



basic education

**Department:
Basic Education
REPUBLIC OF SOUTH AFRICA**

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P2

NOVEMBER 2018

MARKS: 150

TIME: 3 hours

This question paper consists of 16 pages and a 24-page answer book.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. that you used to determine the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. Write neatly and legibly.

QUESTION 1

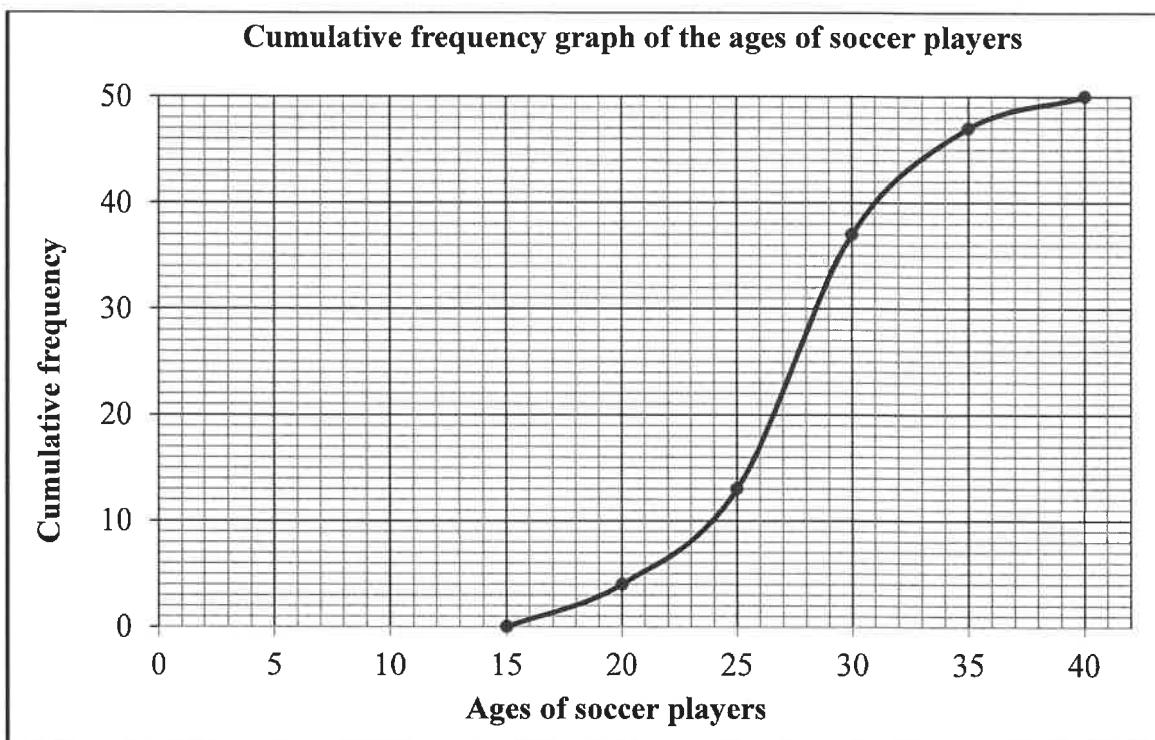
A school held a sports day. One of the items on the programme was an obstacle race. Teams of 10 parents and learners participated in this race. The table below shows the time taken, in minutes, by each member of a particular team to complete the race.

4	12	13	16	17	18	20	22	22	25
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- 1.1 How long, in minutes, did it take for the fastest member of this team to complete the race? (1)
- 1.2 Determine the mean time taken by this team. (2)
- 1.3 Calculate the standard deviation for the data. (1)
- 1.4 How many members of the team completed the obstacle race outside of two standard deviations of the mean? (3)
- 1.5 It took another team a total time of $x+5$ minutes to complete the race. Calculate the value of x if the overall mean of the two teams combined was 18 minutes. (3)
[10]

QUESTION 2

- 2.1 A survey was conducted of the ages of players at a soccer tournament. The results are shown in the cumulative frequency graph (ogive) below.



- 2.1.1 How many players took part in the soccer tournament? (1)
- 2.1.2 Determine the number of players between the ages of 24 and 31 years old. (2)
- 2.1.3 Complete the frequency column of the table below in the ANSWER BOOK.

CLASS INTERVAL	FREQUENCY	CUMULATIVE FREQUENCY
$15 \leq x < 20$		4
$20 \leq x < 25$		13
$25 \leq x < 30$		37
$30 \leq x < 35$		47
$35 \leq x < 40$		50

- 2.1.4 Use the grid provided in the ANSWER BOOK to draw a frequency polygon for the data. (4)

- 2.2 Two Grade 11 Mathematics classes have the same number of learners. The five-number summaries of the marks obtained by these classes for a test are shown below.

CLASS A (30 ; 48 ; 65 ; 82 ; 90)

CLASS B (50 ; 58 ; 65 ; 75 ; 90)

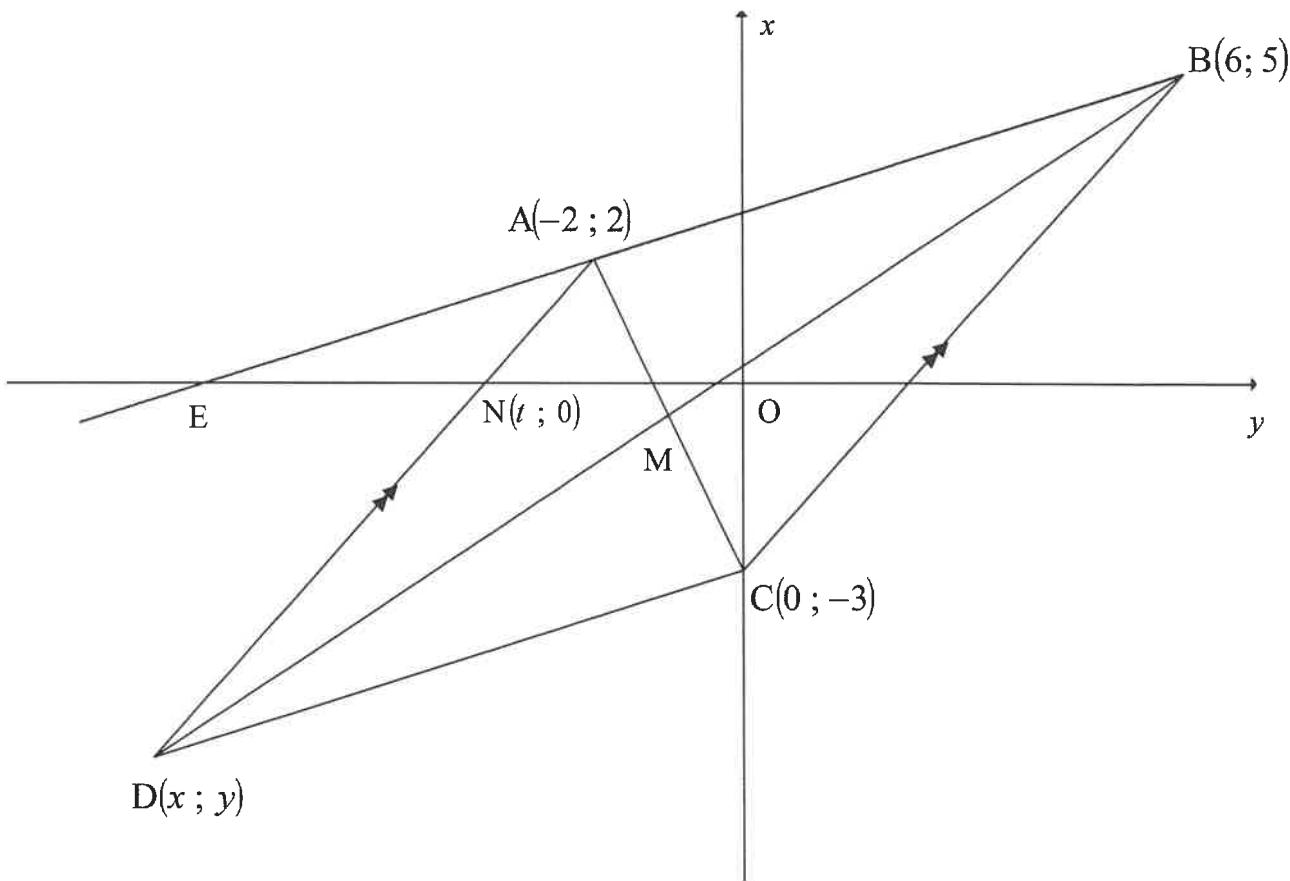
The parents of learners in CLASS A and CLASS B observe that both classes have the same median and the same maximum mark and therefore claim that there is no difference in the performance between these classes.

Do you agree with this claim? Use at least TWO different arguments to justify your answer.

(3)
[13]

QUESTION 3

In the diagram, $A(-2 ; 2)$, $B(6 ; 5)$, $C(0 ; -3)$ and $D(x ; y)$ are the vertices of a quadrilateral having $AD \parallel BC$. BA produced has an x -intercept at E . BD and AC intersect at M . $N(t ; 0)$ is a point on AD .

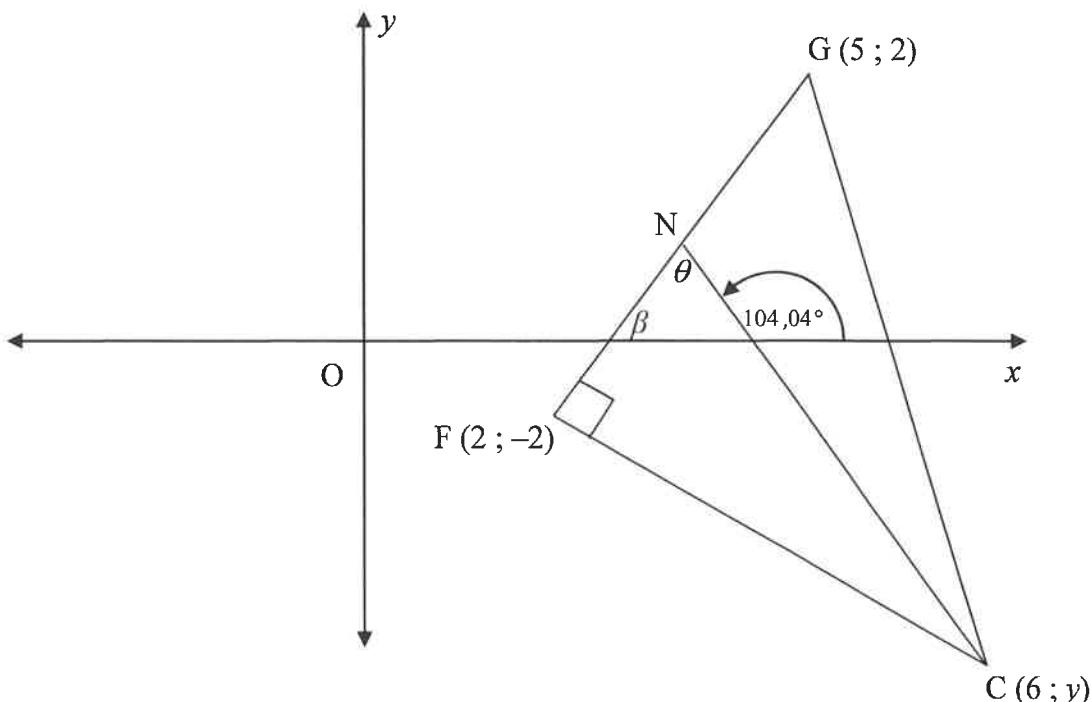


- 3.1 Calculate the gradient of BC . (2)
 - 3.2 Determine the equation of AD . (3)
 - 3.3 Determine the value of t . (2)
 - 3.4 Calculate the length of AN . (2)
 - 3.5 If DC is defined by $y = \frac{3}{8}x - 3$, determine the coordinates of D . (4)
 - 3.6 Prove that $ABCD$ is a parallelogram. (3)
 - 3.7 Calculate the coordinates of M . (3)
- [19]

QUESTION 4

In the diagram, $F(2 ; -2)$, $G(5 ; 2)$ and $C(6 ; y)$ are the vertices of $\triangle FGC$. $FG \perp FC$.
 N is a point on FG such that the inclination of NC is $104,04^\circ$.

