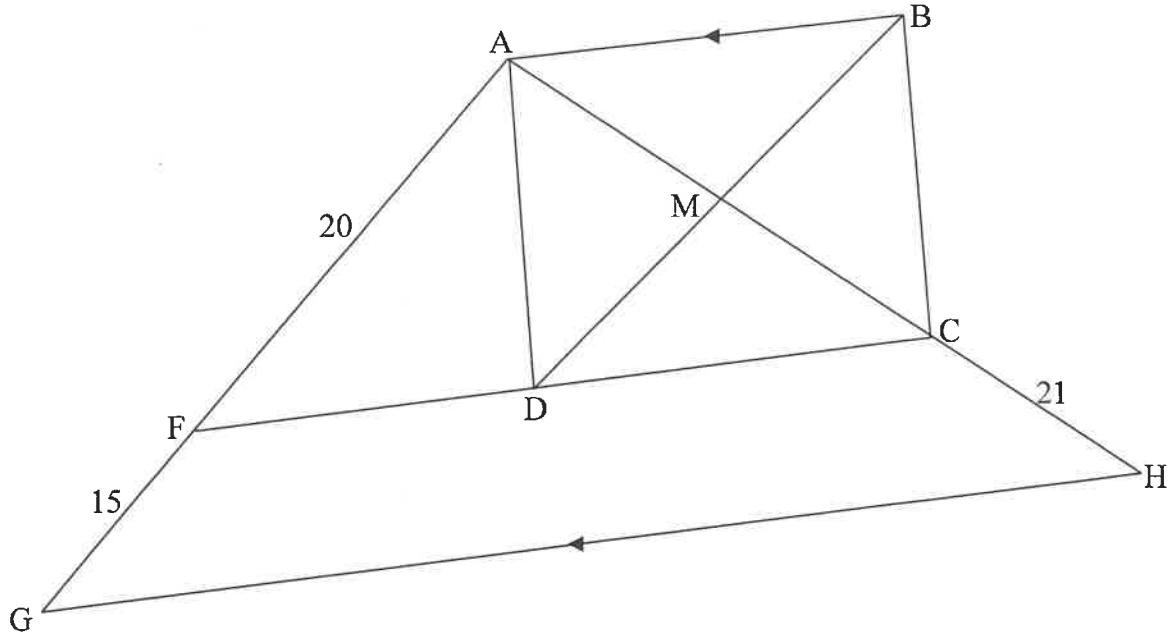
QUESTION/*VRAAG* 8

8.1



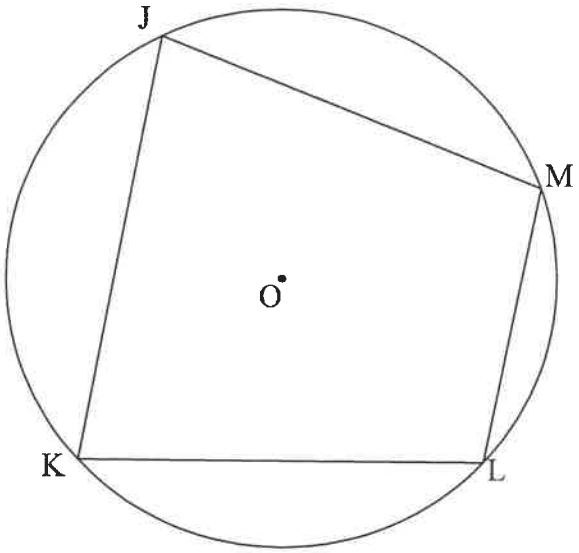
	Solution/*Oplossing*	**Marks** ***Punte***
8.1.1		(2)
8.1.2		(2)

	Solution/*Oplossing*	**Marks** ***Punte***
8.1.3		(1)
8.1.4		(2)
8.1.5		(3)

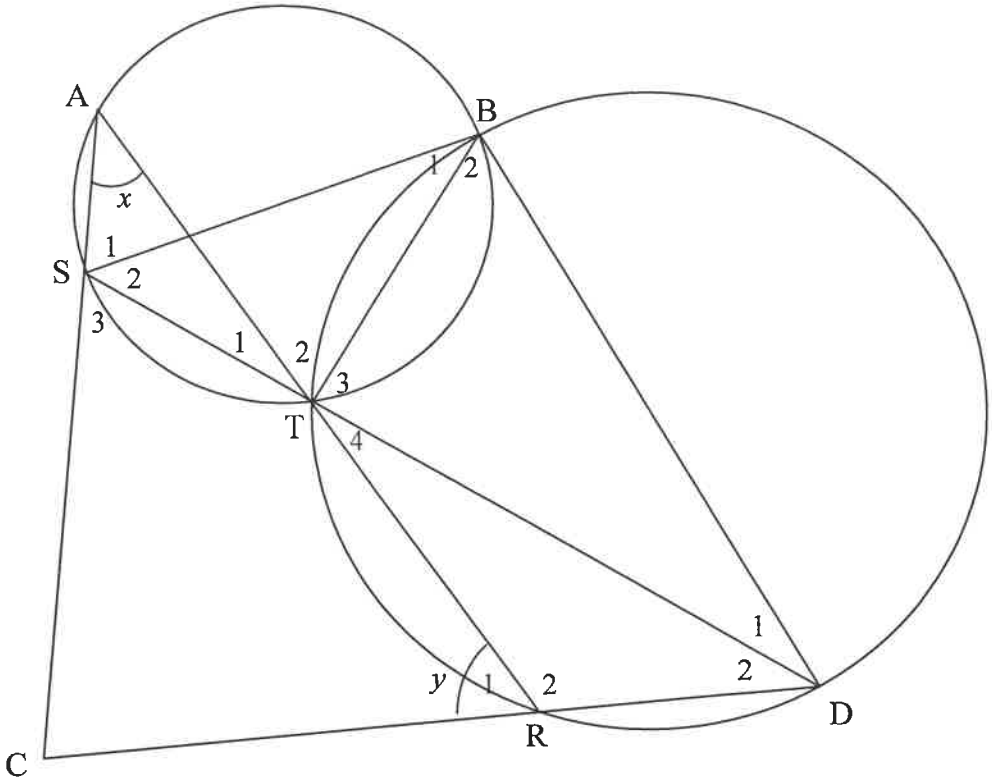


	Solution/*Oplossing*	**Marks *Punte***
8.2.1		(1)
8.2.2		(5)
		[16]

QUESTION/*VRAAG* 9



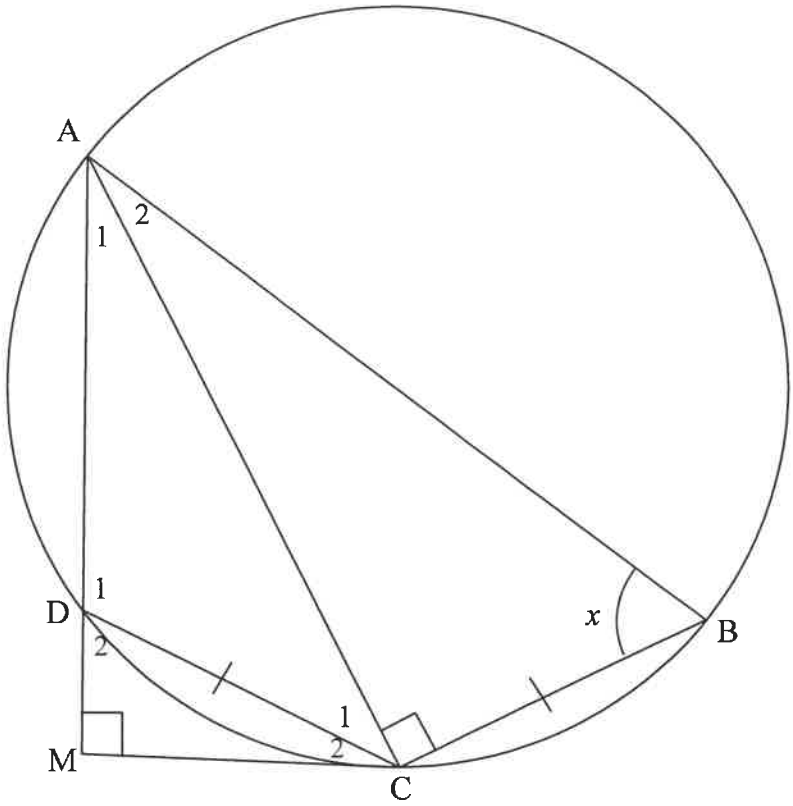
	Solution/*Oplossing*	**Marks** ***Punte***
9.1		(5)



