



# basic education

Department:  
Basic Education  
**REPUBLIC OF SOUTH AFRICA**

**NASIONALE  
SENIOR SERTIFIKAAT**

**GRAAD 12**

**WISKUNDE V2**

**NOVEMBER 2018**

**PUNTE: 150**

**TYD: 3 uur**

**Hierdie vraestel bestaan uit 15 bladsye, 1 inligtingsblad  
en 'n antwoordeboek van 31 bladsye.**

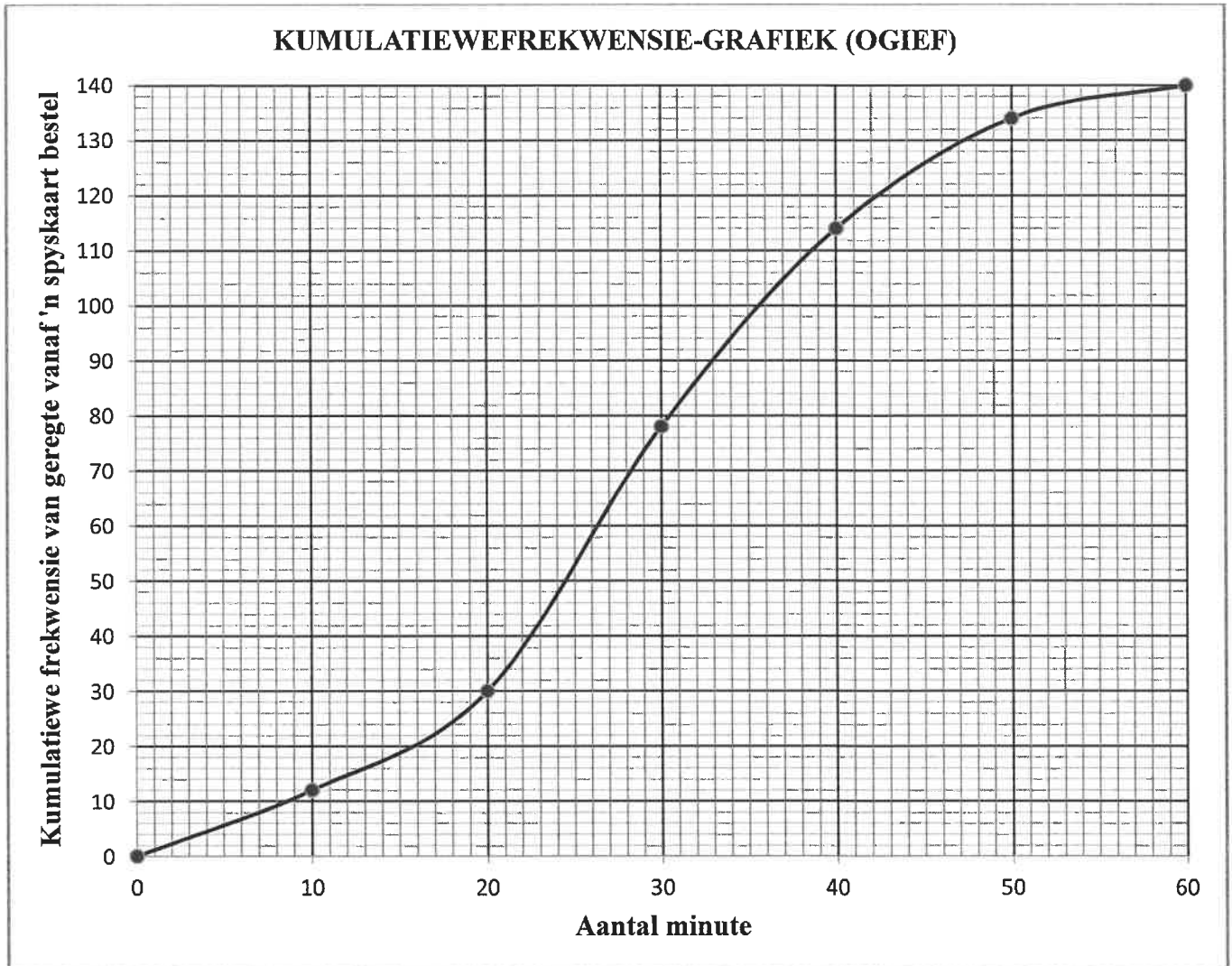
**INSTRUKSIES EN INLIGTING**

Lees die volgende instruksies aandagtig deur voordat die vraestel beantwoord word.

1. Hierdie vraestel bestaan uit 10 vrae.
2. Beantwoord AL die vrae in die SPESIALE ANTWOORDEBOEK wat verskaf word.
3. Dui ALLE berekeninge, diagramme, grafieke, ens. wat jy gebruik in die beantwoording van die vrae, duidelik aan.
4. Slegs antwoorde sal NIE noodwendig volpunte verdien NIE.
5. Jy kan 'n goedgekeurde wetenskaplike sakrekenaar gebruik (nieprogrammeerbaar en niegrafies), tensy anders vermeld.
6. Indien nodig, rond antwoorde tot TWEE desimale plekke af, tensy anders gemeld.
7. Diagramme is NIE noodwendig volgens skaal geteken NIE.
8. 'n Inligtingsblad met formules is aan die einde van die vraestel ingesluit.
9. Skryf netjies en leesbaar.

**VRAAG 1**

1.1 Die kumulatiewefrekwensie-grafiek (ogief) hieronder toon die totale aantal geregte wat oor 'n tydperk van 1 uur vanaf 'n spyskaart bestel is.



- 1.1.1 Skryf die totale aantal voedselitems neer wat gedurende hierdie uur vanaf die spyskaart bestel is. (1)
- 1.1.2 Skryf die modale klas van die data neer. (1)
- 1.1.3 Hoe lank het dit geneem om die eerste 30 voedselitems te bestel? (1)
- 1.1.4 Hoeveel voedselitems is in die laaste 15 minute bestel? (2)
- 1.1.5 Bepaal die 75<sup>ste</sup> persentiel van die data. (2)
- 1.1.6 Bereken die interkwartielvariasiewydte(-omvang) van die data. (2)

- 1.2 Reggie werk deelyds as 'n kelner by 'n plaaslike restaurant. Die bedrag geld (in rand) wat hy oor 'n tydperk van 15 dae met fooritjies ('tips') gemaak het, word hieronder gegee.

|     |     |     |     |     |
|-----|-----|-----|-----|-----|
| 35  | 70  | 75  | 80  | 80  |
| 90  | 100 | 100 | 105 | 105 |
| 110 | 110 | 115 | 120 | 125 |