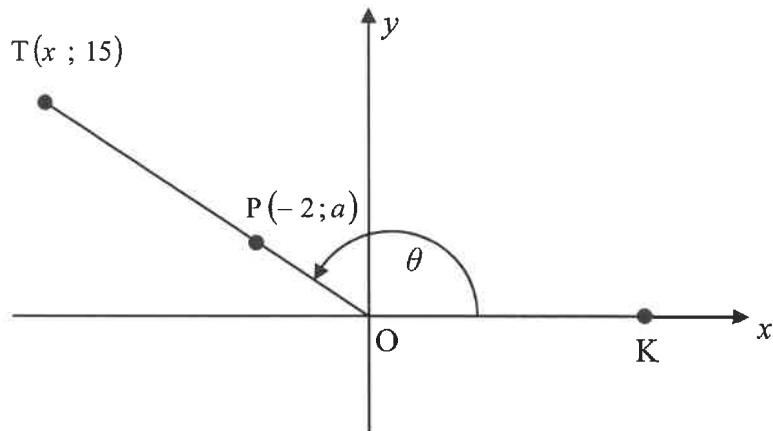
The angle of inclination of FG is β and $\hat{FNC} = \theta$.



- 4.1 Calculate the gradient of FG . (2)
 - 4.2 Calculate the value of y . (3)
 - 4.3 Calculate the size of θ . (3)
 - 4.4 Calculate the length of NC . (4)
- [12]**

QUESTION 5

- 5.1 In the diagram below, $T(x ; 15)$ is a point in the Cartesian plane such that $OT=17$ units. $P(-2 ; a)$ lies on OT . K is a point on the positive x -axis and $\hat{TOK}=\theta$.



Determine, with the aid of the diagram, the following:

5.1.1 The value of x (2)

5.1.2 $\tan \theta$ (1)

5.1.3 $\cos(180^\circ - \theta)$ (2)

5.1.4 $\sin^2 \theta$ (2)

5.1.5 The value of a (3)

- 5.2 Simplify WITHOUT using a calculator:

$$\frac{\sin 120^\circ \cdot \cos 210^\circ \cdot \tan 315^\circ \cdot \cos 27^\circ}{\sin 63^\circ \cdot \cos 540^\circ} \quad (7)$$

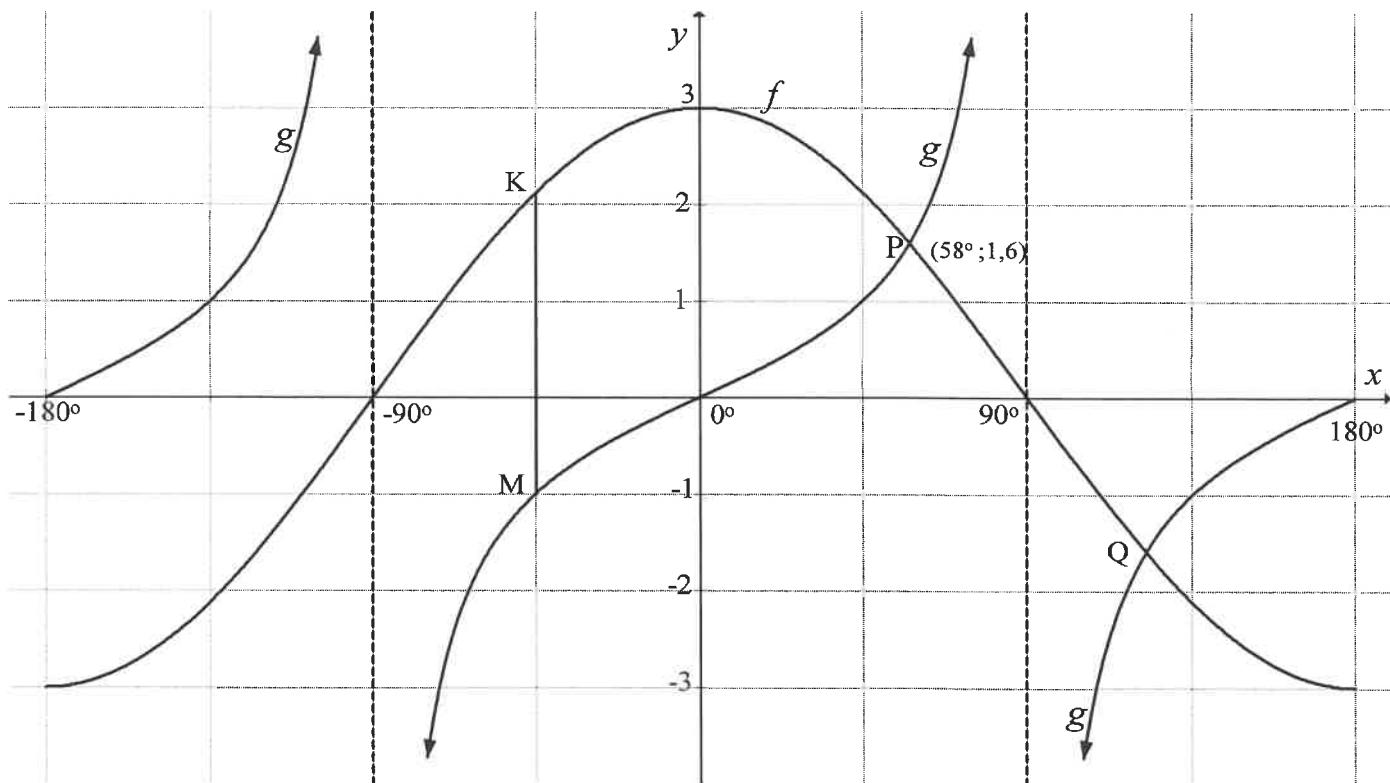
- 5.3 Prove the identity:

$$\frac{1}{\cos \theta} - \frac{\cos \theta}{1 + \sin \theta} = \tan \theta \quad (5)$$

- 5.4 Determine the general solution of $3 \sin x = 2 \tan x$ (6)
[28]

QUESTION 6

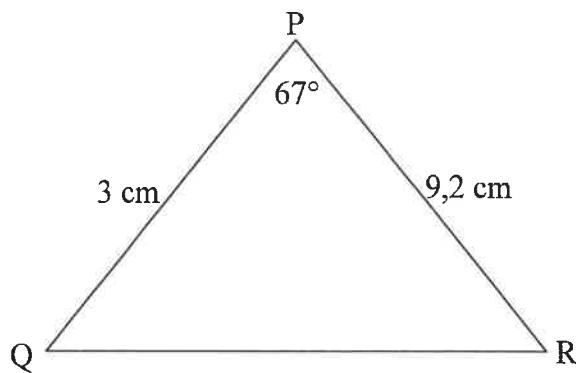
The graphs of the functions $f(x) = a \cos b\theta$ and $g(x) = c \tan \theta$ for $x \in [-180^\circ; 180^\circ]$ are sketched below. The graphs intersect at $P(58^\circ; 1,6)$ and Q .



- 6.1 Write down the range of f . (2)
 - 6.2 If $M(-45^\circ; -1)$ lies on g , determine the value of c . (1)
 - 6.3 Write down the values of a and b . (2)
 - 6.4 Determine the coordinates of Q . (2)
 - 6.5 K lies on f such that KM is parallel to the y -axis.
Calculate the length of KM . (2)
 - 6.6 If the system of axes is shifted 45° to the left and the graphs remain fixed, write down the equation that is now represented by graph f . (2)
- [11]

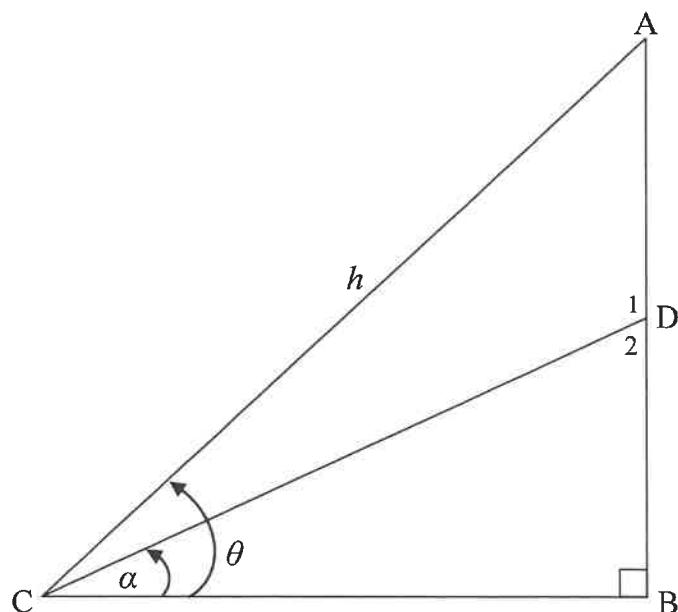
QUESTION 7

- 7.1 In the diagram, $\hat{P} = 67^\circ$, $PQ = 3 \text{ cm}$ and $PR = 9,2 \text{ cm}$.
Determine the length of QR .



(3)

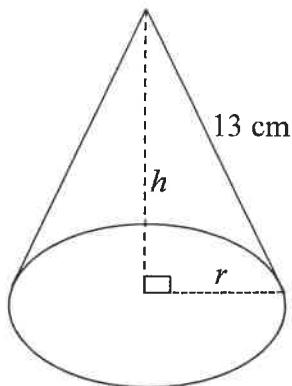
- 7.2 In the diagram below, $D\hat{C}B = \alpha$, $AC = h$ units and $A\hat{C}B = \theta$.



- 7.2.1 Determine size of $A\hat{C}D$ in terms of θ and α . (1)
- 7.2.2 Prove that $AD = \frac{h \sin(\theta - \alpha)}{\cos \alpha}$ (4)
- 7.2.3 Determine the length of AD if $h = 17$ units, $\theta = 58^\circ$ and $\alpha = 23^\circ$. (2)
- 7.2.4 Calculate the area of $\triangle ADC$. (3)
[13]

QUESTION 8

The diagram below shows a cone with a perpendicular height of h cm, a radius of r cm and a slant height of 13 cm.



$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

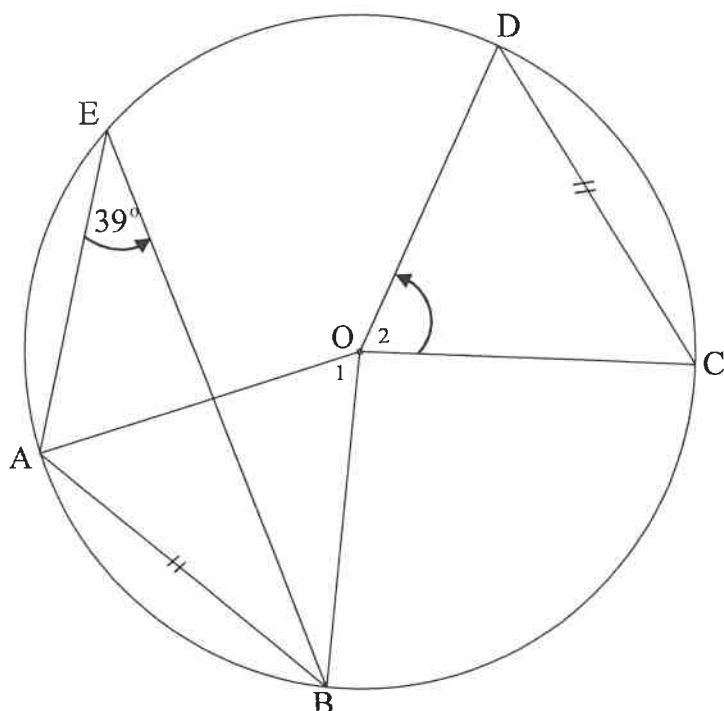
$$\begin{aligned}\text{Total surface area of the cone} \\ = \pi r^2 + \pi r s\end{aligned}$$

- 8.1 Show that the volume of the cone is given by $V = \frac{169\pi h - \pi h^3}{3}$ (4)
- 8.2 If $h = 12$ cm, determine the total surface area of the cone. (3)
[7]

Give reasons for your statements and calculations in QUESTIONS 9, 10 and 11.