	Solution/*Oplossing*	**Marks** ***Punte***
9.2.1(a)		(2)
9.2.1(b)		(2)

	Solution/*Oplossing*	**Marks** ***Punte***
9.2.2		(3)
9.2.3		(4)
		[16]

QUESTION/*VRAAG* 10



	Solution/*Oplossing*	**Marks** ***Punte***
10.1.1		(5)

	Solution/*Oplossing*	**Marks** ***Punte***
10.1.2		(3)
10.2.1		(6)
10.2.2		(2)
		[16]

	Additional space/*Bykomende ruimte*	**Marks** ***Punte***

	Additional space/*Bykomende ruimte*	**Marks** ***Punte***

	Additional space/*Bykomende ruimte*	**Marks** ***Punte***

	Additional space/*Bykomende ruimte*	**Marks** ***Punte***

TOTAL/*TOTAAL*: 150

RE-MARK/RE-CHECK *HERMERK/HERSIEN*				
Question *Vraag*	**Marks** *Punte*			**Initials** *Voorletters*
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
TOTAL *TOTAAL*				
HASH TOTAL *KAF-TOTAAL*				

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2018

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

These marking guidelines consist of 23 pages.
Hierdie nasienriglyne bestaan uit 23 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement** **(A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ***('n Punt vir 'n bewering is onafhanklik van die rede)***
R	**A mark for the correct reason** **(A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ***('n Punt word slegs vir die rede toegeken as die bewering korrek is)***
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

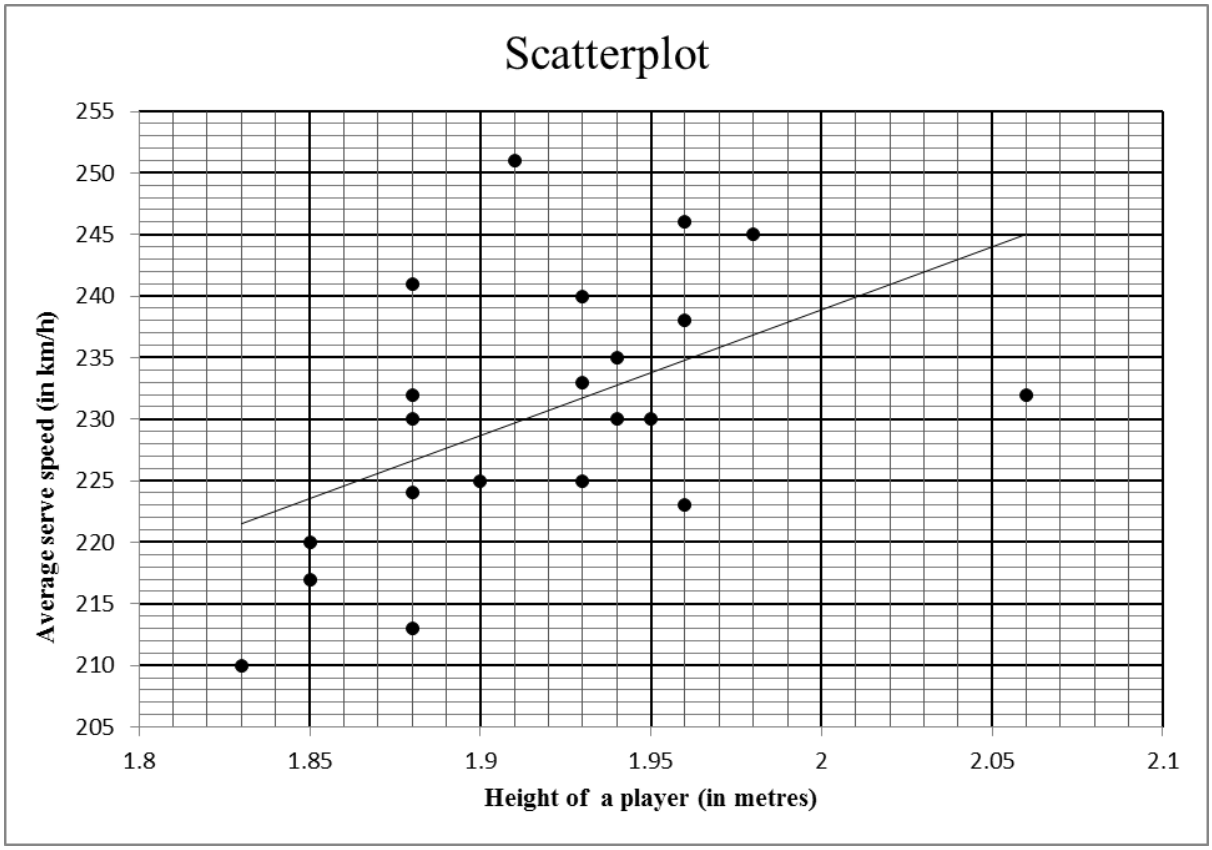
QUESTION/*VRAAG* 1

1.1.1	140 items	answer (1)
1.1.2	Modal class/*modale klas:* $20 < x \leq 30$ minutes **OR/*OF*** $20 \leq x < 30$ minutes	answer (1) answer (1)
1.1.3	Number of minutes taken = 20 minutes	answer (1)
1.1.4	140 – 126 [Accept: 124 to 128] 14 orders (12 to 16) Answer only: Full marks	126 answer (2)
1.1.5	75^{th} percentile is at 105 items =37 minutes [accept 36 – 38 minutes] Answer only: Full marks	105 answer (2)
1.1.6	Lower quartile is at 35 items =21,5 min [accept 21 – 23 min] IQR = 37 – 21,5 = 15,5 min [accept 13 – 17 min] Answer only: Full marks	lower quartile (Q_1) answer (2)

35	70	75	80	80
90	100	100	105	105
110	110	115	120	125

1.2.1(a)	$\bar{x} = \frac{1420}{15}$ $= \text{R}94{,}666.. = R94{,}67$ Answer only: Full marks	1420 answer (2)
1.2.1(b)	$\sigma = R22{,}691... = \text{R}22{,}69$	answer (2)
1.2.2(a)	They both collected the **same (equal) amount** in tips, i.e. R1 420 over the 15-day period. *Hulle albei het **dieselfde bedrag** met fooitjies ontvang, nl. R1 420 oor die 15 dae-tydperk*	answer (1)
1.2.2(b)	Mary's standard deviation is smaller than Reggie's which suggests that there was **greater variation in the amount of tips that Reggie collected** each day compared to the number of tips that Mary collected each day. *Marie se standaardafwyking is kleiner as Reggie s'n wat beteken dat daar **groter variasie/verspreiding in die fooitjies was wat Reggie** elke dag **ontvang** het in vergelyking met die getal fooitjies wat Marie elke dag ontvang het.*	explanation (1)
		[15]