- 1.2.1 Bereken:

- (a) Die gemiddeld van die data (2)
- (b) Die standaardafwyking van die data (2)

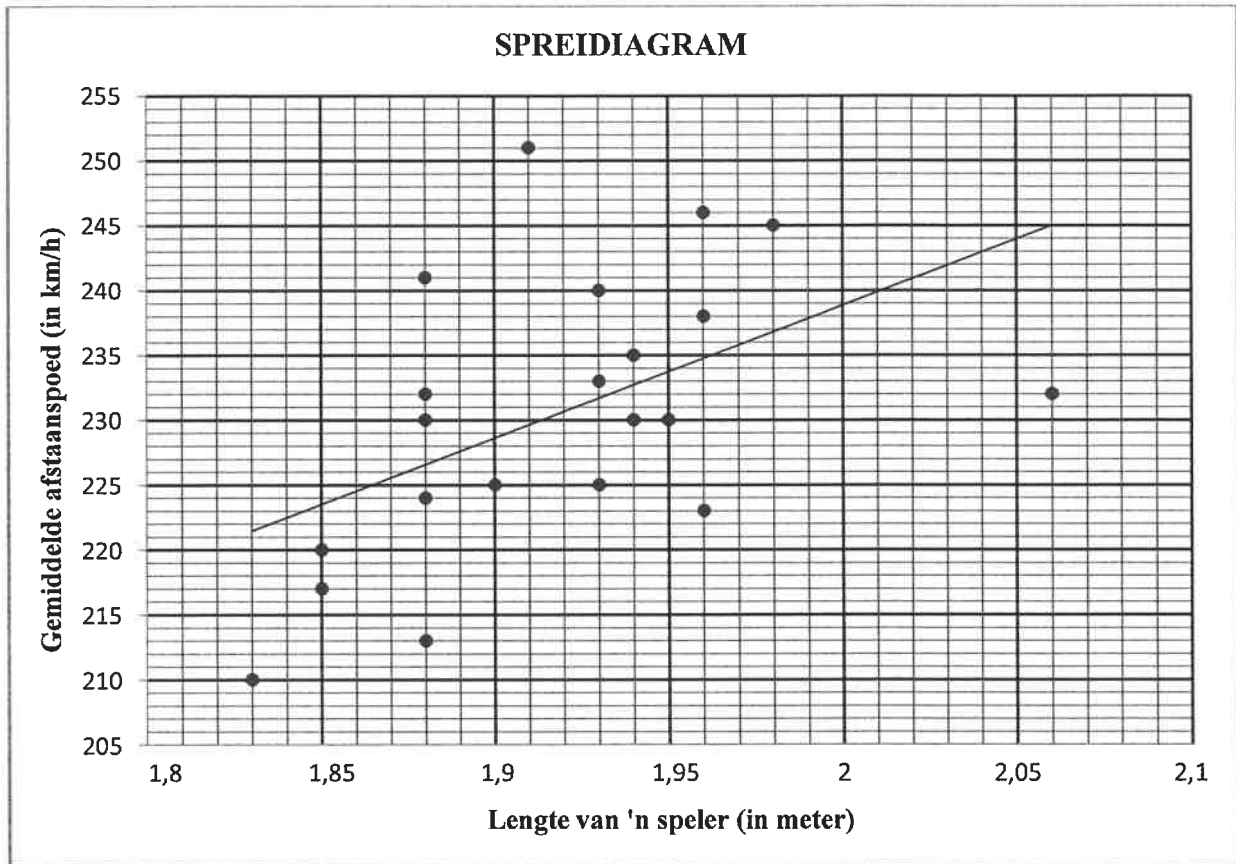
- 1.2.2 Marie werk ook deelyds as 'n kelnerin by dieselfde restaurant. Oor dieselfde 15 dag-tydperk het Mary dieselfde gemiddelde bedrag met fooritjies as Reggie ingesamel, maar haar standaardafwyking was R14.

Gebruik die beskikbare inligting en lewer kommentaar op die:

- (a) Totale bedrag in fooritjies wat ELKEEN van hulle oor die 15 dag-tydperk ingesamel het (1)
- (b) Verspreiding wat ELKEEN van hulle met daaglikse fooritjies oor hierdie tydperk ontvang het (1)
- [15]

**VRAAG 2**

'n Bekende vraag onder professionele tennisspelers is of die spoed van 'n tennisafslaan (in km/h) van die lengte van 'n speler (in meter) afhang. Die lengtes van 21 tennisspelers en die gemiddelde spoed van hulle afslane is tydens 'n toernooi aangeteken. Die data word in die spreidiagram hieronder voorgestel. Die kleinstekwadrate-regressielyn is ook getrek.