QUESTION 9

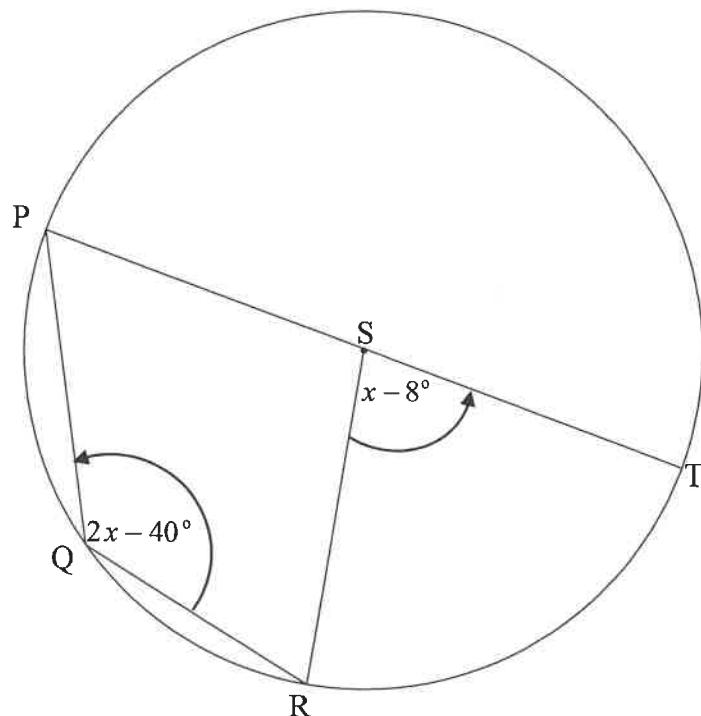
- 9.1 In the figure, O is the centre of the circle. A, B, C, D and E lie on the circle such that chord AB and chord DC are equal in length and $\hat{AEB} = 39^\circ$.



9.1.1 Determine the size of \hat{O}_1 . (2)

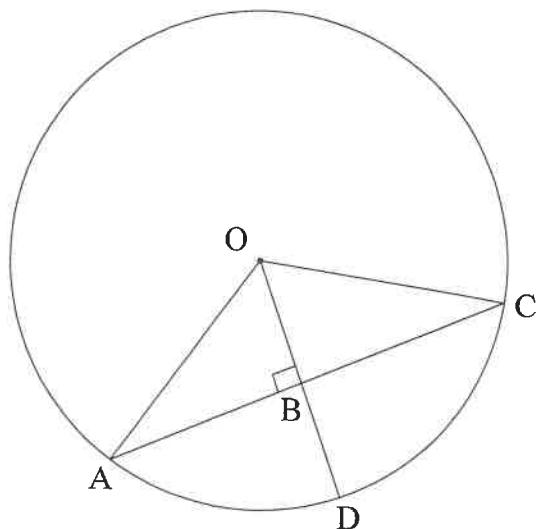
9.1.2 Determine the size of \hat{O}_2 . (2)

- 9.2 In the diagram, S is the centre of circle $PQRT$. PT is a diameter.
 $\hat{RST} = x - 8^\circ$ and $\hat{PQR} = 2x - 40^\circ$.



Determine the value of x . (4)

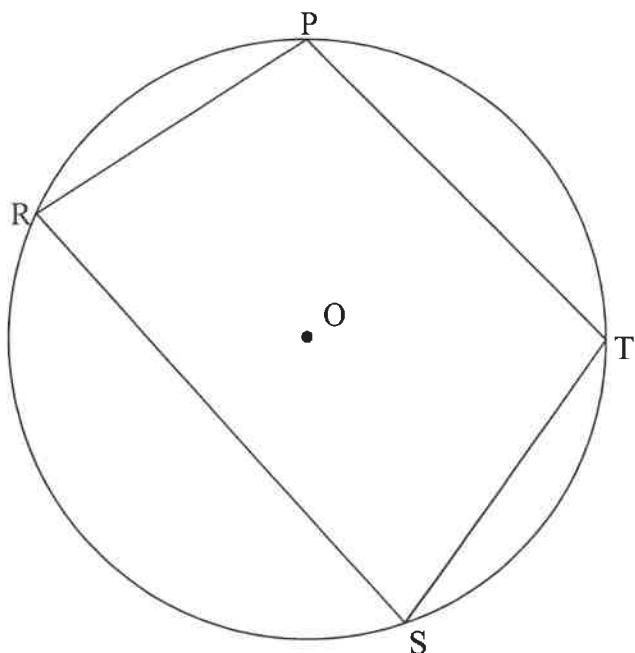
- 9.3 In the diagram, O is the centre of the circle. Chord AC is perpendicular to radius OD at B . $OB = 2x$ units and $AC = 8x$ units.



Show that the length of BD is $2x(\sqrt{5} - 1)$ units. (5)
[13]

QUESTION 10

- 10.1 In the diagram below, O is the centre of the circle and PTSR is a cyclic quadrilateral.

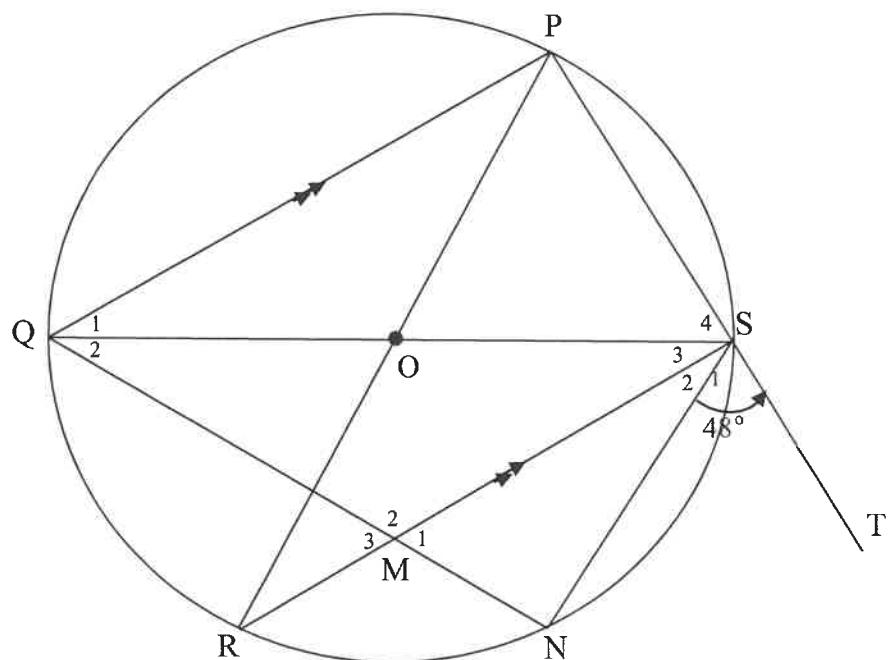


Prove the theorem that states that $\hat{P} + \hat{S} = 180^\circ$.

(5)

- 10.2 In the figure, QS and PR are diameters of the circle with centre O such that $PQ \parallel SR$. PS is produced to T . N is a point on the circle such that $\hat{Q}_1 = \hat{Q}_2$. SN is drawn.

RS intersects QN at M . $\hat{S}_1 = 48^\circ$



- 10.2.1 Determine, with reasons, the size of:

(a) \hat{Q}_1 (3)

(b) \hat{R} (2)

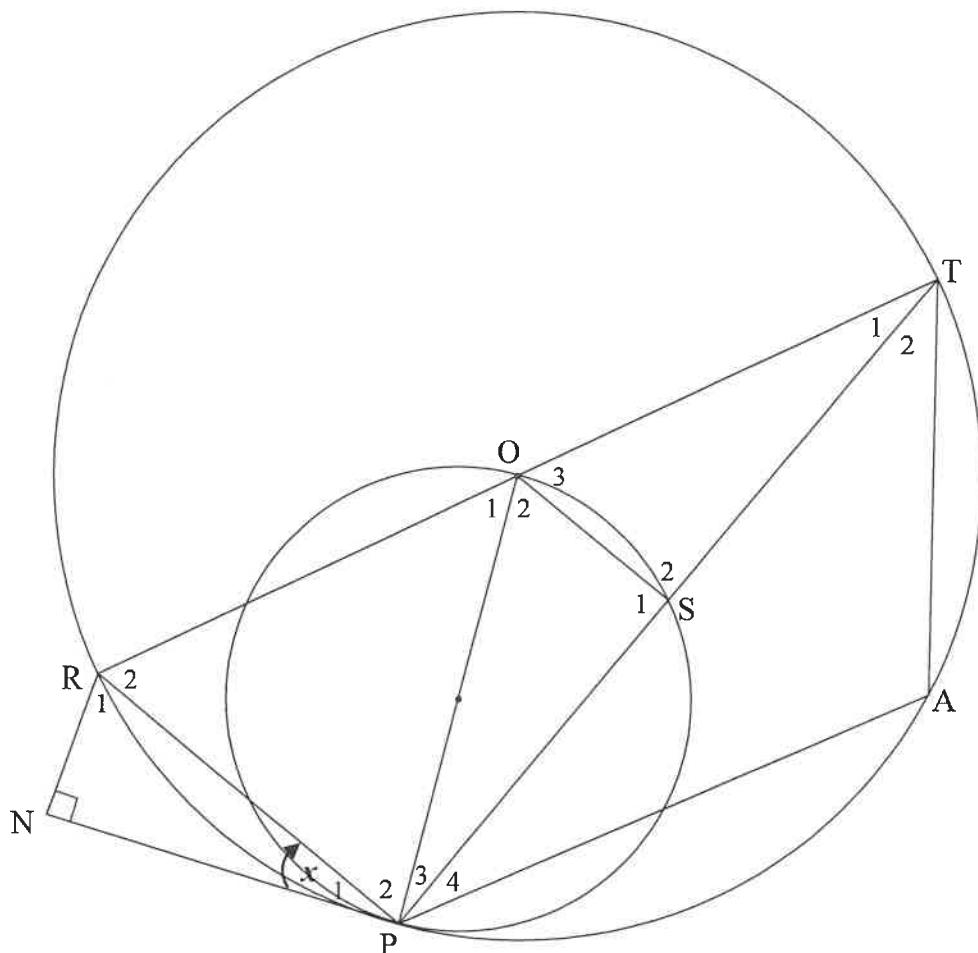
(c) \hat{M}_1 (2)

- 10.2.2 Prove that ST is a tangent to the circle passing through M, N and S . (2)
[14]

QUESTION 11

O is the centre of the larger circle RTAP. OP is the diameter of the smaller circle PSO. NP is a tangent to both circles at P. $RN \perp NP$.

Let $\hat{P}_1 = x$.



11.1 Prove that PR bisects \hat{ORN} . (5)

11.2 Prove that $\hat{ROS} = \hat{PAT}$. (5)

[10]

TOTAL: **150**



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NATIONAL
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*NASIONALE
SENIOR SERTIFIKAAT*

GRADE/GRAAD 11

MATHEMATICS P2/WISKUNDE V2

NOVEMBER 2018

MARKING GUIDELINES/ NASIENRIGLYNE

MARKS/PUNTE: 150

This marking guideline consists of 28 pages.
Hierdie nasienriglyne bestaan uit 28 bladsye.

NOTE:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking memorandum.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.
- As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, sien die deurgehaalde antwoord na.
- Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing.
- Dit is onaanvaarbaar om waardes/antwoorde te veronderstel om 'n probleem op te los.