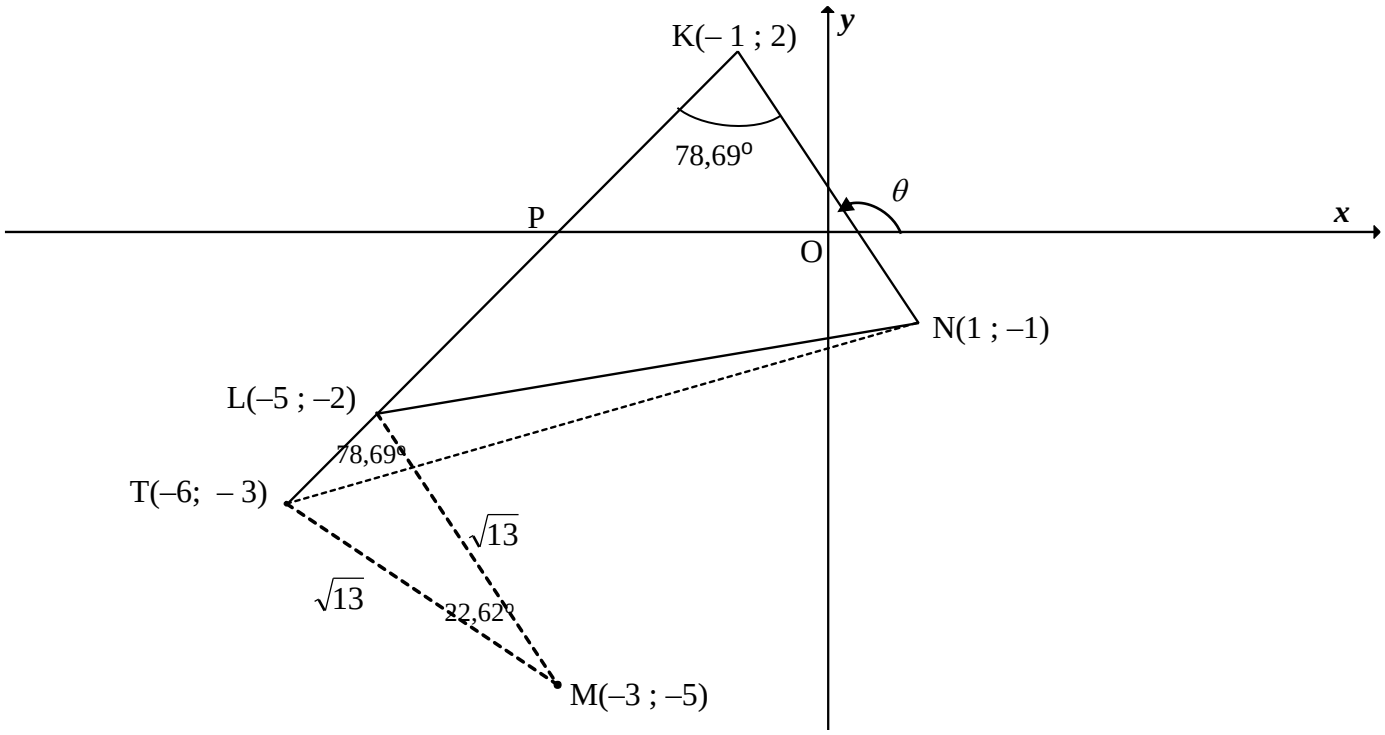
QUESTION/*VRAAG* 2



2.1	251 km/h	answer (1)
2.2.1	r = 0,52 **OR** C	answer (1)
2.2.2	The points are **fairly scattered** and the least squares regression line is increasing. *Die punte is **redelik verspreid** en die kleinstekwadrate-regressielyn neem toe.*	reason (1)
2.3	There is a weak positive relation hence the height could have an influence *Daar is 'n swak positiewe verband, tog kan die lengte 'n invloed hê.* **OR/*OF*** There is no conclusive evidence that the height of a player will influence his/her tennis serve speed. *Daar is geen duidelike bewys dat die lengte van die speler sy/haar afslaanspoed kan beïnvloed nie.* **OR/*OF*** There is no conclusive evidence that a taller person will serve faster than a shorter person. *Daar is geen duidelike bewys dat 'n langer speler vinniger sal afslaan as 'n korter een nie.*	answer (1) answer (1) answer (1)

2.4	For (0 ; 27,07), it means that the player has a height of 0 m but can serve at a speed of 27,07 km/h. **It is impossible for a person to have a height of 0 m**. *(0 ; 27,07) beteken dat 'n speler 'n lengte van 0* m *kan hê en teen 'n spoed van 27,07* km/h *kan afslaan.* ***Dit is onmoontlik om 'n lengte van 0*** **m** ***te hê.*** **OR/OF** This means that the **player does not exist and therefore cannot serve and have a serve speed.** *Dit beteken dat die* ***speler nie bestaan nie en daarom nie kan afslaan en 'n afslaanspoed hê nie.***	explanation (1) explanation (1)
		[5]

QUESTION/*VRAAG* 3

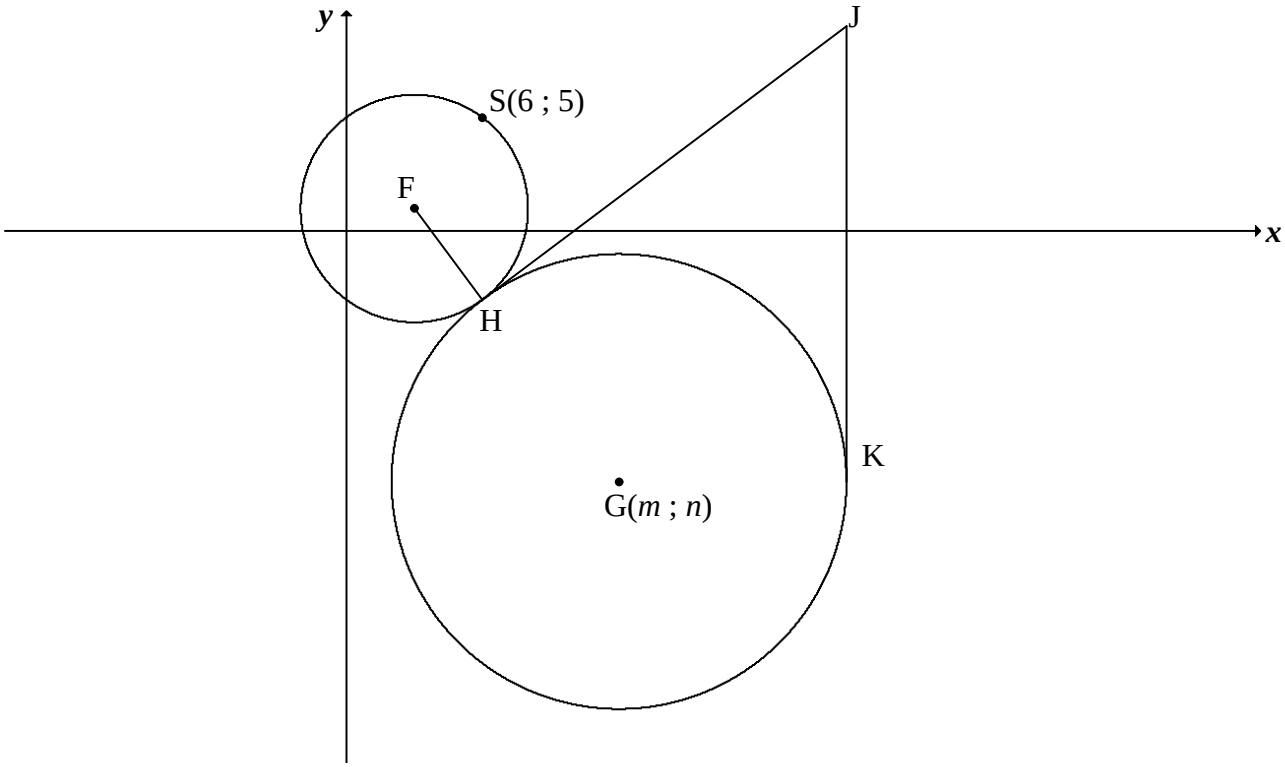


3.1.1	$m_{KN} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{KN} = \frac{2-(-1)}{-1-1}$ $= -\frac{3}{2}$ Answer only: Full marks	✓ correct substitution ✓ answer (2)
3.1.2	$\tan\theta = m_{KN} = -\frac{3}{2}$ $\theta = 180° - 56{,}31°$ $\theta = 123{,}69°$ Answer only: Full marks	✓ $\tan\theta = m_{KN} = -\frac{3}{2}$ ✓ answer (2)
3.2	Inclination KL = $123{,}69° - 78{,}69° = 45°$ [ext $\angle\Delta$] $\tan 45° = m_{KL} = 1$	✓ S ✓ $\tan 45° = m_{KL} = 1$ (2)
3.3	$y = x + c$ $2 = -1 + c$ $c = 3$ $y = x + 3$ **OR/*OF*** $y - y_1 = 1(x - x_1)$ $y - 2 = 1(x - (-1))$ $y = x + 3$	✓ substitute (– 1 ; 2) and m ✓ equation (2) ✓ substitute (– 1 ; 2) and m ✓ equation (2)