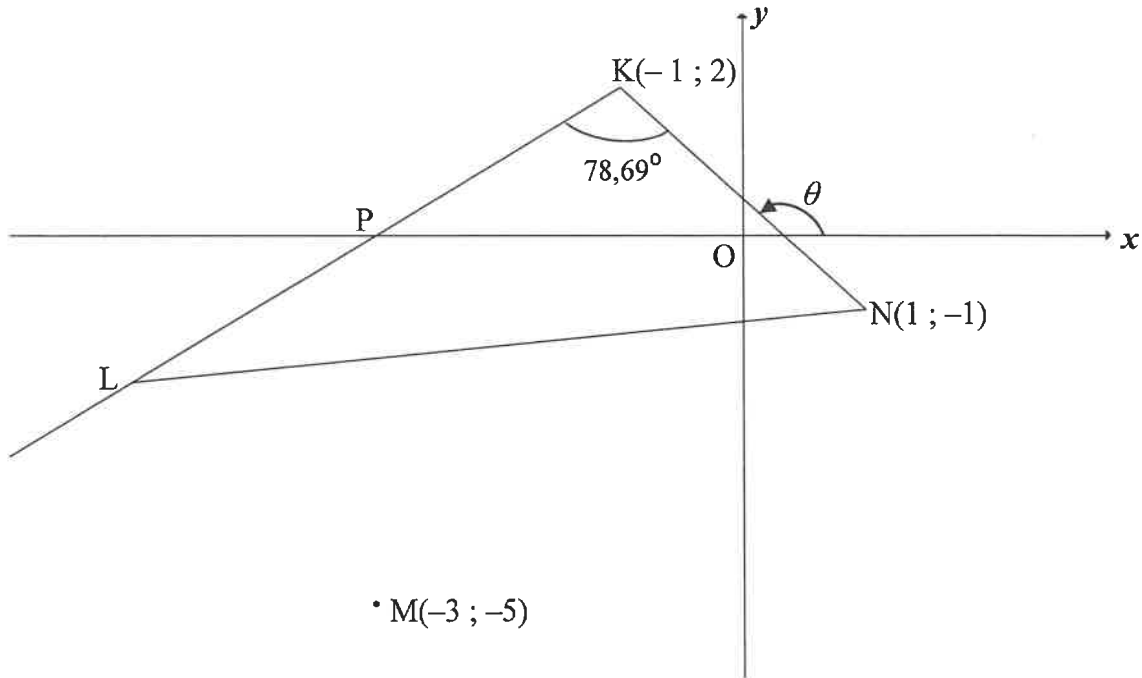


- 2.1 Skryf die vinnigste gemiddelde afstaanspoed (in km/h) wat in hierdie toernooi bereik is, neer. (1)
- 2.2 Beskou die volgende korrelasiekoëffisiënte:
- A.  $r = 0,93$                       B.  $r = -0,42$                       C.  $r = 0,52$
- 2.2.1 Watter EEN van die korrelasiekoëffisiënte gegee, pas die beste by die geplotte data? (1)
- 2.2.2 Gebruik die spreidiagram en kleinstekwadrate-regressielyn om jou antwoord op VRAAG 2.2.1 te motiveer. (1)
- 2.3 Waarop dui die data ten opsigte van die spoed van 'n tennisafslaan (in km/h) en die lengte van 'n speler (in meter)? (1)
- 2.4 Die vergelyking van die regressielyn word as  $\hat{y} = 27,07 + bx$  gegee. Verduidelik waarom, in hierdie konteks, die kleinstekwadrate-regressielyn NIE die y-as by (0 ; 27,07) kan sny NIE. (1)

**[5]**

**VRAAG 3**

In die diagram is  $K(-1; 2)$ ,  $L$  en  $N(1; -1)$  hoekpunte van  $\triangle KLN$  sodat  $\hat{LKN} = 78,69^\circ$ .  $KL$  sny die  $x$ -as by  $P$ .  $KL$  word verleng. Die inklinasie van  $KN$  is  $\theta$ . Die koördinate van  $M$  is  $(-3; -5)$ .

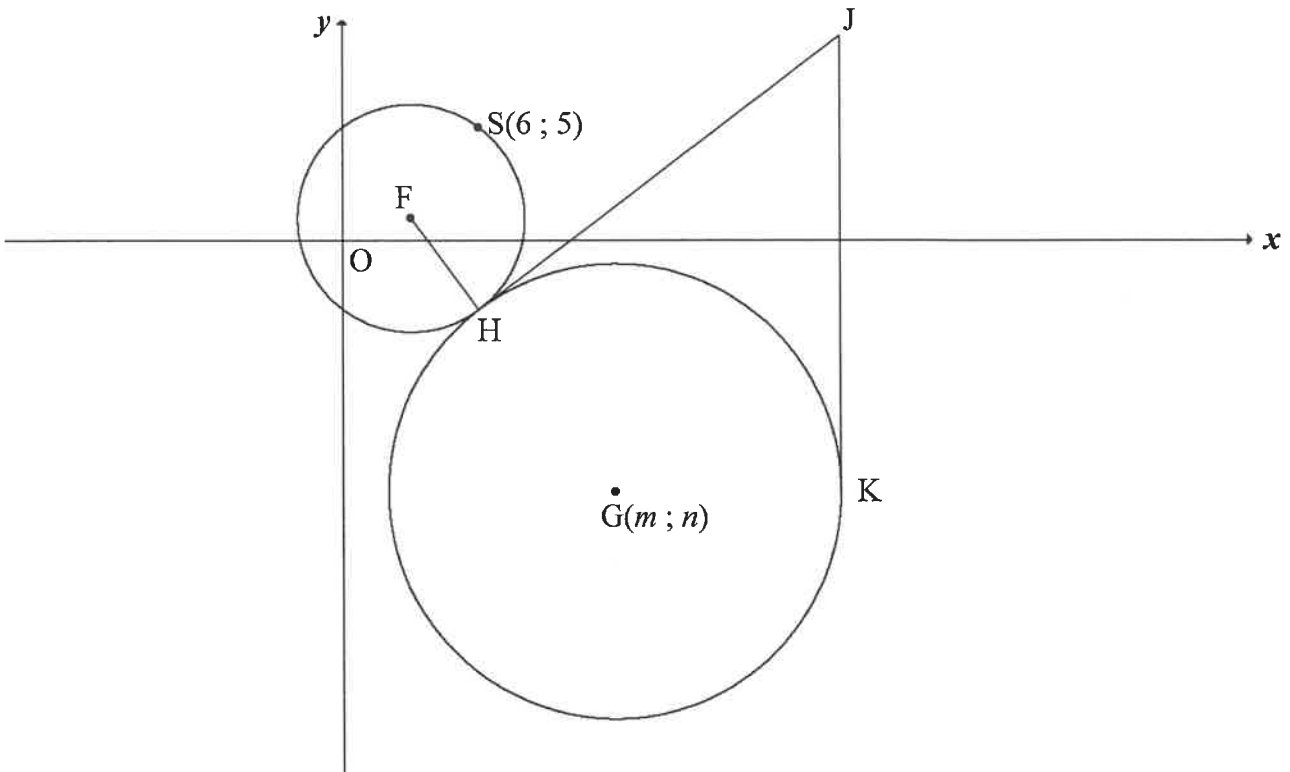


- 3.1 Bereken:
- 3.1.1 Die gradiënt van  $KN$  (2)
- 3.1.2 Die grootte van  $\theta$ , die inklinasie van  $KN$  (2)
- 3.2 Toon dat die gradiënt van  $KL$  gelyk is aan 1. (2)
- 3.3 Bepaal die vergelyking van die reguitlyn  $KL$  in die vorm  $y = mx + c$ . (2)
- 3.4 Bereken die lengte van  $KN$ . (2)
- 3.5 Daar word verder gegee dat  $KN = LM$ .
- 3.5.1 Bereken die moontlike koördinate van  $L$ . (5)
- 3.5.2 Bepaal die koördinate van  $L$  as gegee word dat  $KLMN$  'n parallelogram is. (3)
- 3.6  $T$  is 'n punt op  $KL$  verleng.  $TM$  word so getrek dat  $TM = LM$ . Bereken die oppervlakte van  $\triangle KTN$ . (4)

**[22]**

**VRAAG 4**

In die diagram is  $(x - 3)^2 + (y - 1)^2 = r^2$  die vergelyking van die sirkel met middelpunt F. S(6 ; 5) is 'n punt op die sirkel met middelpunt F. 'n Ander sirkel met middelpunt G(m ; n) in die 4<sup>de</sup> kwadrant raak die sirkel met middelpunt F, by H sodat FH : HG = 1 : 2. Die punt J lê in die eerste kwadrant sodat HJ 'n gemeenskaplike raaklyn aan beide hierdie sirkels is. JK is 'n raaklyn aan die groter sirkel by K.