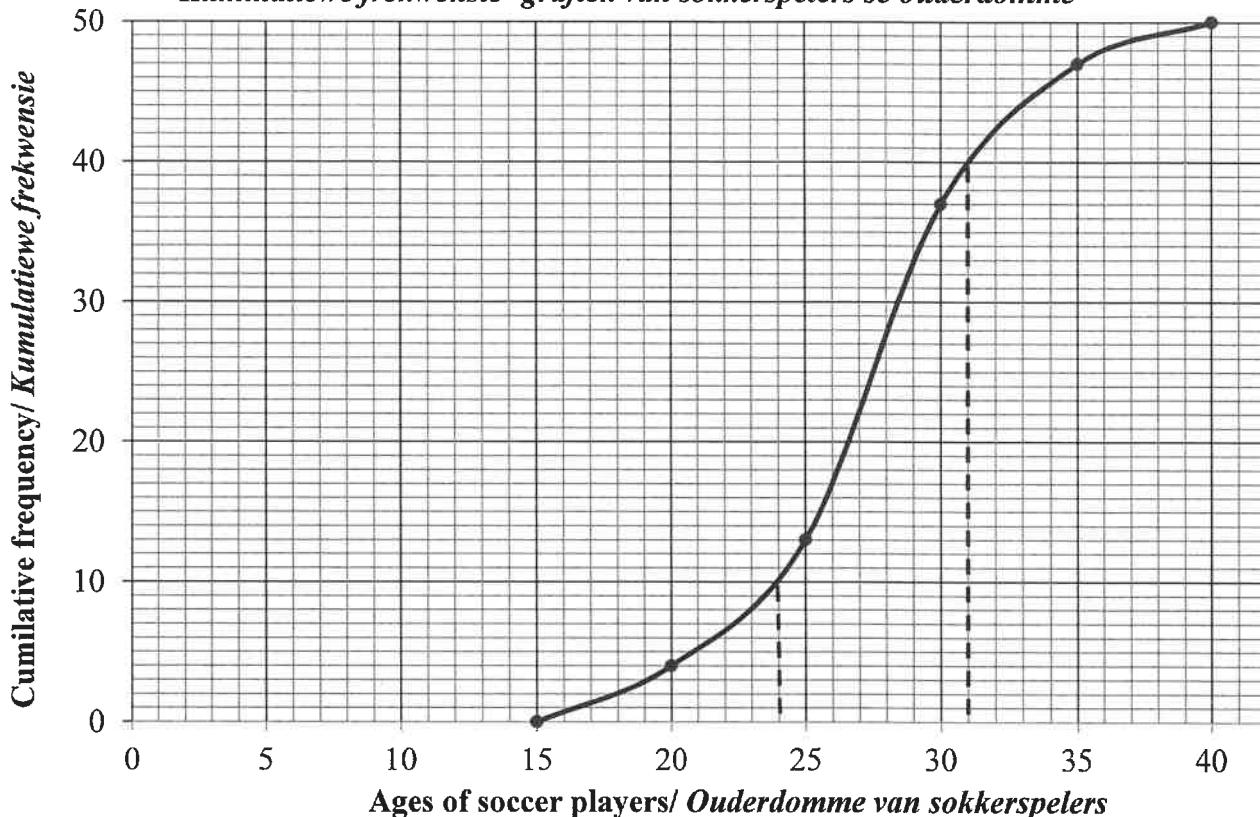
QUESTION/VRAAG 1

4	12	13	16	17	18	20	22	22	25
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1.1	4 minutes/ minute	✓ answer/ antwoord (1)
1.2	Mean/ gemiddeld = $\frac{169}{10} = 16,9$	✓ 169 ✓ answer/ antwoord (2)
1.3	Standard deviation/ Standardafwyking = 5,79	✓ answer/ antwoord (1)
1.4	$(16,9 - 2 \times 5,79; 16,9 + 2 \times 5,79)$ $(5,32; 28,48)$ \therefore 1 member of the team completed the obstacle race outside of 2 standard deviations of the mean./ <i>1 lid van die span het die hundernisbaan buite twee standardafwykings van die gemiddeld voltooi.</i>	✓ $\bar{x} - 2\sigma$ ✓ $\bar{x} + 2\sigma$ ✓ answer/ antwoord (3)
1.5	$\frac{169 + x + 5}{20} = 18$ $x = 18 \times 20 - 174$ $x = 186$	✓ $169 + x + 5$ ✓ dividing by 20/ deel deur 20 ✓ answer/ antwoord (3) [10]

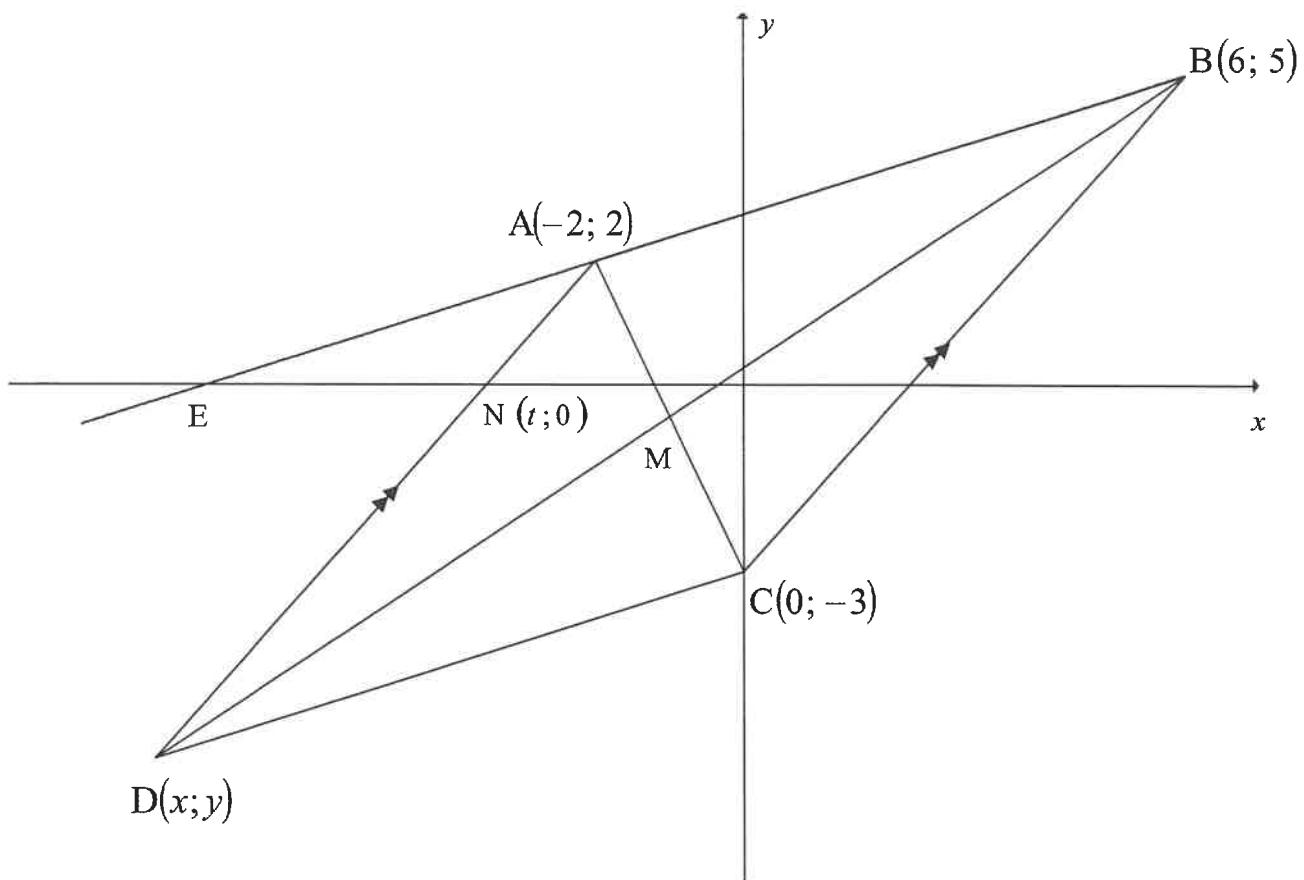
QUESTION/VRAAG 2

**Cumulative frequency graph of the ages of soccer players/
Kumulatiewe frekwensie -grafiek van sokkerspelers se ouderdomme**



2.1.1	50 players/ spelers	✓ answer/ antwoord (1)																		
2.1.2	$40 - 10 = 30$ players/ spelers	✓ 40 and/ en 10 ✓ answer/ antwoord (2)																		
2.1.3	<table border="1"> <thead> <tr> <th>Class interval/ <i>Klas interval</i></th> <th>Frequency/ <i>Frekwensie</i></th> <th>Cumulative frequency <i>Kumulatiewe frekwensie</i></th> </tr> </thead> <tbody> <tr> <td>$15 \leq x < 20$</td> <td>4</td> <td>4</td> </tr> <tr> <td>$20 \leq x < 25$</td> <td>9</td> <td>13</td> </tr> <tr> <td>$25 \leq x < 30$</td> <td>24</td> <td>37</td> </tr> <tr> <td>$30 \leq x < 35$</td> <td>10</td> <td>47</td> </tr> <tr> <td>$35 \leq x < 40$</td> <td>3</td> <td>50</td> </tr> </tbody> </table>	Class interval/ <i>Klas interval</i>	Frequency/ <i>Frekwensie</i>	Cumulative frequency <i>Kumulatiewe frekwensie</i>	$15 \leq x < 20$	4	4	$20 \leq x < 25$	9	13	$25 \leq x < 30$	24	37	$30 \leq x < 35$	10	47	$35 \leq x < 40$	3	50	✓ two correct values/ twee korrekte waardes ✓ three correct values/ drie korrekte waardes ✓ all correct values/ al die waardes korrek (3)
Class interval/ <i>Klas interval</i>	Frequency/ <i>Frekwensie</i>	Cumulative frequency <i>Kumulatiewe frekwensie</i>																		
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2.1.4	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: left; padding-bottom: 5px;">Class interval/ Klas- interval</th><th style="text-align: left; padding-bottom: 5px;">Class midpoint/ Klas- middelpunt</th><th style="text-align: left; padding-bottom: 5px;">Frequency/ Frekwensie</th></tr> </thead> <tbody> <tr><td>$15 \leq x < 20$</td><td>17,5</td><td>4</td></tr> <tr><td>$20 \leq x < 25$</td><td>22,5</td><td>9</td></tr> <tr><td>$25 \leq x < 30$</td><td>27,5</td><td>24</td></tr> <tr><td>$30 \leq x < 35$</td><td>32,5</td><td>10</td></tr> <tr><td>$35 \leq x < 40$</td><td>37,5</td><td>3</td></tr> </tbody> </table>	Class interval/ Klas- interval	Class midpoint/ Klas- middelpunt	Frequency/ Frekwensie	$15 \leq x < 20$	17,5	4	$20 \leq x < 25$	22,5	9	$25 \leq x < 30$	27,5	24	$30 \leq x < 35$	32,5	10	$35 \leq x < 40$	37,5	3	
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	<p>Frequency polygon of the ages of soccer players/ Frekwensie- veelhoek van sokkerspelers se oudermme</p> <p>Frequency/ Frekwensie</p> <p>Ages of soccer players/ Ouderdomme van sokkerspelers</p>	<ul style="list-style-type: none"> ✓ using midpoints / gebruik middelpunte ✓ plotting the points correctly/ korrekte punte geplot ✓ points joined by straight line/ punte verbind met 'n reguitlyn ✓ grounding at/ geanker by (12,5;0) and/ en (42,5 ; 0) 																		
2.2	<p>The claim is not valid. / Die bewering is nie geldig nie</p> <p>Range of class/ Omvang van klas A = 60 Range of class/ Omvang van klas B = 40</p> <p>The range of class A is bigger than the range of class B. Therefore the marks of class A are more spread out than the class B./ <i>Die omvang van klas A is groter as die omvang van klas B. Dus is die punte in klas A meer verspreid as klas B</i></p> <p>At least 25% of class A have lower marks than any learner in class B./ <i>ten minste 25% van klas A het laer punte as enige leerder in klas B.</i></p> <p>Class A performed worse at the bottom end. / <i>Klas A het slechter gevorder aan die onderste groep</i></p>	<ul style="list-style-type: none"> ✓ claim not valid/ bewering nie geldig nie ✓ comment on the overall spread/ kommentaar oor die algehele verspreiding ✓ comparison of the lower marks/ vergelyk laer punte 																		