3.4	$KN = \sqrt{(1+1)^2 + (-1-2)^2}$ $KN = \sqrt{13}$ or 3,61 Answer only: Full marks	substitute K and N into distance formula answer (2)
3.5.1	$(x+3)^2 + (y+5)^2 = 13$ ...(1) L is a point on KL $y = x + 3$ …(2) (2) in (1): $(x+3)^2 + (x+3+5)^2 = 13$ $x^2 + 6x + 9 + x^2 + 16x + 64 = 13$ $2x^2 + 22x + 60 = 0$ $x^2 + 11x + 30 = 0$ $(x+5)(x+6) = 0$ $x = -5$ or $x = -6$ $y = -2$ or $y = -3$ L(–5 ; –2) or (–6 ; –3) **OR/OF** $(x+3)^2 + (y+5)^2 = 13$ ...(1) L is a point on KL $y = x + 3$ $\therefore x = y - 3$ …(2) (2) in (1): $(y-3+3)^2 + (y+5)^2 = 13$ $y^2 + y^2 + 10y + 25 = 13$ $2y^2 + 10y + 12 = 0$ $y^2 + 5y + 6 = 0$ $(y+2)(y+3) = 0$ $y = -2$ or $y = -3$ $x = -5$ or $x = -6$ L(–5 ; –2) or (–6 ; –3)	equation (1) substituting eq (2) standard form x-values y-values (5) equation (1) substituting eq (2) standard form y-values (both) x-values (both) (5)
3.5.2	Midpoint of KM: (– 2 ; – 1,5) $\therefore \frac{x_L + 1}{2} = -2$ and $\frac{y_L - 1}{2} = -\frac{3}{2}$ $\therefore$ L(– 5 ; – 2) **OR/*OF*** $m_{KN} = m_{LM}$ $\frac{y-(-5)}{x-(-3)} = -\frac{3}{2}$ $2(x+3+5) = -3(x+3)$ $2x + 16 = -3x - 9$ $5x = -25$ Answer only: Full marks $x = -5$ $\therefore$ L(– 5 ; – 2)	midpoint of KM x value y value (3) $m_{LM} = m_{KN}$ x value y value (3)

	OR/*OF* N→M: $(x\,;y) \rightarrow (x-4\,;y-4)$ ∴ L(– 1 – 4 ; 2 – 4) **OR/OF** ∴ L(– 5 ; – 2) N→K: $(x\,;y) \rightarrow (x-2\,;y+3)$ ∴ L(– 3 – 2 ; – 5 + 3) ∴ L(– 5 ; – 2)	transformation *x* value *y* value (3)
3.6	T(–6 ; –3) (from Question 3.5.1) $KT = \sqrt{(-1-(-6))^2 + (2-(-3))^2}$ $= \sqrt{50}$ $KN = \sqrt{13}$ (CA from 3.4) $\text{Area of } \Delta KTN = \frac{1}{2} KT.KN \sin L\hat{K}N$ $= \frac{1}{2}\sqrt{50}.\sqrt{13}\sin 78{,}69°$ = 12,50 square units **OR/*OF*** In ΔKLM: $\frac{TL}{\sin 22{,}62°} = \frac{\sqrt{13}}{\sin 78{,}69°}$ $TL = 1{,}414..$ $KL = \sqrt{(-1-(-5))^2 + (2-(-2))^2}$ $= \sqrt{32}$ ∴ KT = 7,0708… $\text{Area of } \Delta KTN = \frac{1}{2} KT.KN \sin L\hat{K}N$ $= \frac{1}{2}(7{,}0708).\sqrt{13}\sin 78{,}69°$ = 12,50 square units	coordinates of T length of KT substitution into area rule answer (4) length of TL length of KT substitution into area rule answer (4)
		[22]

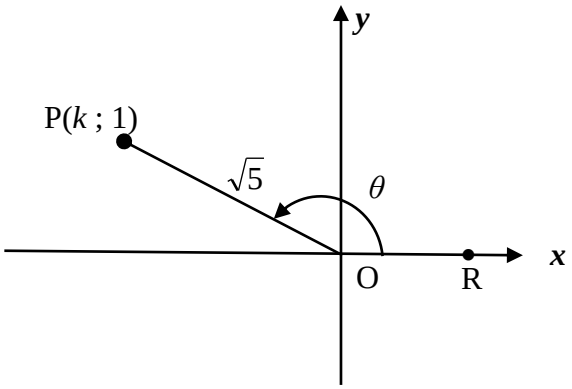
QUESTION/*VRAAG* 4



4.1	F(3;1)	✓*x* value ✓ *y* value (2)
4.2	$FS = \sqrt{(6-3)^2 + (5-1)^2}$ FS = 5	✓ substitution of F & S ✓ answer (2)
4.3	FH(FS) : HG = 1 : 2 $\therefore$ HG = 2 FH = 10	✓ HG = 10 (1)
4.4	Tangents from common/same point / *Raaklyne vanaf gemeenskaplike of dieselfde punt*	✓ answer (1)
4.5.1	$F\hat{H}J = 90°$ [tan $\perp$ radius / *rkl* $\perp$ *radius*] $FJ^2 = 20^2 + 5^2$ [Pyth theorem/*stelling*] FJ = $\sqrt{425}$ or $5\sqrt{17}$ or 20,62	✓ S ✓ R ✓ S ✓ answer (4)
4.5.2	$(x-m)^2 + (y-n)^2 = 100$	✓ answer (1)

4.5.3	K(22; *n*) [radius ⊥ tangent] GK = HG = 10 [radii] FH = FS = 5 [radii] *m* = 22 – 10 *m* = 12 F, H and G are collinear [HJ is a common tangent] *F, H en G is saamlynig* *[HJ is 'n gemeemskaplike raaklyn]* $FG^2 = (12-3)^2 + (n-1)^2$ $15^2 = 81 + (n-1)^2$ $(n-1)^2 = 144$ $n-1 = \pm 12$ **OR/*OF*** $n \neq 13$ or $n = -11$ $\therefore$ G(12; –11) **OR/*OF*** K(22; *n*) [radius ⊥ tangent] GK = HG = 10 [radii] FH = FS = 5 [radii] *m* = 22 – 10 *m* = 12 Let J(22 ; *y*): $FJ^2 = (22-3)^2 + (y-1)^2$ $425 = 361 + y^2 - 2y + 1$ $0 = y^2 - 2y - 63$ $0 = (y-9)(y+7)$ $\therefore$ *y* = 9 or/of $y \neq -7$ $\therefore$ *n* = 9 – 20 = –11 $\therefore$ G(12; –11)	$n^2 - 2n - 143 = 0$ $(n+11)(n-13) = 0$ $n = -11$ or $n \neq 13$ K(22; *n*) value of *m* subst. of F and G in distance formula FG = 15 simplification/ standard form value of *n* coordinates of G (7) K(22; *n*) value of *m* subst. of F and J in distance formula FJ = $\sqrt{425}$ standard form value of *n* coordinates of G (7)
		[18]

QUESTION/*VRAAG* 5

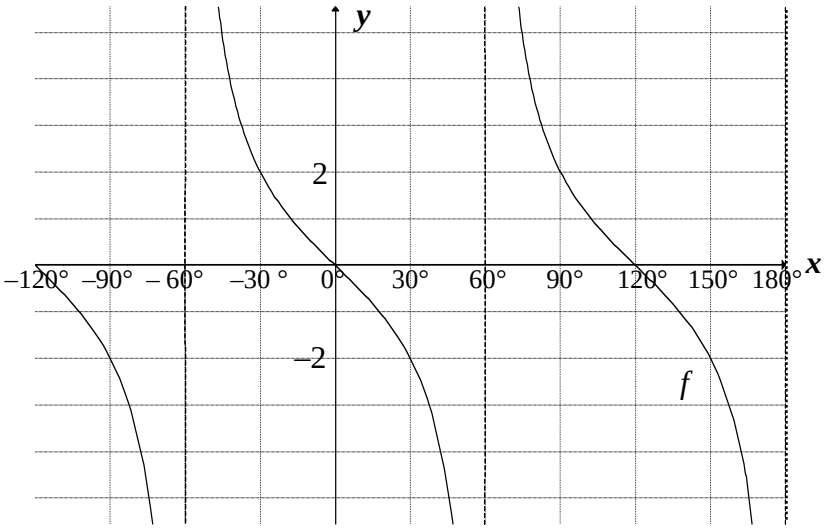


5.1.1	$k^2 = (\sqrt{5})^2 - 1^2$ $= 4$ $k = -2$ Answer only: full marks	substitution into theorem of Pythagoras answer (2)
5.1.2(a)	$\tan\theta = -\frac{1}{2}$	answer (1)
5.1.2(b)	$\cos(180° + \theta) = -\cos\theta$ $= \frac{2}{\sqrt{5}}$ Answer only: full marks	reduction answer (2)
5.1.2(c)	$\sin(\theta + 60°) = \frac{a+b}{\sqrt{20}}$ LHS= $\sin\theta\cos 60° + \cos\theta\sin 60°$ $= \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{2}\right) + \left(-\frac{2}{\sqrt{5}}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{1-2\sqrt{3}}{2\sqrt{5}}$ $= \frac{1-2\sqrt{3}}{\sqrt{20}}$	expansion subst of sin θ subst of cos θ both special $\angle$s $\frac{1-2\sqrt{3}}{2\sqrt{5}}$ (5)
5.1.3	$\tan\theta = -\frac{1}{2}$ $\therefore \theta = 180° - 26{,}57°$ $\therefore \theta = 153{,}43°$ $\tan(2\theta - 40°) = \tan[(2\times 153{,}43°) - 40°]$ $= \tan 266{,}87°$ $= 18{,}3$	θ substitution answer (3)