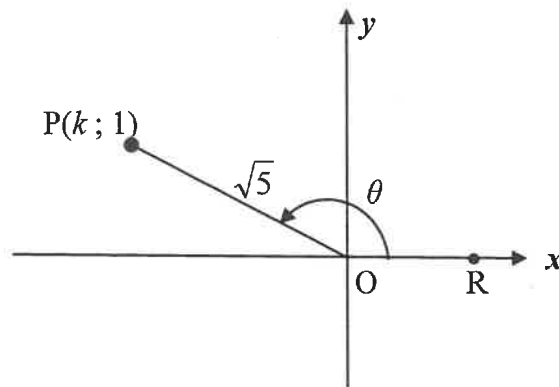


- 4.1 Skryf die koördinate van F neer. (2)
- 4.2 Bereken die lengte van FS. (2)
- 4.3 Skryf die lengte van HG neer. (1)
- 4.4 Gee 'n rede waarom JH = JK. (1)
- 4.5 Bepaal:
  - 4.5.1 Die afstand FJ, met redes, as gegee word dat JK = 20 (4)
  - 4.5.2 Die vergelyking van die sirkel met middelpunt G in terme van m en n in die vorm  $(x - a)^2 + (y - b)^2 = r^2$  (1)
  - 4.5.3 Die koördinate van G, indien verder gegee word dat die vergelyking van die raaklyn JK aan  $x = 22$  (7)

[18]

**VRAAG 5**

- 5.1 In die diagram is  $P(k; 1)$  'n punt in die 2<sup>de</sup> kwadrant en is  $\sqrt{5}$  eenhede vanaf die oorsprong. R is 'n punt op die positiewe  $x$ -as en stomphoek  $\widehat{R\hat{O}P} = \theta$ .



- 5.1.1 Bereken die waarde van  $k$ . (2)
- 5.1.2 **Sonder om 'n sakrekenaar te gebruik**, bereken die waarde van:
- (a)  $\tan \theta$  (1)
- (b)  $\cos(180^\circ + \theta)$  (2)
- (c)  $\sin(\theta + 60^\circ)$  in die vorm  $\frac{a+b}{\sqrt{20}}$  (5)
- 5.1.3 **Gebruik 'n sakrekenaar** om die waarde van  $\tan(2\theta - 40^\circ)$  korrek tot EEN desimale plek te bereken. (3)
- 5.2 Bewys die volgende identiteit:  $\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2 \tan 2x$  (5)
- 5.3 Evalueer, **sonder om 'n sakrekenaar te gebruik**:  $\sum_{A=38^\circ}^{52^\circ} \cos^2 A$  (5)

**[23]**



**VRAAG 6**

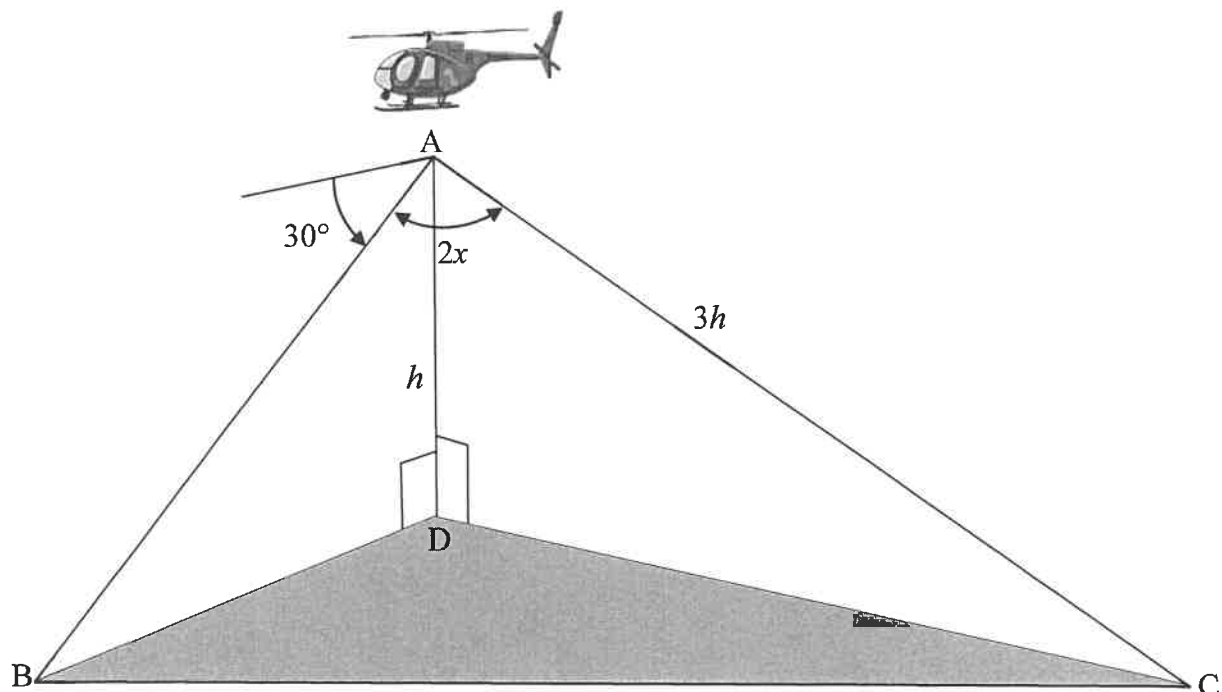
Beskou:  $f(x) = -2 \tan \frac{3}{2}x$

- 6.1 Skryf die periode van  $f$  neer. (1)
- 6.2 Die punt  $A(t; 2)$  lê op die grafiek. Bepaal die algemene oplossing van  $t$ . (3)
- 6.3 Op die rooster wat in die ANTWOORDEBOEK verskaf is, skets die grafiek van  $f$  vir die interval  $x \in [-120^\circ; 180^\circ]$ . Toon duidelik ALLE asimptote, afsnitte met die asse en eindpunt(e) van die grafiek aan. (4)
- 6.4 Gebruik die grafiek om te bepaal vir watter waarde(s) van  $x$  is  $f(x) \geq 2$  vir  $x \in [-120^\circ; 180^\circ]$ . (3)
- 6.5 Beskryf die transformasie van grafiek  $f$  om die grafiek van  $g(x) = -2 \tan\left(\frac{3}{2}x + 60^\circ\right)$  te vorm. (2)

**[13]**

**VRAAG 7**

'n Loods vlieg in 'n helikopter. By punt A, wat  $h$  meter direk bokant punt D op die grond is, neem hy 'n vreemde voorwerp by punt B waar. Die loods bepaal dat die dieptehoek vanaf A na B,  $30^\circ$  is. Hy bepaal ook dat die kontrolekamer by punt C,  $3h$  meter vanaf A is en dat  $\hat{BAC} = 2x$ . Punte B, C en D is in dieselfde horisontale vlak. Hierdie scenario word in die diagram hieronder getoon.