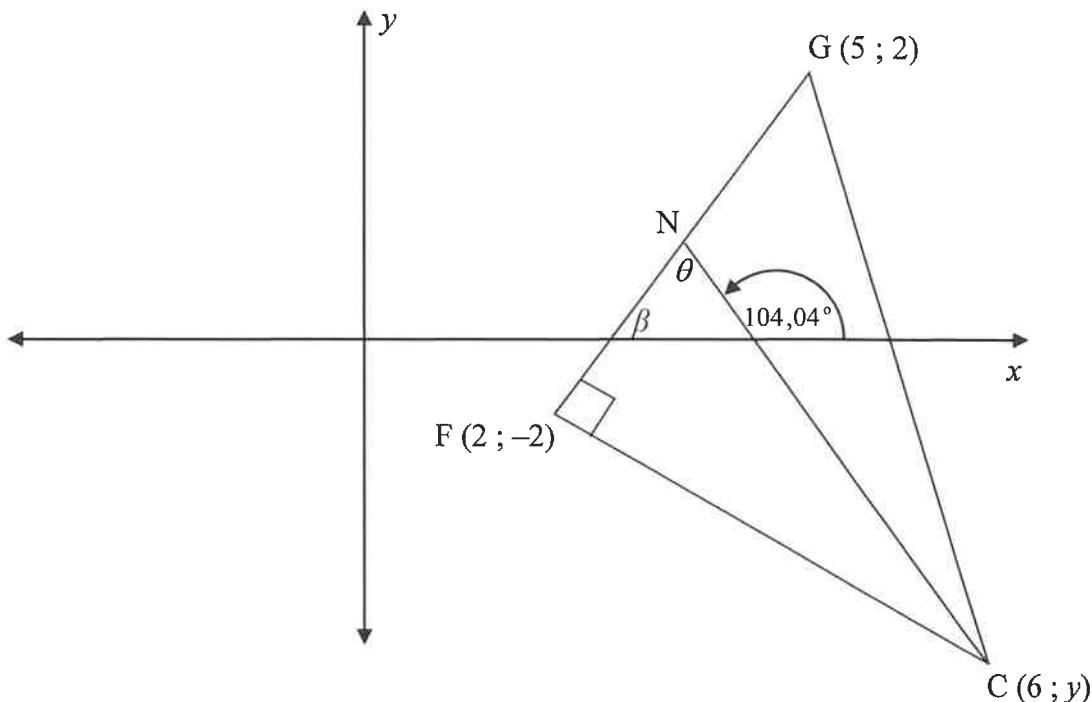
QUESTION/VRAAG 3

3.1	$B(6;5) \quad C(0;-3)$ $m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-3 - 5}{0 - 6}$ $= \frac{-8}{-6}$ $= \frac{4}{3}$	OR/ OF $m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{5 - (-3)}{6 - 0}$ $= \frac{8}{6}$ $= \frac{4}{3}$	✓ subst into correct grad.form / verv in gradform. ✓ answer/ antwoord (2)
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3.2	$m_{AD} = m_{BC} = \frac{4}{3} \text{ (AD} \parallel \text{BC)}$ $y = \frac{4}{3}x + c$ $2 = \frac{4}{3}(-2) + c$ $\frac{14}{3} = c$ $\therefore y = \frac{4}{3}x + \frac{14}{3}$ <p>OR/OF</p> $m_{AD} = \frac{4}{3} \text{ (AD} \parallel \text{BC)}$ $y - 2 = \frac{4}{3}(x - (-2))$ $y = \frac{4}{3}x + \frac{14}{3}$ $\therefore y = \frac{4}{3}x + \frac{14}{3}$	✓ $m_{AD} = \frac{4}{3}$ ✓ subst of m and point $(-2; 2)$ / verv. m en punt $(-2; 2)$ ✓ answer/ antwoord (3)
3.3	$y = \frac{4}{3}x + \frac{14}{3}$ $0 = \frac{4}{3}t + \frac{14}{3}$ $\frac{-14}{3} = \frac{4}{3}t$ $t = \frac{-14}{4} = \frac{-7}{2}$	✓ subst/ verv. $y=0$ ✓ answer/ antwoord (2)
3.4	$\begin{aligned} AN &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left((-2) - \left(-\frac{7}{2}\right)\right)^2 + (2 - 0)^2} \\ &= \sqrt{\frac{25}{4}} \\ &= \frac{5}{2} \end{aligned}$	✓ subst. in distance formula/ verv. in afstand formule ✓ answer/ antwoord (2)

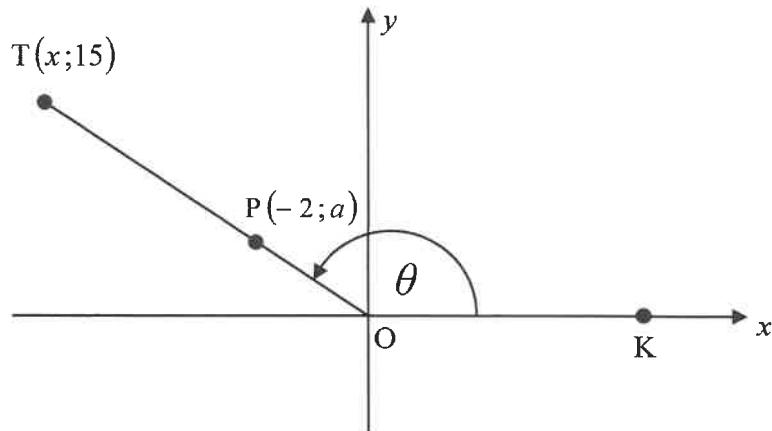
<p>3.5</p> $\frac{3}{8}x - 3 = \frac{4}{3}x + \frac{14}{3}$ $\frac{23}{24}x = -\frac{23}{3}$ $x = -8$ $y = \frac{4}{3}(-8) + \frac{14}{3}$ $= -6$ $D(-8; -6)$	<p>✓ equating/ vergelyk</p> <p>✓ simplification/ vereenv.</p> <p>✓ x- value/ waarde</p> <p>✓ y- value/ waarde</p> <p>(4)</p>
<p>3.6</p> $m_{AB} = \frac{5-2}{6-(-2)} = \frac{3}{8}$ $m_{AB} = m_{DC}$ $\therefore AB \parallel DC$ <p>but/maar AD BC</p> <p>\therefore ABCD is a parallelogram [opp sides are / teenoorst sye is]</p> <p>OR/OF</p> $AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{((-2) - (-8))^2 + (2 - 6)^2}$ $= \sqrt{100}$ $= 10$ $BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(6 - 0)^2 + (5 - (-3))^2}$ $= \sqrt{100}$ $= 10$ $\therefore AD = BC$ <p>but/maar AD BC</p> <p>\therefore ABCD is a parallelogram [2 opp sides are = and / teenoorst sye is = en]</p> <p>OR/OF</p>	<p>✓ $m_{AB} = \frac{3}{8}$</p> <p>✓ $AB \parallel DC$</p> <p>✓ reason/ rede</p> <p>(3)</p> <p>✓ length of AD/ lengte van AD</p> <p>✓ length of BC/ lengte van BC</p> <p>✓ reason/ rede</p> <p>(3)</p>

	<p>M is the midpoint of AC <i>M is die middelpunt van AC</i></p> $M\left(\frac{(-2)+0}{2}; \frac{2+(-3)}{2}\right)$ $M\left(-1; -\frac{1}{2}\right)$ <p>M is the midpoint of BD <i>M is die middelpunt van BD</i></p> $M\left(\frac{(-8)+6}{2}; \frac{(-6)+5}{2}\right)$ $M\left(-1; -\frac{1}{2}\right)$ <p>\therefore ABCD is a parallelogram</p> <p style="text-align: right;">[diagonals bisect each other <i>hoeklyne halveer mekaar</i>]</p>	<p>✓ midpoint of AC/ <i>middelpunt van AC</i></p> <p>✓ midpoint of BD/ <i>middelpunt van AC</i></p> <p>✓ reason/ rede</p> <p>(3)</p>
3.7	<p>M is the midpoint of AC [diagonals bisect] <i>M is die middelpunt van AC [hoeklyne halveer]</i></p> $M\left(\frac{(-2)+0}{2}; \frac{2+(-3)}{2}\right)$ $M\left(-1; -\frac{1}{2}\right)$	<p>✓ Substitution into the correct formula/ <i>Verv. in korrekte form.</i></p> <p>✓ x- value / waarde ✓ y- value / waarde</p> <p>(3)</p> <p>[19]</p>

QUESTION/VRAAG 4

4.1	$m_{FG} = \frac{2 - (-2)}{5 - 2}$ $= \frac{4}{3}$	✓ subst. into correct gradient form./ vervang in gradiënt formule ✓ answer (2)
4.2	$m_{FC} = \frac{-3}{4}$ (FC \perp FG) $\frac{y + 2}{6 - 2} = \frac{-3}{4}$ $y = -5$	✓ $m_{FC} = \frac{-3}{4}$ ✓ equating gradients/ stel gradiënte gelyk ✓ answer/ antwoord (3)
OR/OF		

	$m_{FC} \times m_{FG} = -1 \text{ (FC} \perp \text{FG)}$ $\frac{y+2}{6-2} \times \frac{4}{3} = -1$ $4(y+2) = -12$ $y+2 = -3$ $y = -5$	✓ $m_{FC} \times m_{FG} = -1$ ✓ substitution/ verv. ✓ answer/ antwoord (3)
4.3	$\tan \beta = \frac{4}{3}$ $\beta = 53,13^\circ$ $\theta = 104,04^\circ - 53,13^\circ$ [ext \angle of Δ / buite \angle van Δ] $\theta = 50,91^\circ$	✓ $\tan \beta = \frac{4}{3}$ ✓ $\beta = 53,13^\circ$ ✓ answer/ antwoord (3)
4.4	$FC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(6-2)^2 + (-5-(-2))^2}$ $= \sqrt{16+9}$ $= 5$ $\sin \theta = \frac{FC}{NC}$ $\sin 50,91^\circ = \frac{5}{NC}$ $NC = \frac{5}{\sin 50,91^\circ}$ $= 6,44 \text{ unit}$	✓ subst. into distance formula/ verv. in afst. form. ✓ length of FC / lengte van FC ✓ $\sin 50,91^\circ = \frac{5}{NC}$ ✓ answer/ antwoord (4) [12]

QUESTION/VRAAG 5

5.1.1	$x^2 + y^2 = r^2$ [Pythagoras] $(x)^2 + (15)^2 = 17^2$ $x^2 = 64$ $x = -8$ (P is in quadrant 2/ is in kwadrant 2)	✓ subst in pyth/ verv in pyth ✓ answer/ antwoord (2)
5.1.2	$\tan \theta = \frac{15}{-8}$	✓ answer/ antwoord (1)
5.1.3	$\cos(180^\circ - \theta)$ $= -\cos \theta$ $= -\left(\frac{-8}{17}\right)$ $= \frac{8}{17}$	✓ $-\cos \theta$ ✓ answer/ antwoord (2)
5.1.4	$\sin^2 \theta$ $= \left(\frac{15}{17}\right)^2$ $= \frac{225}{289}$	✓ substitution/ vervanging ✓ answer/ antwoord (2)