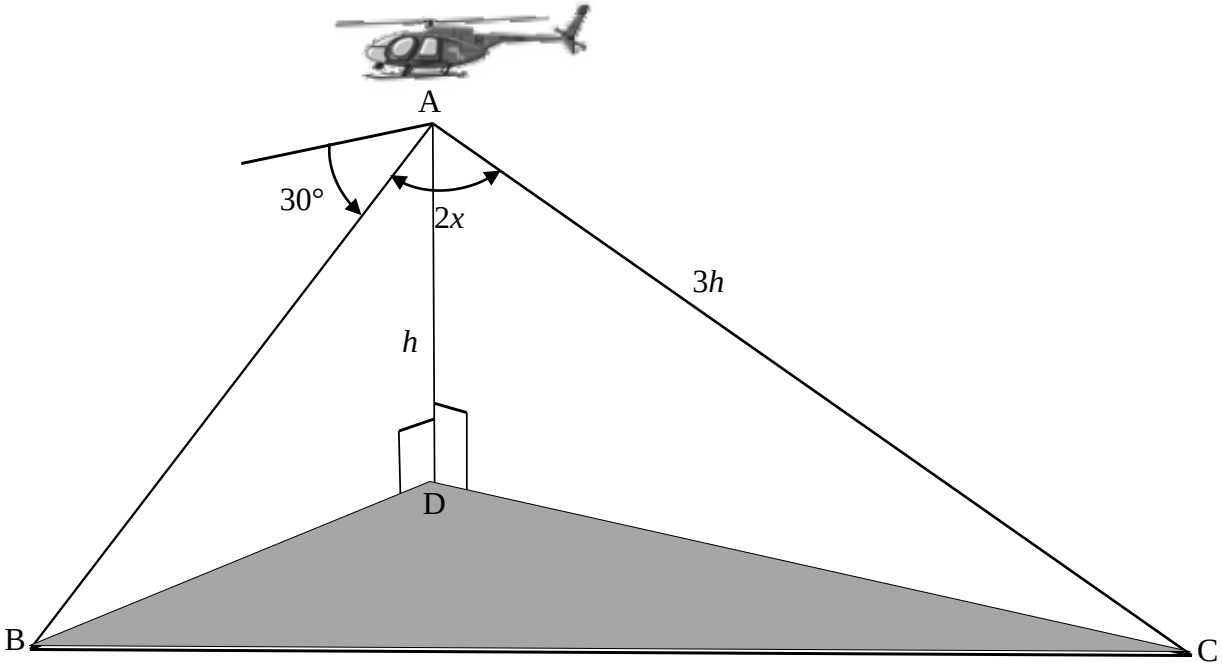
5.2	LHS $= \dfrac{\cos x+\sin x}{\cos x-\sin x} - \dfrac{\cos x-\sin x}{\cos x+\sin x}$ RHS $= 2\tan 2x$	
	$= \dfrac{(\cos x+\sin x)^2 - (\cos x-\sin x)^2}{(\cos x-\sin x)(\cos x+\sin x)}$	✓ single fraction
	$= \dfrac{\cos^2 x+2\sin x\cos x+\sin^2 x-\cos^2 x+2\sin x\cos x-\sin^2 x}{\cos^2 x-\sin^2 x}$	✓ expansion
	$= \dfrac{2(2\sin x\cos x)}{\cos^2 x-\sin^2 x}$	✓ simplification (both)
	$= \dfrac{2\sin 2x}{\cos 2x}$	✓ double $\angle$ identity ✓ double $\angle$ identity
	$= 2\tan 2x$	
	= RHS	(5)
	OR/*OF*	
	LHS $= \dfrac{\cos x+\sin x}{\cos x-\sin x} - \dfrac{\cos x-\sin x}{\cos x+\sin x}$ RHS $= 2\tan 2x$	
	$= \dfrac{(\cos x+\sin x)^2 - (\cos x-\sin x)^2}{(\cos x-\sin x)(\cos x+\sin x)}$	✓ single fraction
	$= \dfrac{(\cos x+\sin x+\cos x-\sin x)(\cos x+\sin x-\cos x+\sin x)}{\cos^2 x-\sin^2 x}$	✓ difference of two squares
	$= \dfrac{(2\cos x)(2\sin x)}{\cos^2 x-\sin^2 x}$	✓ simplification (both)
	$= \dfrac{2(2\sin x\cos x)}{\cos^2 x-\sin^2 x}$	
	$= \dfrac{2\sin 2x}{\cos 2x}$	✓ double $\angle$ identity ✓ double $\angle$ identity
	$= 2\tan 2x$	(5)
	= RHS	
	OR/*OF*	
	RHS $= 2\tan 2x$	
	$= \dfrac{2\sin 2x}{\cos 2x}$	
	$= \dfrac{2(2\sin x.\cos x)}{\cos^2 x-\sin^2 x}$	✓ double $\angle$ identity ✓ double $\angle$ identity
	$= \dfrac{4\sin x.\cos x}{\cos^2 x-\sin^2 x}$	
	$= \dfrac{1+2\sin x.\cos x-(1-2\sin x.\cos x)}{\cos^2 x-\sin^2 x}$	✓ identity & method
	$= \dfrac{(\cos x+\sin x)^2-(\cos x-\sin x)^2}{(\cos x+\sin x)(\cos x-\sin x)}$	✓ factorising numerator and denominator
	$= \dfrac{(\cos x+\sin x)^2}{(\cos x+\sin x)(\cos x-\sin x)} - \dfrac{(\cos x-\sin x)^2}{(\cos x+\sin x)(\cos x-\sin x)}$	✓ writing as 2 terms
	$= \dfrac{\cos x+\sin x}{\cos x-\sin x} - \dfrac{\cos x-\sin x}{\cos x+\sin x} = \text{LHS}$	(5)

5.3	$\sum_{A=38^\circ}^{52^\circ} \cos^2 A$	
	$= \cos^2 38^\circ + \cos^2 39^\circ + \cos^2 40^\circ + ... + \cos^2 51^\circ + \cos^2 52^\circ$	✓ expansion
	$= \sin^2 52^\circ + \sin^2 51^\circ + \sin^2 50^\circ + ... + \cos^2 51^\circ + \cos^2 52^\circ$	✓ co ratio
	$= 7(1) + \cos^2 45^\circ$	✓ $\cos^2 45^\circ$
	$= 7 + \left(\frac{\sqrt{2}}{2}\right)^2$ or $= 7 + \left(\frac{1}{\sqrt{2}}\right)^2$	✓ $7 \times$ identity
	$= 7\frac{1}{2}$	✓ answer (5)
	OR/*OF*	
	$\sum_{A=38^\circ}^{52^\circ} \cos^2 A$	
	$= \cos^2 38^\circ + \cos^2 39^\circ + \cos^2 40^\circ + ... + \cos^2 51^\circ + \cos^2 52^\circ$	✓ expansion
	$= (\cos^2 38^\circ + \sin^2 52^\circ) + (\cos^2 39^\circ + \sin^2 51^\circ) ... + \cos^2 45^\circ$	✓ pairing
	$= 7(1) + \cos^2 45^\circ$	✓ $\cos^2 45^\circ$
	$= 7 + \left(\frac{\sqrt{2}}{2}\right)^2$ or $= 7 + \left(\frac{1}{\sqrt{2}}\right)^2$	✓ $7 \times$ identity
	$= 7\frac{1}{2}$	✓ answer (5)
		[23]