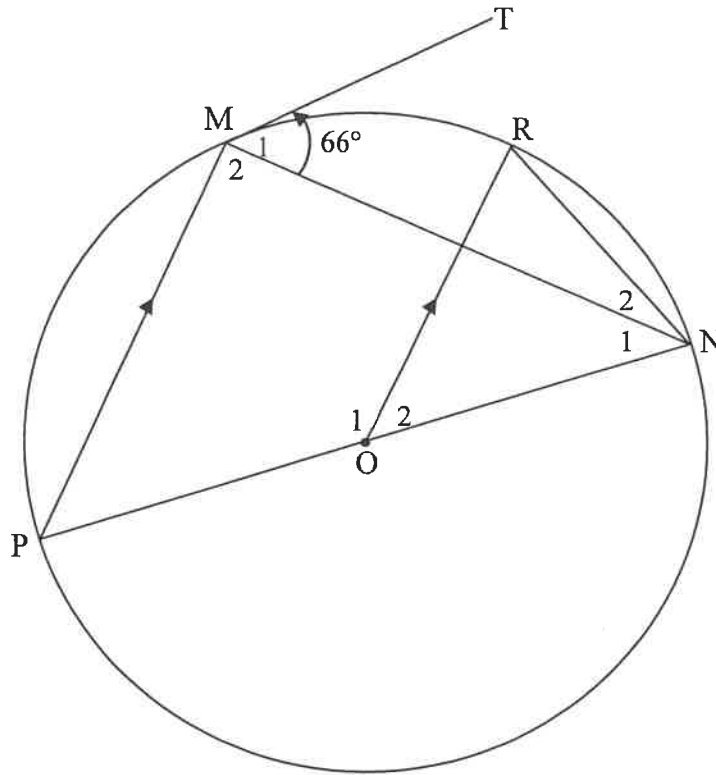
7.1 Bepaal die afstand AB in terme van  $h$ . (2)

7.2 Toon dat die afstand tussen die vreemde voorwerp by punt B en die kontrolekamer by punt C deur  $BC = h\sqrt{25 - 24\cos^2 x}$  gegee word. (4)

[6]

**VRAAG 8**

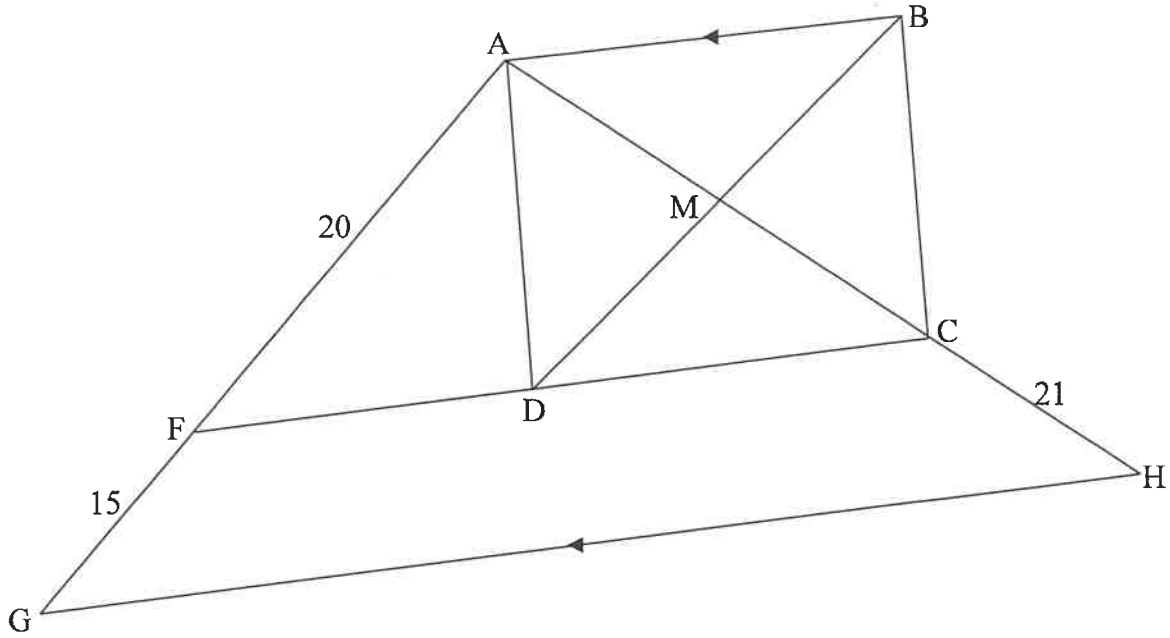
8.1 PON is 'n middellyn van die sirkel met middelpunt O. TM is 'n raaklyn aan die sirkel by M, 'n punt op die sirkel. R is 'n ander punt op die sirkel sodat  $OR \parallel PM$ . NR en MN is getrek. Laat  $\hat{M}_1 = 66^\circ$ .