5.1.5	$\tan \theta = \frac{a}{-2} = \frac{15}{-8}$ $\frac{a}{-2} = \frac{15}{-8}$ $a = \frac{15}{4}$	✓ $\tan \theta = \frac{a}{-2}$ ✓ equating/ stel gelyk ✓ answer/ antwoord (3)
OR/OF	$m = \frac{15}{-8}$ $y = \frac{15}{-8}x$ $a = \frac{15}{-8}(-2)$ $a = \frac{15}{4}$	✓ $y = \frac{15}{-8}x$ ✓ substitution of $P(-2; a)$ / vervanging van $P(-2; a)$ ✓ answer/ antwoord (3)
5.2	$\text{LHS} = \frac{\sin 120^\circ \cdot \cos 210^\circ \cdot \tan 315^\circ \cdot \cos 27^\circ}{\cos 540^\circ \cdot \sin 63^\circ}$ $= \frac{\sin 60^\circ \cdot (-\cos 30^\circ) \cdot (-\tan 45^\circ) \cdot \sin 63^\circ}{\cos 180^\circ \cdot \sin 63^\circ}$ $= \frac{\frac{\sqrt{3}}{2} \cdot \frac{-\sqrt{3}}{2} \cdot (-1)}{-1}$ $= -\frac{3}{4}$	✓ $\sin 60^\circ / \cos 30^\circ$ ✓ $-\cos 30^\circ$ ✓ $-\tan 45^\circ$ ✓ $\sin 63^\circ / \cos 27^\circ$ ✓ $\cos 180^\circ$ ✓ special angle ratios/ spesiale hoekverhoudings ✓ answer/ antwoord (7)

<p>5.3</p> $ \begin{aligned} \text{LHS} &= \frac{1}{\cos \theta} - \frac{\cos \theta}{1 + \sin \theta} \\ &= \frac{1 + \sin \theta - \cos^2 \theta}{\cos \theta (1 + \sin \theta)} \\ &= \frac{1 + \sin \theta - (1 - \sin^2 \theta)}{\cos \theta (1 + \sin \theta)} \\ &= \frac{\sin \theta (1 + \sin \theta)}{\cos \theta (1 + \sin \theta)} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta \\ &= \text{RHS} \end{aligned} $	<ul style="list-style-type: none"> ✓ common denominator/ <i>gemene noemer</i> ✓ $1 - \sin^2 \theta$ ✓ simplification/ <i>vereenv.</i> ✓ factors/ <i>faktore</i> ✓ $\frac{\sin \theta}{\cos \theta}$
<p>OR/OF</p> $ \begin{aligned} \text{LHS} &= \frac{1}{\cos \theta} - \frac{\cos \theta}{1 + \sin \theta} \\ &= \frac{1 + \sin \theta - \cos^2 \theta}{\cos \theta (1 + \sin \theta)} \\ &= \frac{(1 - \cos^2 \theta) + \sin \theta}{\cos \theta (1 + \sin \theta)} \\ &= \frac{\sin^2 \theta + \sin \theta}{\cos \theta (1 + \sin \theta)} \\ &= \frac{\sin \theta (1 + \sin \theta)}{\cos \theta (1 + \sin \theta)} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta \\ &= \text{RHS} \end{aligned} $	<ul style="list-style-type: none"> ✓ common denominator/ <i>gemene noemer</i> ✓ $1 - \cos^2 \theta$ ✓ simplification/ <i>vereenv.</i> ✓ factors/ <i>faktore</i> ✓ $\frac{\sin \theta}{\cos \theta}$

$ \begin{aligned} \text{LHS} &= \frac{1}{\cos \theta} - \frac{\cos \theta}{1 + \sin \theta} \\ &= \frac{1 + \sin \theta - \cos^2 \theta}{\cos \theta (1 + \sin \theta)} \\ &= \frac{\sin^2 \theta + \cos^2 \theta + \sin \theta - \cos^2 \theta}{\cos \theta (1 + \sin \theta)} \\ &= \frac{\sin^2 \theta + \sin \theta}{\cos \theta (1 + \sin \theta)} \\ &= \frac{\sin \theta (1 + \sin \theta)}{\cos \theta (1 + \sin \theta)} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta \\ &= \text{RHS} \end{aligned} $	<ul style="list-style-type: none"> ✓ common denominator/ <i>gemene noemer</i> ✓ $\sin^2 \theta + \cos^2 \theta$ ✓ simplification/ <i>vereenv.</i> ✓ factors/ <i>faktore</i> ✓ $\frac{\sin \theta}{\cos \theta}$
	(5)

<p>5.4</p> $ \begin{aligned} 3 \sin x &= 2 \tan x. \\ 3 \sin x &= 2 \times \frac{\sin x}{\cos x} \\ 3 \sin x \cos x &= 2 \sin x \\ 3 \sin x \cos x - 2 \sin x &= 0 \\ \sin x (3 \cos x - 2) &= 0 \\ \sin x &= 0 \\ x &= 360^\circ k, \quad k \in \mathbb{Z} \\ \text{or} \\ x &= 180^\circ + 360^\circ k, \quad k \in \mathbb{Z} \\ \text{or} \end{aligned} $	<ul style="list-style-type: none"> ✓ $\frac{\sin x}{\cos x}$ ✓ factors/ <i>faktore</i> ✓ both equations/ <i>beide vergelykings</i> ✓ both general solutions/ <i>beide algemene oplossings</i>
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$$\cos x = \frac{2}{3}$$

$$x = 48,19^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

or

$$x = 311,81^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

✓ both general solutions/ *beide algemene oplossings*
✓ $k \in \mathbb{Z}$

(6)

OR/ OF

$$3 \sin x = 2 \tan x.$$

$$3 \sin x = 2 \times \frac{\sin x}{\cos x}$$

$$3 \sin x \cos x = 2 \sin x$$

$$3 \sin x \cos x - 2 \sin x = 0$$

$$\sin x (3 \cos x - 2) = 0$$

$$\sin x = 0$$

$$x = 180^\circ \cdot k, k \in \mathbb{Z}$$

$$\cos x = \frac{2}{3}$$

$$x = \pm 48,19^\circ + 360^\circ \cdot k, k \in \mathbb{Z}$$

✓ $\frac{\sin x}{\cos x}$

✓ factors/ *faktore*

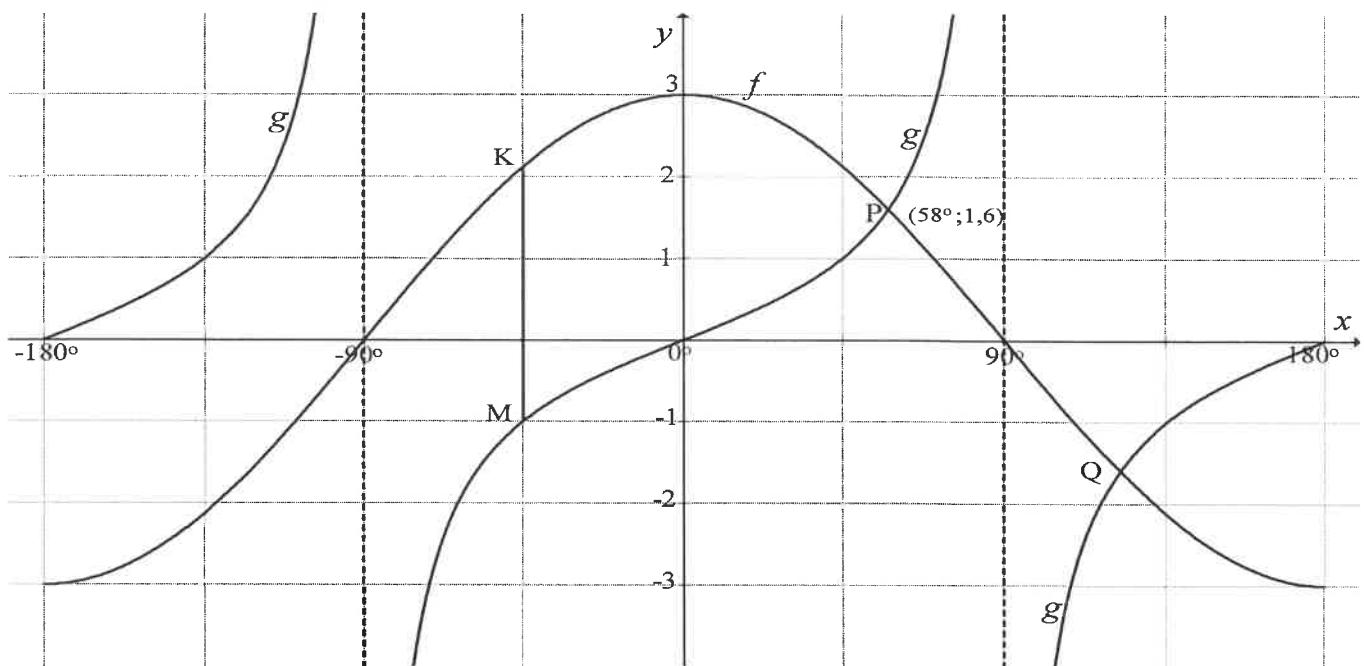
✓ both equations/ *beide vergelykings*

✓ general solution/ *algemene oplossing*

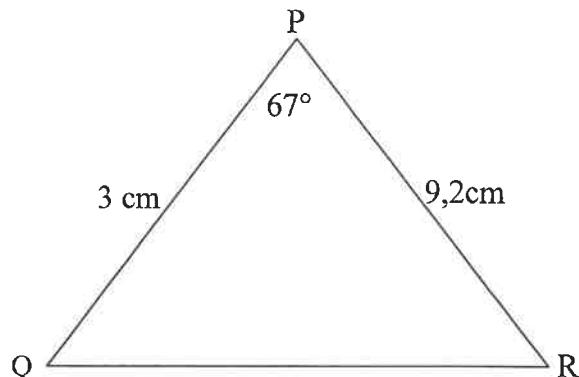
✓ both general solutions/ *beide algemene oplossings*
✓ $k \in \mathbb{Z}$

(6)

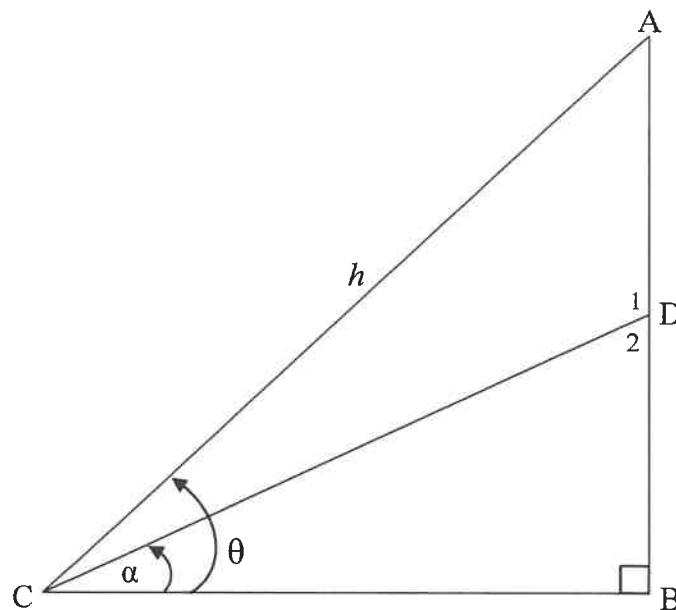
[28]

QUESTION/VRAAG 6

6.1	$-3 \leq y \leq 3$ or/ of $y \in [-3; 3]$	✓ end points/ eindpunte ✓ notation/ notasie (2)
6.2	$c = 1$	✓ answer/ antwoord (1)
6.3	$a = 3, b = 1$	✓ $a = 3$ ✓ $b = 1$ (2)
6.4	$Q(122^\circ; -1,6)$	✓ x - value/ waarde ✓ y - value/ waarde (2)
6.5	$K(-45^\circ; \frac{3\sqrt{2}}{2})$ $M(-45^\circ; -1)$ $KM = \frac{3\sqrt{2}}{2} + 1$ $= \frac{3\sqrt{2} + 2}{2}$ $= 3,12$	✓ coordinates of/ koördinate van K ✓ length of/ lengte van KM (2)
6.6	$f(x) = 3 \cos(\theta - 45^\circ)$	✓ 3 ✓ -45° (2)
		[11]