QUESTION/*VRAAG* 6

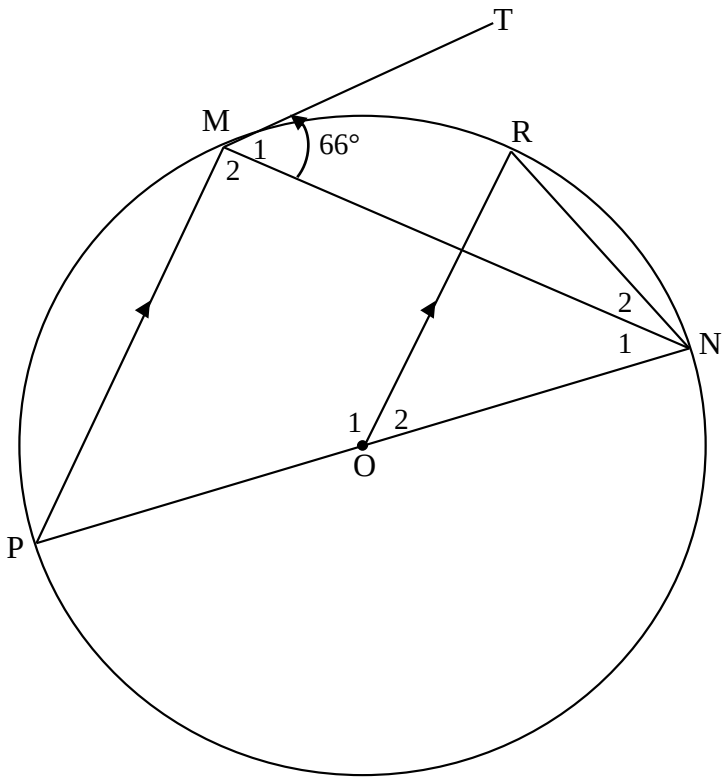
6.1	Period = 120°	✓ answer (1)
6.2	$2 = -2\tan\frac{3}{2}x$ $\tan\left(\frac{3}{2}t\right) = -1$ $\frac{3}{2}t = 135° + k.180°$ **OR/*OF*** $\frac{3}{2}t = -45° + k.180°$ $t = 90° + k.120°\ ;\ k \in Z$ $t = -30° + k.120°\ ;\ k \in Z$ **OR/*OF*** $2 = -2\tan\frac{3}{2}x$ $\tan\left(\frac{3}{2}t\right) = -1$ $\frac{3}{2}t = 135° + k.360°$ **o**r/of $\frac{3}{2}t = 315° + k.360°$ $t = 90° + k.240°$ **o**r.of $t = 210° + k.240°\ ;\ k \in Z$	✓ equating ✓ general solution of $\frac{3}{2}t$ ✓ general solution of t (3) ✓ equating ✓ general solution of $\frac{3}{2}t$ ✓ general solution of t (3)
6.3		✓ asymptotes: $x = \pm 60°$; $x = 180°$ ✓ x-intercepts 0°; ±120° ✓ negative shape ✓ (90° ; 2) or (– 30° ; 2) or (30° ; – 2) or (– 90° ; – 2) (4)
6.4	$x \in (-60° ; -30°]$ or $(60°; 90°]$ **OR/*OF*** $-60° < x \leq -30°$ or $60° < x \leq 90°$	✓ interval ✓ interval ✓ notation (3) ✓ interval ✓ interval ✓ notation (3)
6.5	$g(x) = -2\tan\left[\frac{3}{2}(x + 40°)\right] = f(x + 40°)$ Translation of 40° to the left / skuif met 40° links	✓ Translation of 40° ✓ to the left (2)
		[13]

QUESTION/*VRAAG* 7



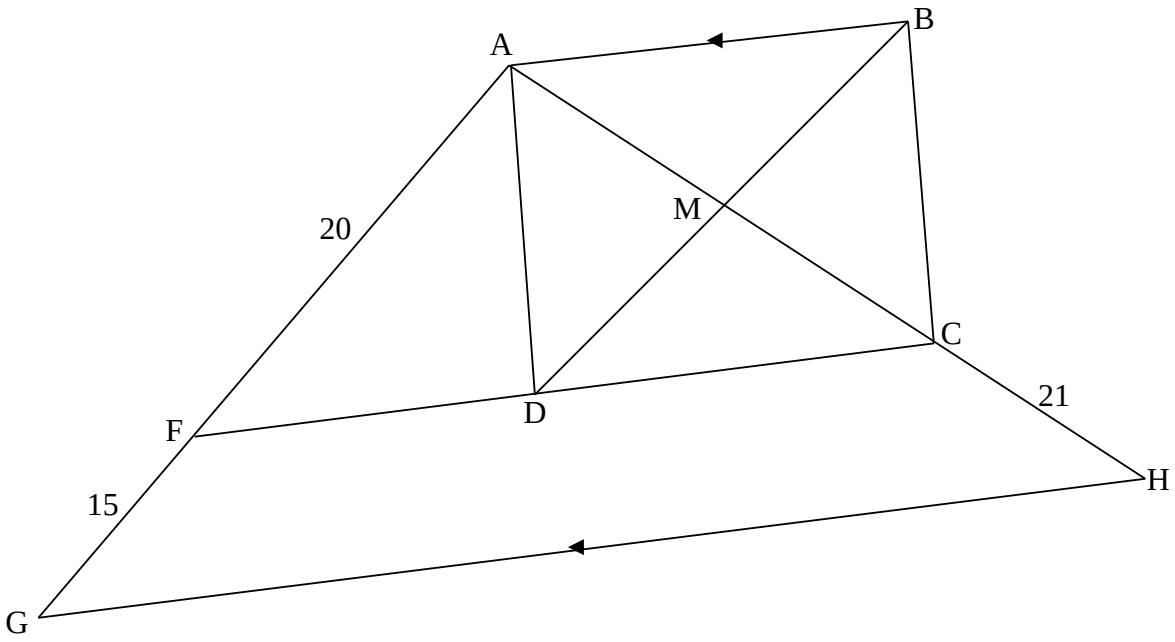
7.1	$A\hat{B}D = 30°$ $\sin 30° = \frac{h}{AB}$ $AB = \frac{h}{\sin 30°}$ **OR** $AB = \frac{h}{\frac{1}{2}}$ **OR** $AB = 2h$ **OR/*OF*** $B\hat{A}D = 60°$ $cos60° = \frac{h}{AB}$ $AB = \frac{h}{\cos 60°}$ **OR** $AB = \frac{h}{\frac{1}{2}}$ **OR** $AB = 2h$	$A\hat{B}D = 30°$ answer (2) $B\hat{A}D = 60°$ answer (2)
7.2	$BC^2 = AB^2 + AC^2 - 2AB.AC\cos B\hat{A}C$ $= (2h)^2 + (3h)^2 - 2(2h)(3h)\cos 2x$ $= 13h^2 - 12h^2(2\cos^2 x - 1)$ $= 13h^2 - 24h^2\cos^2 x + 12h^2$ $= 25h^2 - 24h^2\cos^2 x$ $BC = h\sqrt{25 - 24\cos^2 x}$	use of cosine rule in ΔABC substitution double angle identity $25h^2 - 24h^2\cos^2 x$ (4)
		[6]

QUESTION/*VRAAG* 8



8.1.1	$\hat{P} = \hat{M}_1 = 66°$	[tan chord theorem/*raaklyn koordst*]	✓S ✓R (2)
8.1.2	$\hat{M}_2 = 90°$	[∠ in semi circle/*∠ in halfsirkel*]	✓S ✓R (2)
8.1.3	$\hat{N}_1 = 180° - (90° + 66°)$ $= 24°$	[sum of ∠s of /*som van ∠e* ΔMNP]	✓S (1)
8.1.4	$\hat{O}_2 = \hat{P} = 66°$	[corres. ∠s;/*ooreenk ∠e,* PM || OR]	✓S ✓R (2)
8.1.5	$\hat{R} + \hat{N}_1 + \hat{N}_2 = 180° - 66°$ $= 114°$ $\hat{R} = \hat{N}_1 + \hat{N}_2 = 57°$ $\therefore\ \hat{N}_2 = 33°$	[sum of ∠s of/*som van ∠e* ΔRNO] [∠s opposite = radii/*∠e teenoor = radii*]	✓S ✓S/R ✓S (3)
	OR/*OF* $P\hat{O}R = 114°$ $P\hat{N}R = 57°$ $\therefore\ \hat{N}_2 = 33°$	[∠s on straight line/*∠e op reguitlyn*] [∠ at centre = twice ∠ at circumference/ *midpts∠ = 2 × omtreks∠*]	✓S ✓S/R ✓S (3)