Bereken, met redes, die grootte van ELK van die volgende hoeke:

- 8.1.1  $\hat{P}$  (2)
- 8.1.2  $\hat{M}_2$  (2)
- 8.1.3  $\hat{N}_1$  (1)
- 8.1.4  $\hat{O}_2$  (2)
- 8.1.5  $\hat{N}_2$  (3)

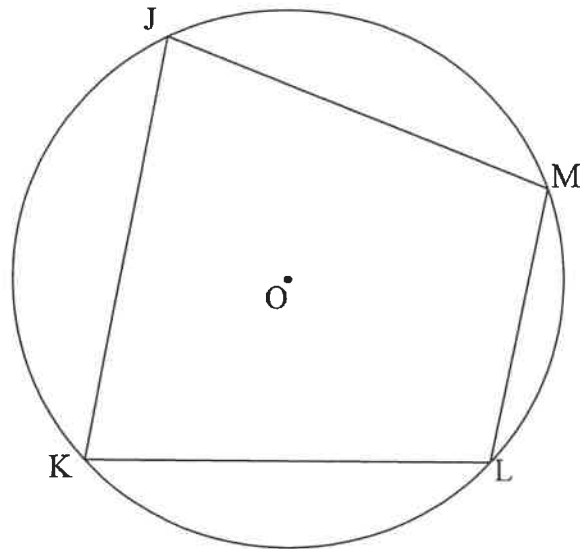
- 8.2 In die diagram is  $\triangle AGH$  geskets. F en C is punte op AG en AH onderskeidelik sodat  $AF = 20$  eenhede,  $FG = 15$  eenhede en  $CH = 21$  eenhede. D is 'n punt op FC sodat ABCD 'n reghoek is met AB, ook ewewydig aan GH. Die hoeklyne van ABCD sny by M, 'n punt op AH.



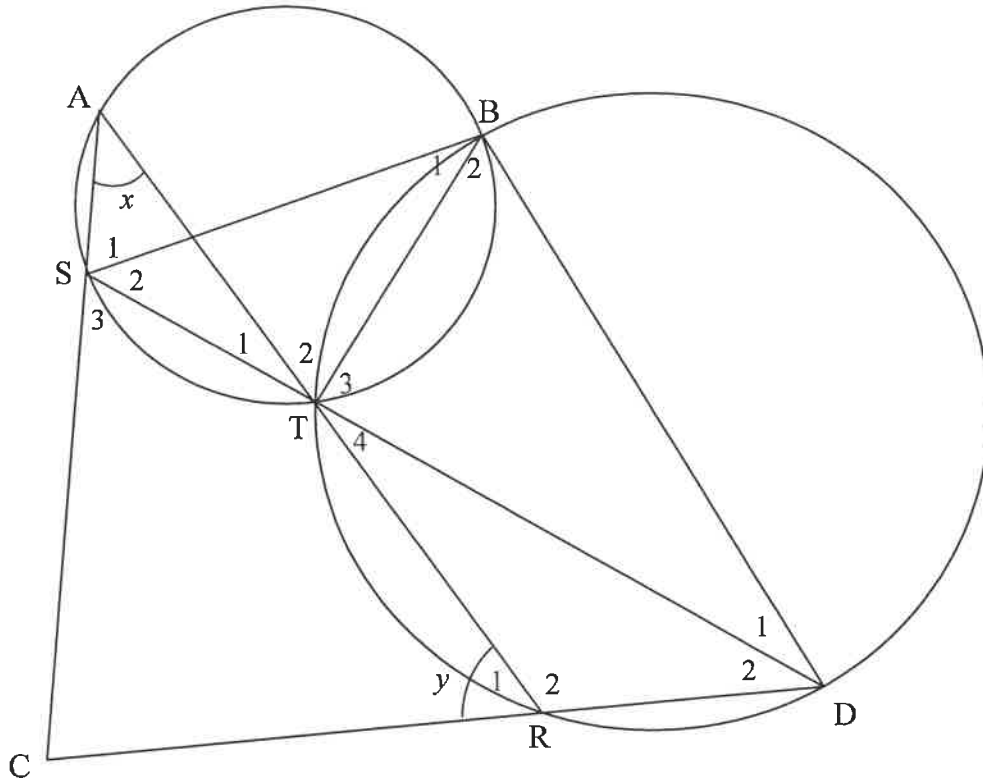
- 8.2.1 Verduidelik waarom  $FC \parallel GH$ . (1)
- 8.2.2 Bereken, met redes, die lengte van DM. (5)
- [16]**

**VRAAG 9**

- 9.1 In die diagram is JKLM 'n koordevierhoek en die sirkel het middelpunt O.  
Bewys die stelling wat beweer dat  $\hat{J} + \hat{L} = 180^\circ$ . (5)



9.2 In die diagram word 'n kleiner sirkel  $ABTS$  en 'n groter sirkel  $BDRT$  gegee.  $BT$  is 'n gemeenskaplike koord. Reguitlyne  $STD$  en  $ATR$  is getrek. Koorde  $AS$  en  $DR$  word verleng om mekaar in  $C$ , 'n punt buite die twee sirkels, te sny.  $BS$  en  $BD$  is getrek.  $\hat{A} = x$  en  $\hat{R}_1 = y$ .



9.2.1 Noem, met 'n rede, 'n ander hoek gelyk aan:

(a)  $x$  (2)

(b)  $y$  (2)

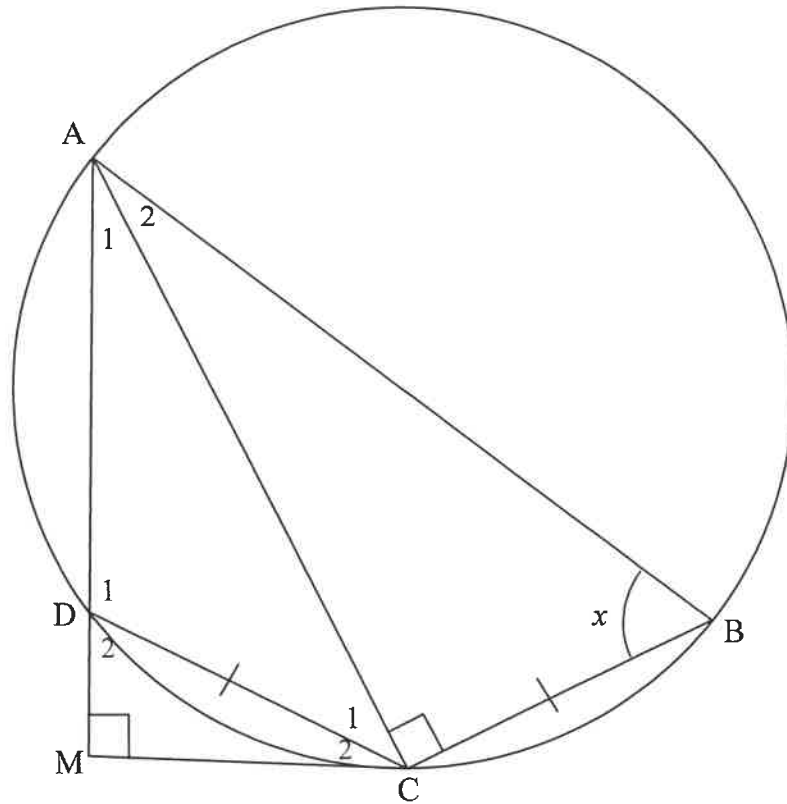
9.2.2 Bewys dat  $SCDB$  'n koordevierhoek is. (3)

9.2.3 Daar word verder gegee dat  $\hat{D}_2 = 30^\circ$  en  $\hat{AST} = 100^\circ$ .  
Bewys dat  $SD$  nie 'n middellyn van sirkel  $BDS$  is nie. (4)

[16]

**VRAAG 10**

In die diagram is  $ABCD$  'n koordevierhoek met  $AC \perp CB$  en  $DC = CB$ .  $AD$  is verleng na  $M$  sodat  $AM \perp MC$ . Laat  $\hat{B} = x$ .