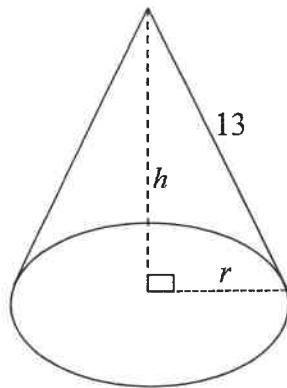
QUESTION/VRAAG 7

7.1	$QR^2 = PR^2 + PQ^2 - 2PR \cdot PQ \cos \hat{P}$ $QR^2 = (3)^2 + (9,2)^2 - 2(3)(9,2) \cos 67^\circ$ $QR = \sqrt{(3)^2 + (9,2)^2 - 2(3)(9,2) \cos 67^\circ}$ $QR = 8,49\text{cm}$	<ul style="list-style-type: none"> ✓ using cos rule/ <i>gebruik cos reël</i> ✓ substitution/ <i>vervanging</i> ✓ answer/ <i>antwoord</i> 	(3)
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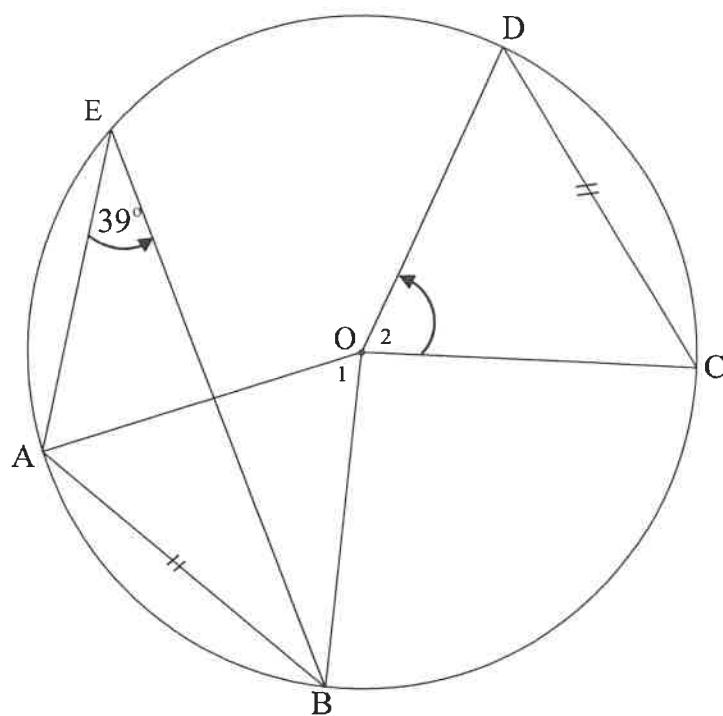


7.2.1	$\hat{A}CD = \theta - \alpha$	✓ answer/antw. (1)
7.2.2	$\hat{D}_1 = 90^\circ + \alpha$ $\frac{\sin(90^\circ + \alpha)}{h} = \frac{\sin(\theta - \alpha)}{AD}$ $\frac{\cos \alpha}{h} = \frac{\sin(\theta - \alpha)}{AD}$ $AD = \frac{h \sin(\theta - \alpha)}{\cos \alpha}$	✓ $\hat{D}_1 = 90^\circ + \alpha$ ✓ $\frac{\sin(90^\circ + \alpha)}{h}$ ✓ $\frac{\sin(\theta - \alpha)}{AD}$ ✓ $\sin(90^\circ + \alpha) = \cos \alpha$ (4)
7.2.3	$AD = \frac{17 \sin(58^\circ - 23^\circ)}{\cos 23^\circ}$ $AD = 10,59 \text{ units}$	✓ subst/verv. ✓ answer/antw. (2)
7.2.4	Area of $\Delta ADC = \frac{1}{2} \times AD \times AC \times \sin \hat{A}$ $= \frac{1}{2} \times 10,59 \times 17 \times \sin 32^\circ$ $= 47,70 \text{ unit}^2$ <p>OR/ OF</p>	✓ correct area rule/ korrekte area reël ✓ 32° ✓ answer/antw. (3)

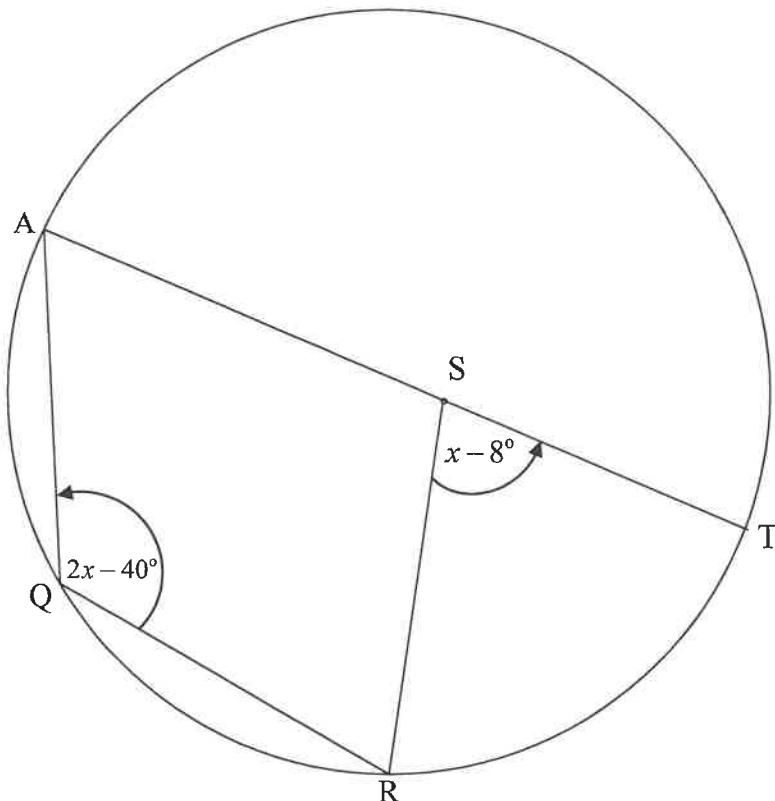
$\sin 58^\circ = \frac{AB}{17}$ $AB = 17 \sin 58^\circ$ $= 14,41682\dots$ $BD = 14,41682\dots - 10,59289\dots = 3,82393\dots$ $\sin 23^\circ = \frac{3,82393\dots}{CD}$ $CD = \frac{3,82393\dots}{\sin 23^\circ}$ $= 9,78660\dots$ $\text{Area of } \triangle ADC = \frac{1}{2} \times CD \times AC \times \sin 35^\circ$ $= \frac{1}{2} \times 9,78660\dots \times 17 \times \sin 35^\circ$ $= 47,71 \text{ unit}^2$	✓ length of BD / <i>lengte van BD</i> ✓ length of CD/ <i>lengte van CD</i> ✓ answer/antw. (3)
	[28]

QUESTION/VRAAG 8

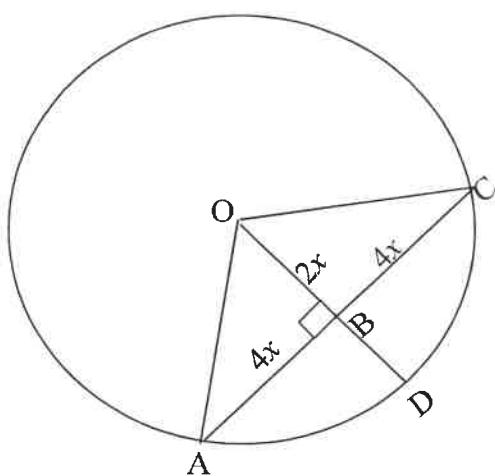
8.1 $r^2 = 13^2 - h^2$ (Pythagoras) $r^2 = 169 - h^2$ $V = \frac{1}{3} \pi r^2 h$ $= \frac{1}{3} \pi r^2 h$ $= \frac{1}{3} \pi (169 - h^2) h$ $= \frac{169\pi h - \pi h^3}{3}$	✓ using theorem of pythagoras/ gebruik stelling van pythagoras ✓ $r^2 = 169 - h^2$ ✓ substitution/ vervanging ✓ simplification/ vereenvoudig (4)
8.2 $r = \sqrt{13^2 - 12^2}$ (Pythagoras) $= 5$ Total surface area/ buite oppervlakte $= \pi r^2 + \pi r s$ $= \pi(5^2) + \pi(5)(13)$ $= 90\pi$ $= 282,74 \text{ cm}^2$	✓ value of/ waarde van r ✓ subst. / verv. ✓ answer/ antwoord (3) [7]

QUESTION/VRAAG 9

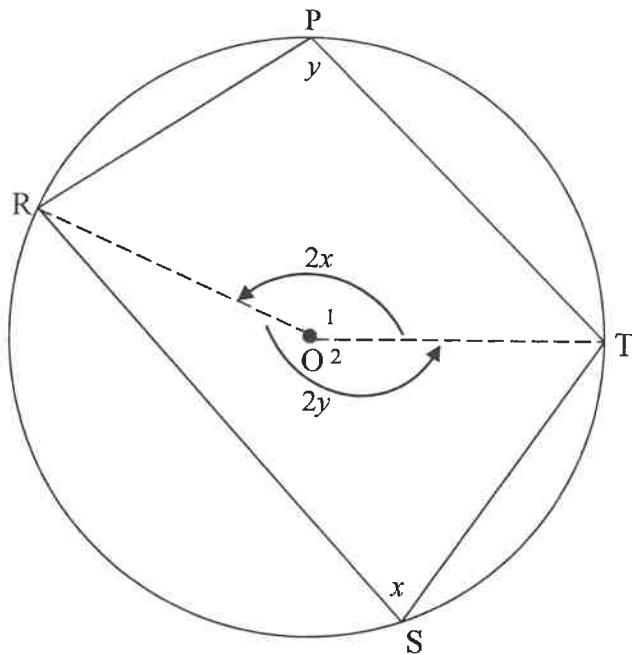
9.1.1	$\hat{O}_1 = 78^\circ$ [angle at centre = $2 \times \angle$ at circumference] [middelpuntshoek = $2 \times$ omtrekshoek]	✓ S ✓ R (2)
9.1.2	$\hat{O}_2 = 78^\circ$ [equal chords; equal \angle^s / gelyke koorde; gelyke hoeke]	✓ S ✓ R (2)