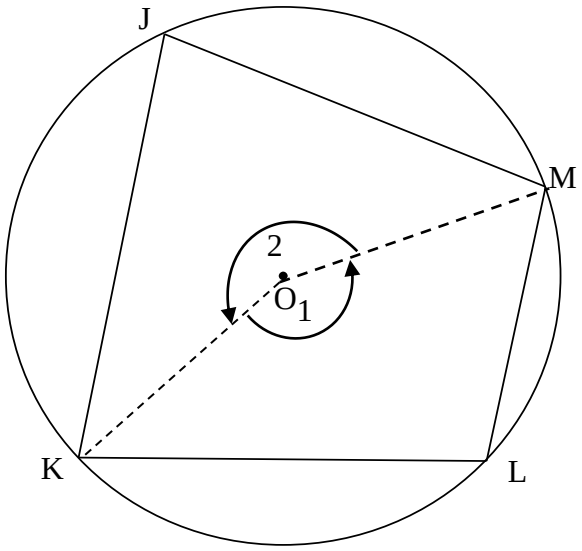
8.2



8.2.1	FC || AB || GH [opp sides of **rectangle**/*teenoorst sye v reghoek*]	✓ R (1)
8.2.2	$\frac{AC}{CH} = \frac{AF}{FG}$ [line || one side of Δ] **OR** [prop theorem; FC || GH] *[lyn || een sy van Δ]* ***OF*** *[eweredighst; FC || GH]* $\frac{AC}{21} = \frac{20}{15}$ $AC = \frac{20 \times 21}{15}$ $= 28$ DB = AC = 28 [diags of rectangle =/*hoeklyne v reghoek* =] DM = $\frac{1}{2}$DB = 14 [diags of rectangle bisect/*hoekl v reghoek halveer*]	✓S ✓R ✓ AC ✓ S ✓ S (5)
		[16]

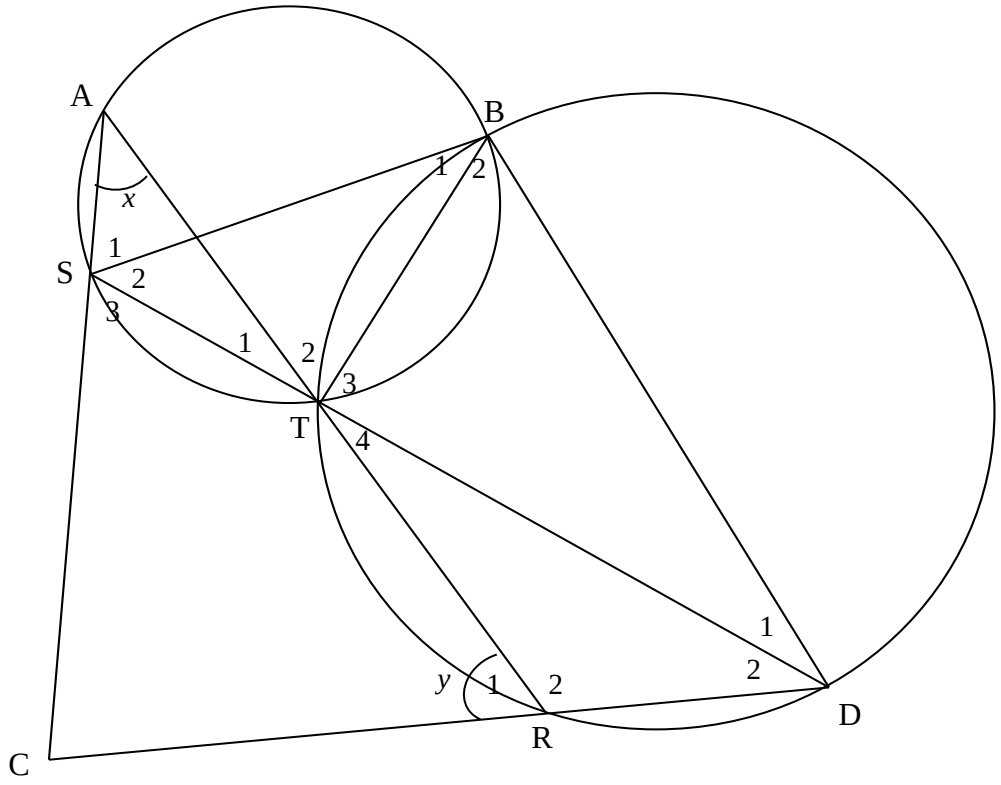
QUESTION/*VRAAG* 9

9.1



9.1	Constr/*Konstr*.: Draw KO and MO/*Trek KO en MO* Proof: $\hat{O}_1 = 2\hat{J}$ [∠ at centre = twice ∠ at circumference] *[midpts∠ = 2 × omtreks∠]* $\hat{O}_2 = 2\hat{L}$ [∠ at centre = twice ∠ at circumference] $\hat{O}_1 + \hat{O}_2 = 360°$ [∠s around a point /*∠e om 'n punt*] $\therefore 2\hat{J} + 2\hat{L} = 360°$ $\therefore 2(\hat{J} + \hat{L}) = 360°$ $\therefore \hat{J} + \hat{L} = 180°$ **OR/*OF*** Constr/*Konstr*.: Draw KO and MO/*Trek KO en MO* Proof: Let $\hat{J} = x$ $\hat{O}_1 = 2x$ [∠ at centre = twice ∠ at circumference] *[midpts∠ = 2 × omtreks∠]* $\hat{O}_2 = 360° - 2x$ [∠s around a point /*∠e om 'n punt*] $\therefore \hat{L} = 180° - x$ [∠ at centre = twice ∠ at circumference] $\therefore \hat{J} + \hat{L} = 180°$	construction S/R S S/R S (5) construction S R S/R S (5)

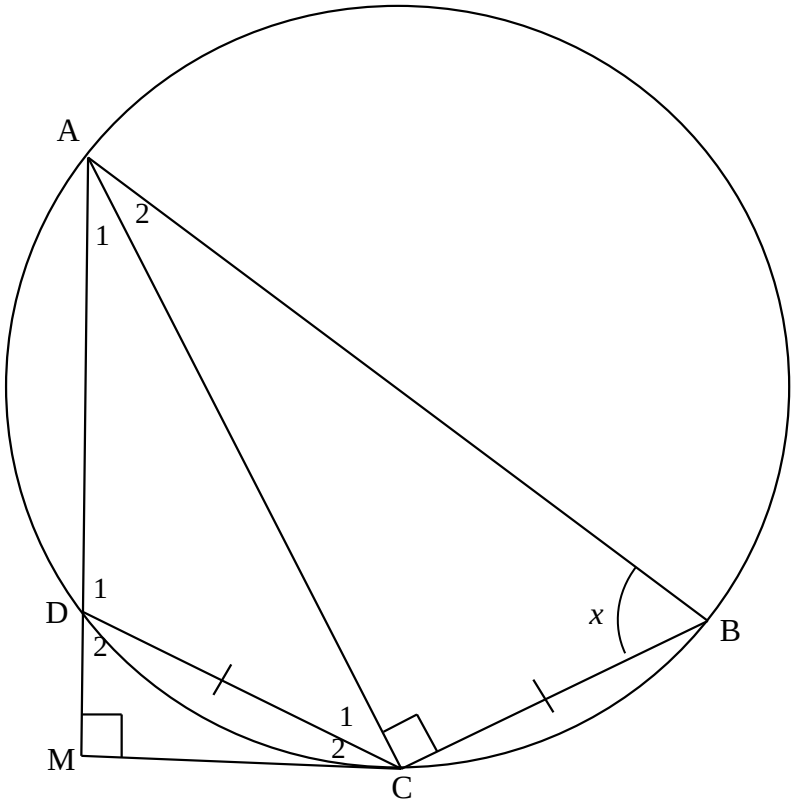
9.2



9.2.1(a)	$\hat{B}_1 = x$ [∠s in same seg/*∠e in dieselfde segm*]	S R (2)
9.2.1(b)	$\hat{B}_2 = y$ [ext ∠ of cyclic quad/*buite∠ koordevh*]	S R (2)
9.2.2	$\hat{C} = 180° - (x + y)$ [sum of ∠s of/*som v ∠e*, ΔACR] $S\hat{B}D + \hat{C} = x + y + 180° - (x + y)$ $S\hat{B}D + \hat{C} = 180°$ SCDB is a cyclic quad [converse opp angles of cyclic quad] [*omgekeerde teenoorst ∠e koordevh*] **OR/*OF*** $\hat{S}_1 = \hat{T}_2$ [∠s in same segment/*∠e in dies. segment*] $\hat{T}_2 = \hat{D}_1 + \hat{D}_2 = B\hat{D}R$ [ext ∠ of cyc quad/*buite∠ koordevh*] $\therefore \hat{S}_1 = B\hat{D}R$ $\therefore$ SCDB is cyc quad [ext ∠ of quad = opp ∠/*buite∠ = tos ∠*]	S S R (3) S S R (3)