10.1 Bewys dat:

10.1.1  $MC$  'n raaklyn aan die sirkel by  $C$  is (5)

10.1.2  $\triangle ACB \cong \triangle CMD$  (3)

10.2 Bewys vervolgens, of andersins, dat:

10.2.1  $\frac{CM^2}{DC^2} = \frac{AM}{AB}$  (6)

10.2.2  $\frac{AM}{AB} = \sin^2 x$  (2)

[16]

**TOTAAL: 150**

## INLICHTINGSBLAD

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \Delta ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

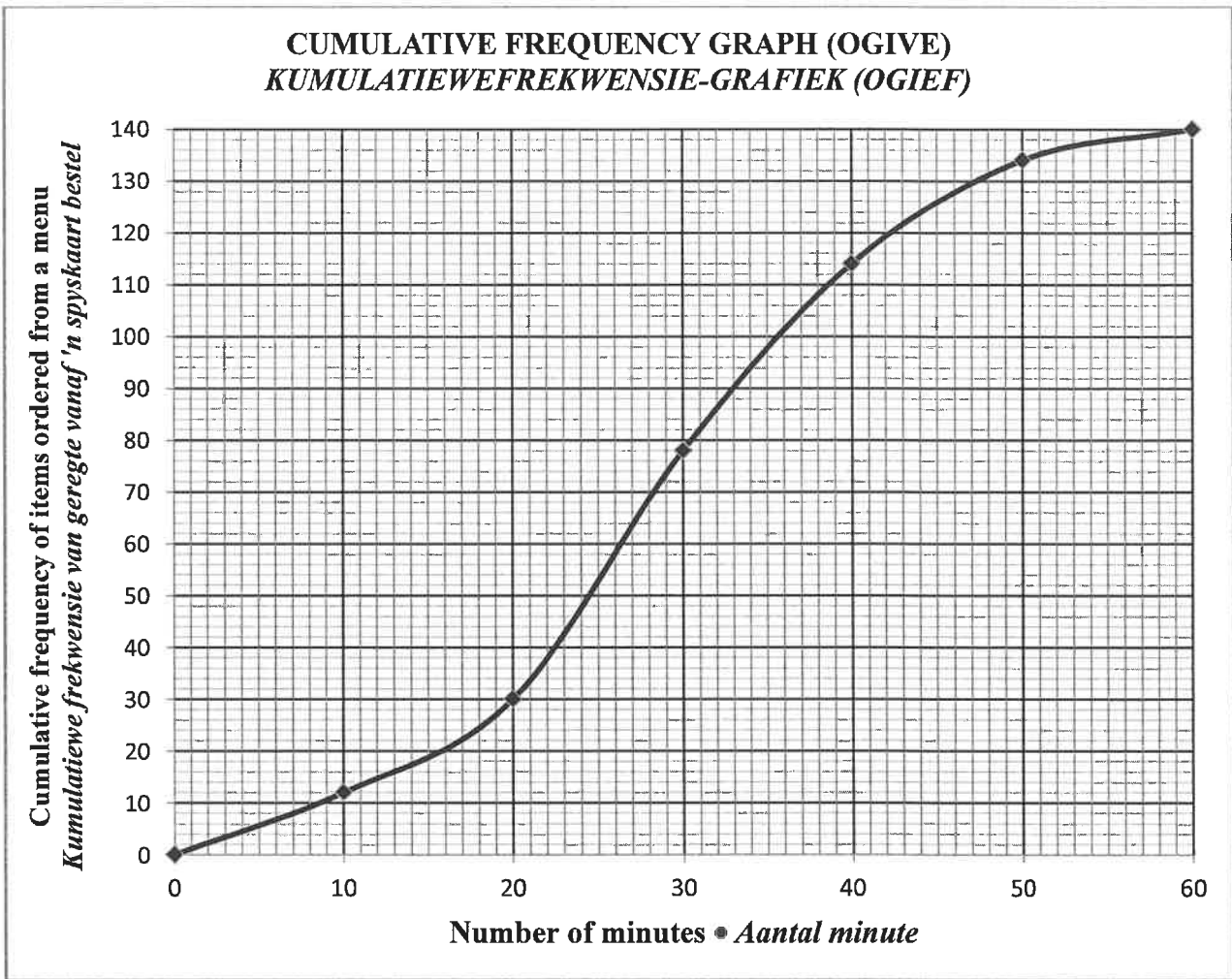
$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$





| PLEASE FOLLOW THESE INSTRUCTIONS CAREFULLY   | VOLG ASSEBLIEF HIERDIE INSTRUKSIES NOUKEURIG  |
|--|---|
| 1. Clearly write your examination number and centre number in the space provided and attach your barcode label in the space provided.  | 1. Skryf jou eksamennommer en sentrumnummer duidelik in die ruimtes verskaf en plak jou stafieskodeplakker in die ruimte verskaf.   |
| 2. Remember that your own name (or the name of your school) may NOT appear anywhere on or in this answer book.   | 2. Onthou dat jou eie naam (of die naam van jou skool) NIE op of in hierdie antwoordeboek mag voorkom NIE.  |
| 3. Answer ALL questions in the spaces provided.  | 3. Beantwoord ALLE vrae in die ruimtes wat verskaf is.  |
| 4. NO pages may be torn from this answer book.   | 4. GEEN bladsye mag uit hierdie antwoordeboek geskeur word NIE.   |
| 5. Read the instructions printed on your timetable carefully as well as any other instructions which may be given in each question paper.  | 5. Lees die instruksies, wat op jou eksamenrooster gedruk is, sorgvuldig deur, asook enige ander instruksies wat op elke vraestel gegee word.   |
| 6. Candidates may NOT retain an answer book or remove it from the examination room.  | 6. GEEN antwoordeboek mag deur die kandidaat behou of uit die eksamenlokaal verwyder word NIE.  |
| 7. Answers must be written in black/blue ink as distinctly as possible. Do NOT write in the margins.   | 7. Skryf die antwoorde so duidelik moontlik met swart/blou ink. Laat die kantlyne oop.  |
| 8. Write the numbers of the questions you have answered on the front cover of the answer book where marks are to be recorded.  | 8. Skryf die nommers van die vrae wat jy beantwoord het op die voorblad van die antwoordeboek waar die punte aangebring word.   |
| 9. If you require additional space for your answers:<br>9.1 Use the additional space provided at the end of the answer book.<br>9.2 When answering a question in the additional space, clearly indicate the question number in the column on the LHS.<br>9.3 Rule off after each answer. | 9. In geval jy addisionele ruimte benodig vir jou antwoorde:<br>9.1 Gebruik die addisionele ruimte wat aan die einde van die antwoordeboek verskaf word.<br>9.2 As 'n vraag in die addisionele ruimte beantwoord word, dui duidelik die vraagnommer in die kolom aan die LK aan.<br>9.3 Trek 'n lyn na elke antwoord. |
| 10. Draw a neat line through any work/rough work that must not be marked.  | 10. Trek 'n netjiese lyn deur enige werk/rofwerk wat nie nagesien moet word nie.  |