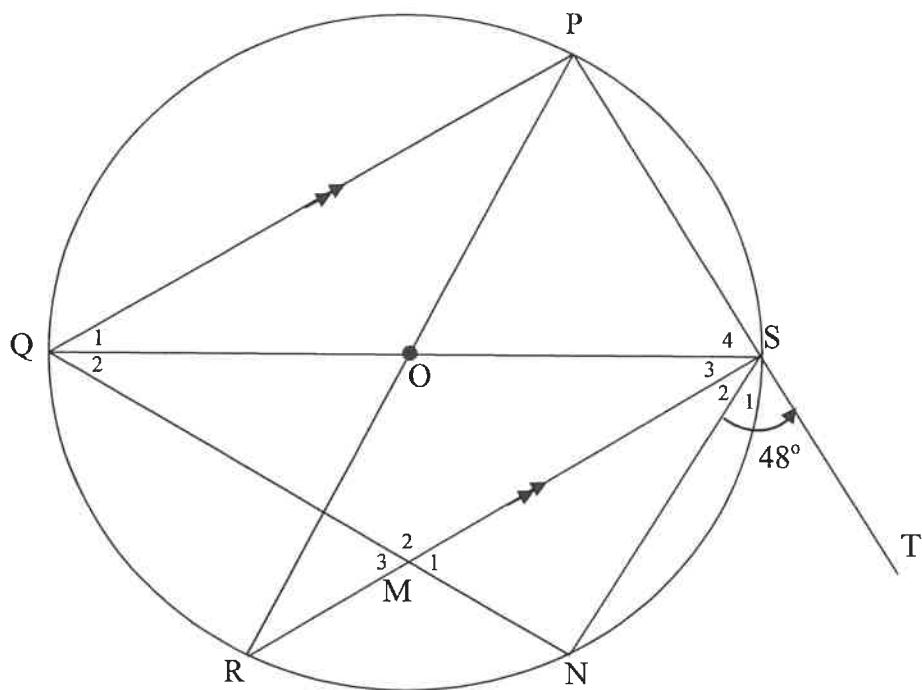
<p>9.2</p> $x - 8^\circ + 180^\circ = 2(2x - 40^\circ) \quad \begin{bmatrix} \text{angle at centre} = 2 \times \angle \text{ at circumference}/ \\ \text{middelpuntshoek} = 2 \times \text{omtrekshoek} \end{bmatrix}$ $4x - 80^\circ = 172^\circ + x$ $3x = 252^\circ$ $x = 84^\circ$ <p>OR/OF</p> <p>Join T and R/ verbind T en R</p> $\hat{T} = 180^\circ - (2x - 40^\circ) \quad \begin{bmatrix} \text{opp } \angle's \text{ of cyclic quad}/ \\ \text{teenoorste } \angle^e \text{ van koordevierhoek} \end{bmatrix}$ $\hat{R} = \hat{T} = 220^\circ - 2x \quad \begin{bmatrix} \angle^s \text{ opp. = sides } / \angle^s \text{ teenoor gelyke sye} \end{bmatrix}$ $x - 8^\circ + 220^\circ - 2x + 220^\circ - 2x = 180^\circ \quad \begin{bmatrix} \text{sum of int } \angle^s \text{ of } \Delta \\ \text{som binne } \angle^e \text{ van } \Delta \end{bmatrix}$ $-3x = -252^\circ$ $x = 84^\circ$	<p>✓ S ✓ R</p> <p>✓ simplification/ vereenvoudiging</p> <p>✓ answer/ antwoord</p> <p>(4)</p> <p>✓ S ✓ R</p> <p>✓ S</p> <p>✓ answer/ antwoord</p> <p>(4)</p>
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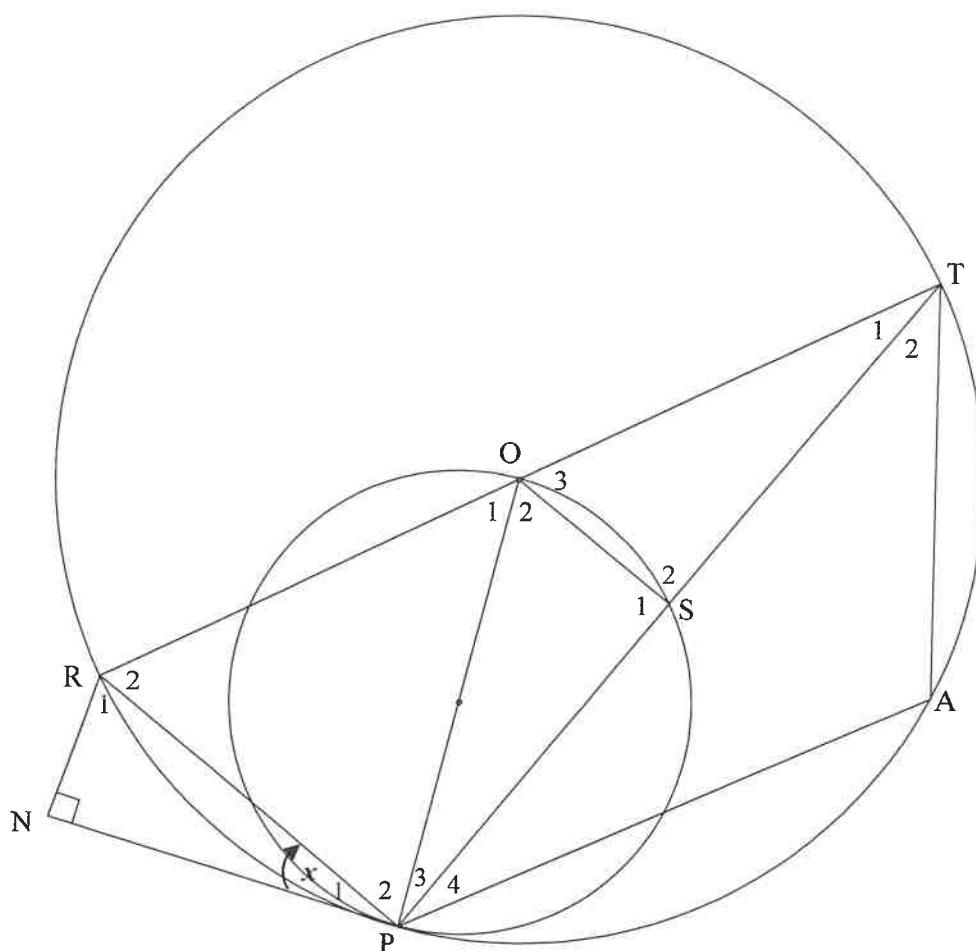
9.3	$AB = BC = 4x \quad \left[\text{line from centre} \perp \text{to chord/lyn van middelpunt} \perp \text{aan koord} \right]$ $OA^2 = (4x)^2 + (2x)^2 \quad [\text{Pythagoras}]$ $OA = \sqrt{16x^2 + 4x^2}$ $= \sqrt{20x^2}$ $= 2\sqrt{5}x$ $OD = OA = 2\sqrt{5}x \quad [\text{radii}]$ $BD = 2\sqrt{5}x - 2x$ $= 2x(\sqrt{5} - 1)$	✓ S ✓ R ✓ Substitution/ vervanging ✓ length of OA / lente van OA ✓ $BD = 2\sqrt{5}x - 2x$ (5) [13]
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QUESTION/VRAAG 10

10.1	<p>Construction: Draw radii OR and OT <i>Konstruksie: teken raduse OR en OT</i></p> <p>Let/ laat: $\hat{S} = x$ and/en $\hat{P} = y$</p> $\hat{O}_1 = 2\hat{S} \quad \left[\text{angle at centre} = 2 \text{ times angle at circumference/} \right. \\ \left. \text{middelpuntshoek} = 2 \text{ keer omtrekshoek} \right]$ $\hat{O}_1 = 2x$ <p>Similarly/ in die selfde manier: $\hat{O}_2 = 2y$</p> $2x + 2y = 360^\circ \quad [\text{angles around a pt / hoeke om'n punt}]$ $x + y = 180^\circ$ $\therefore \hat{S} + \hat{P} = 180^\circ$	<p><input checked="" type="checkbox"/> construction/ konstruksie</p> <p><input checked="" type="checkbox"/> S <input checked="" type="checkbox"/> R</p> <p><input checked="" type="checkbox"/> S</p> <p><input checked="" type="checkbox"/> S/R</p>	(5)
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10.2.1(a)	$\hat{Q} = \hat{S}_1 = 48^\circ$ [ext \angle of cyclic quad/ buite \angle van 'n koodervierhoek] $\hat{Q}_1 = \hat{Q}_2 = 24^\circ$ [QS bisects/ halveer $P\hat{Q}N$]	\checkmark S \checkmark R \checkmark S (3)
10.2.1(b)	$\hat{R} = \hat{Q}_1 = 24^\circ$ [\angle^s in the same segment/ in dieselfde segment]	\checkmark S \checkmark R (2)
10.2.1(c)	$\hat{M}_1 = \hat{Q} = 48^\circ$ [corresp/ ooreenkoms \angle^s , $PQ \parallel SR$] OR/OF $\hat{S}_3 = \hat{Q}_1 = 24^\circ$ [alt \angle^s / ooreenkoms \angle^s , $PQ \parallel SR$] $\hat{M}_1 = 48^\circ$ [ext \angle of Δ / buite \angle van Δ]	\checkmark S \checkmark R (2) \checkmark S / R \checkmark answer/ antwoord (2)
10.2.2	$\hat{M}_1 = \hat{S}_1 = 48^\circ$ $\therefore ST$ is a tangent to circle MNS . [converse tan–chord theorem] $\therefore ST$ is 'n raaklyn aan MNS [omgekeerd raaklyn-koord st.]	\checkmark S \checkmark R (2) [14]

QUESTION/VRAAG 11

11.1	$\hat{T}_1 = x$ [tan – chord theorem / raaklyn-koord st] $\hat{O}_1 = 2x$ $\left[\begin{matrix} \angle \text{ at centre} = 2 \times \angle \text{ at circumference} \\ \text{middelpuntshoek} = 2 \text{ keer omtrekshoek} \end{matrix} \right]$ $\hat{P}_2 = 90^\circ - x$ [tan \perp diameter/ raaklyn \perp middellyn] $\hat{R}_2 = 90^\circ - x$ [\angle^s opp.=sides / \angle^s teenoor gelyke sye] $\therefore \hat{R}_1 = \hat{R}_2$ PR bisects / halveer \hat{ORN}	\checkmark S/R \checkmark S \checkmark S $\checkmark \hat{R}_2 = 90^\circ - x$ \checkmark S
		(5)
	OR/ OF	

	$\hat{P}T = 90^\circ$ [\angle in the semi circle/ <i>in dieselfde segment</i>] $\hat{T}_1 = x$ [tan – chord theorem/ <i>raaklyn-koord st</i>] $\hat{R}_2 = 90^\circ - x$ [sum int \angle^s of Δ / <i>som binne \angle^s van Δ</i>] $\hat{R}_1 = 90^\circ - x$ [sum int \angle^s of Δ / <i>som binne \angle^s van Δ</i>] $\therefore \hat{R}_1 = \hat{R}_2$ PR bisects/ <i>halveer</i> ORN OR/ OF	✓ S ✓ S/R ✓ S ✓ S ✓ S
	$\hat{T}_1 = x$ [tan – chord theorem/ <i>raaklyn-koord st</i>] $\hat{P}_3 = x$ [\angle^s opp.=sides/ <i>\angle^s teeoor gelyke sye</i>] $\hat{O}_1 = 2x$ [ext \angle of Δ / <i>buite \angle van Δ</i>] $\hat{R}_2 = \hat{P}_2 = \frac{180^\circ - 2x}{2}$ [\angle^s opp.=sides/ <i>\angle^s teeoor gelyke sye</i>] $\hat{R}_2 = 90^\circ - x$ $\hat{R}_1 = 90^\circ - x$ [sum int \angle^s of Δ / <i>som binne \angle^s van Δ</i>] $\therefore \hat{R}_1 = \hat{R}_2$ PR bisects/ <i>halveer</i> ORN	✓ S ✓ S/R ✓ S ✓ S ✓ S ✓ S ✓ S ✓ S ✓ S
11.2	$\hat{P}AT = 90^\circ + x$ [<i>opp \angle^s of cyclic quad/ teenoorst. hoeke van koordevierhoek</i>] $\hat{S}_2 = 90^\circ$ [Line from centre \perp to chord / <i>lyn van mdpt \perp aan koo</i> $\hat{ROS} = 90^\circ + x$ [ext \angle of Δ / <i>buite \angle van Δ</i>] $\therefore \hat{ROS} = \hat{PAT}$ OR/ OF	✓ S ✓ R ✓ S ✓ R ✓ S/R ✓ S

$$\hat{RPT} = 90^\circ \quad [\angle \text{in the semi circle/ in dieselfde segment}]$$

$$\hat{NPT} = 90^\circ + x$$

$$\hat{PAT} = \hat{NPT} = 90^\circ + x \quad [\tan - \text{chord theorem / raaklyn-koord st}]$$

$$\hat{P}_3 = x \quad [\angle^s \text{ opp. = sides / } \angle^s \text{ teeoor gelyke sye}]$$

$$\hat{S}_1 = 90^\circ \quad [\text{Line from centre } \perp \text{ to chord / lyn van mdpt } \perp \text{ aan koord}]$$

$$\therefore \hat{O}_2 = 90^\circ - x \quad [\text{sum of int } \angle^s \text{ of } \Delta]$$

$$\hat{ROS} = \hat{O}_1 + \hat{O}_2$$

$$= 2x + 90^\circ - x$$

$$= 90^\circ + x$$

$$\therefore \hat{ROS} = \hat{PAT}$$

✓S

✓S ✓R

✓S/R

✓S

(5)

[10]

TOTAL/TOTAAL: 150