9.2.3	$\hat{T}_4 = y - 30°$ [ext ∠ of/*buite* ∠ Δ TDR] $\hat{T}_1 = y - 30°$ [vert opp ∠s =/*regoorst* ∠*e* =] $y - 30° + x + 100° = 180°$ [sum of ∠s of/*som v* ∠*e*, Δ AST] $\therefore x + y = 110°$ $S\hat{B}D = 110°$ ∴ SD not diameter [line does not subtend 90° ∠] *SD nie 'n middellyn [lyn onderspan nie 90°∠]* **OR/*OF*** $A\hat{S}T = \hat{C} + \hat{D}_2$ [ext ∠ of/*buite* ∠ Δ SCD] $\hat{C} = 100° - 30° = 70°$ $S\hat{B}D = 180° - 70°$ [opp ∠s cyclic quad/ teenoorst ∠e kdvh] $= 110°$ ∴ SD not diameter [line does not subtend 90° ∠] *SD nie 'n middellyn [lyn onderspan nie 90°∠]*	S S S R (4) S S S R (4)
		[16]

QUESTION/*VRAAG* 10



10.1.1	$\hat{A}_2 = \hat{A}_1 = 90° - x$ [= chords subtend = ∠s = *kde onderspan =∠e*]	✓ S ✓R
	$\hat{D}_2 = x$ [exterior angle of cyclic quad/*buite∠ koordevh.*]	✓ S/R
	$\therefore \hat{C}_2 = 90° - x$ [sum of ∠s of/*som v ∠e,* ΔDCM]	✓ $\hat{C}_2 = 90° - x$
	$\therefore \hat{C}_2 = \hat{A}_1 = 90° - x$ ∴ MC is a tangent to the circle at C [converse: tan chord th] *MC is 'n raaklyn by C [omgekeerde raakl koordst]*	✓ R (5)
	OR/*OF*	
	$\hat{A}_2 = \hat{A}_1 = 90° - x$ [= chords subtend = ∠s/ = *kde onderspan =∠e*]	✓ S ✓ R
	$\hat{C}_1 + \hat{C}_2 = x$ [sum of ∠s of/*som v ∠e,* ΔACM]	✓✓ $\hat{C}_1 + \hat{C}_2 = x$
	$\therefore \hat{C}_1 + \hat{C}_2 = \hat{B} = x$ ∴ MC is a tangent to the circle at C [converse : tan chord th] *MC is 'n raaklyn by C [omgekeerde raakl koordst]*	✓ R (5)
	OR/*OF*	
	In ΔAMC and ΔACB:	
	$\hat{A}_2 = \hat{A}_1 = 90° - x$ [= chords subtend = ∠s/ = *kde onderspan =∠e*]	✓ S ✓ R
	$A\hat{M}C = A\hat{C}B = 90°$ [given]	
	$\therefore \hat{C}_1 + \hat{C}_2 = \hat{B} = x$	✓✓ $\hat{C}_1 + \hat{C}_2 = x$
	∴ MC is a tangent to the circle at C [converse : tan chord th] *MC is 'n raaklyn by C [omgekeerde raakl koordst]*	✓ R (5)

10.1.2	In ΔACB and/*en* ΔCMD $\hat{B} = \hat{D}_2 = x$ [proved **OR** exterior $\angle$ of cyclic quad.] *[bewys **OF** buite $\angle$ v koordevh]* $\hat{A}_2 = \hat{C}_2 = 90° - x$ [proved **OR** sum of $\angle$s in Δ] [*Bewys **OF** som v $\angle$e in Δ]* $\Delta ACB \mid\mid\mid \Delta CMD$ [$\angle, \angle, \angle$] **OR/*OF*** In ΔACB and/*en* ΔCMD $\hat{B} = \hat{D}_2 = x$ [proved **OR** exterior $\angle$ of cyclic quad.] *[bewys **OF** buite $\angle$ v koordevh]* $A\hat{C}B = A\hat{M}C = 90°$ [given/*gegee*] $\Delta ACB \mid\mid\mid \Delta CMD$ [$\angle, \angle, \angle$] **OR/*OF*** In ΔACB and/*en* ΔCMD $\hat{B} = \hat{D}_2 = x$ [proved **OR** exterior $\angle$ of cyclic quad] *[bewys **OF** buite $\angle$ v koordevh]* $\hat{A}_2 = \hat{C}_2 = 90° - x$ [proved **OR** sum of $\angle$s in Δ] *[Bewys **OF** som v $\angle$e in Δ]* $A\hat{C}B = A\hat{M}C = 90°$ [given **OR** sum of $\angle$s in Δ] *[gegee **OF** som v $\angle$e in Δ]* $\Delta ACB \mid\mid\mid \Delta CMD$	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
10.2.1	$\frac{BC}{MD} = \frac{AB}{DC}$ [$\Delta ACB \mid\mid\mid \Delta CMD$] $\frac{DC}{MD} = \frac{AB}{DC}$ [BC = DC] $\therefore DC^2 = AB \times MD$ In ΔAMC and/*en* ΔCMD $\hat{M}$ is common/*gemeen* $\hat{A}_1 = \hat{C}_2$ [tan chord th /*raaklyn koordst*] **OR/*OF*** $\hat{C}_1 + \hat{C}_2 = \hat{B} = \hat{D} = x$ [tan chord th /*raaklyn koordst* **OR/*OF*** exterior $\angle$ of cyclic quad/ *buite $\angle$ v kdvh*] $\Delta AMC \mid\mid\mid \Delta CMD$ [$\angle, \angle, \angle$] $\frac{AM}{CM} = \frac{CM}{MD}$ $\therefore CM^2 = AM \times MD$ $\therefore \frac{CM^2}{DC^2} = \frac{AM \times MD}{AB \times MD}$ $= \frac{AM}{AB}$	✓ $\frac{BC}{MD} = \frac{AB}{DC}$ ✓ $DC^2 = AB \times MD$ ✓ S ✓ S ✓ $CM^2 = AM \times MD$ ✓ $\frac{AM \times MD}{AB \times MD}$ (6)

	OR/*OF*	
	$\frac{AC}{MC}=\frac{AB}{DC}$ [$\Delta ACB \,|||\, \Delta CMD$] $\therefore CM\times AB = AC\times DC$	✓ $\frac{AC}{MC}=\frac{AB}{DC}$
	In ΔAMC and/*en* ΔACB $\hat{C}=\hat{M}=90°$ [given] $\hat{A}_1=\hat{A}_2$ [proven]	✓ S ✓ S
	OR/*OF*	
	$A\hat{C}M=\hat{B}=x$ [proven] $\Delta AMC \,|||\, \Delta ACB$ [$\angle,\angle,\angle$] $\frac{AC}{AM}=\frac{BC}{MC}$ $\therefore AC\times MC = AM\times BC$ $\therefore AC=\frac{BC.AM}{MC}$	✓ AC.MC= AM.BC
	$CM\times AB=\frac{BC.AM}{MC}\times DC$ $CM^2=\frac{DC.AM}{AB}\times DC$ [BC = DC] $\frac{CM^2}{DC^2}=\frac{AM}{AB}$	✓ equating ✓ S (6)
10.2.2	In ΔDMC: $\frac{CM}{DC}=\sin x$ $\frac{CM^2}{DC^2}=\sin^2 x$ $\frac{AC}{AB}=\frac{CM}{DC}$ $\therefore \frac{AM}{AB}=\sin^2 x$	✓ trig ratio ✓ square both sides (2)
	OR/*OF*	
	In ΔABC: $\sin x=\frac{AC}{AB}$ In ΔAMC: $\sin x=\frac{AM}{AC}$ $\sin x.\sin x=\frac{AC}{AB}\times\frac{AM}{AC}=\frac{AM}{AB}$	✓ 2 equations for $\sin x$ ✓ product (2)
		[16]

TOTAL/TOTAAL: 150