**QUESTION/VRAAG 1**



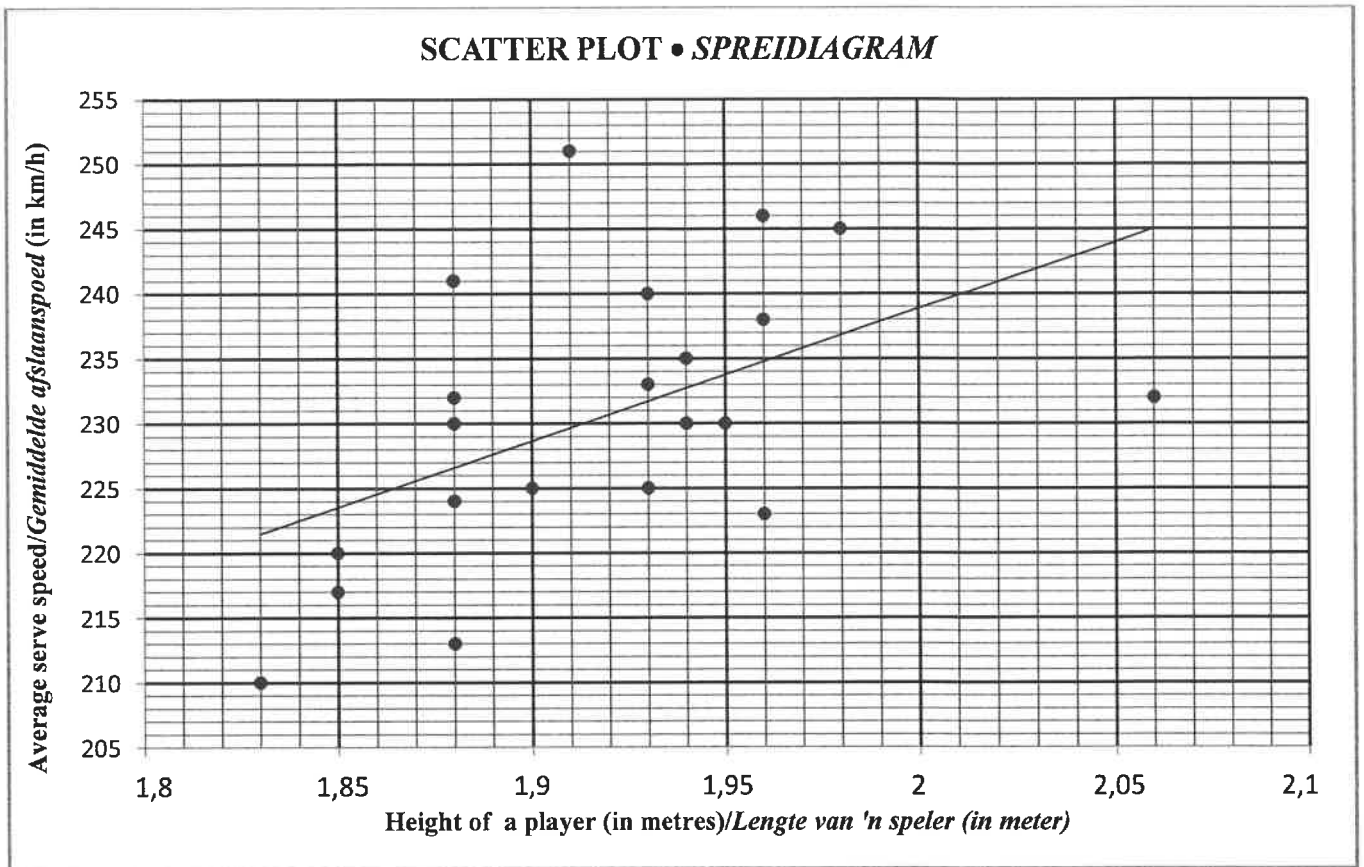
|       | <b>Solution/Oplissing</b> | <b>Marks<br/>Punte</b> |
|-------|---------------------------|------------------------|
| 1.1.1 |                           | (1)                    |
| 1.1.2 |                           | (1)                    |
| 1.1.3 |                           | (1)                    |

|       | <b>Solution/<i>Oplossing</i></b> | <b>Marks<br/><i>Punte</i></b> |
|-------|----------------------------------|-------------------------------|
| 1.1.4 |                                  | (2)                           |
| 1.1.5 |                                  | (2)                           |
| 1.1.6 |                                  | (2)                           |

|     |     |     |     |     |
|-----|-----|-----|-----|-----|
| 35  | 70  | 75  | 80  | 80  |
| 90  | 100 | 100 | 105 | 105 |
| 110 | 110 | 115 | 120 | 125 |

|          | <b>Solution/Oplissing</b> | <b>Marks<br/>Punte</b> |
|----------|---------------------------|------------------------|
| 1.2.1(a) |                           | (2)                    |
|          |                           |                        |
|          |                           |                        |
|          |                           |                        |
|          |                           |                        |
| 1.2.1(b) |                           | (2)                    |
|          |                           |                        |
|          |                           |                        |
|          |                           |                        |
|          |                           |                        |
| 1.2.2(a) |                           | (1)                    |
|          |                           |                        |
|          |                           |                        |
|          |                           |                        |
|          |                           |                        |
| 1.2.2(b) |                           | (1)                    |
|          |                           |                        |
|          |                           |                        |
|          |                           |                        |
|          |                           |                        |
|          |                           | <b>[15]</b>            |

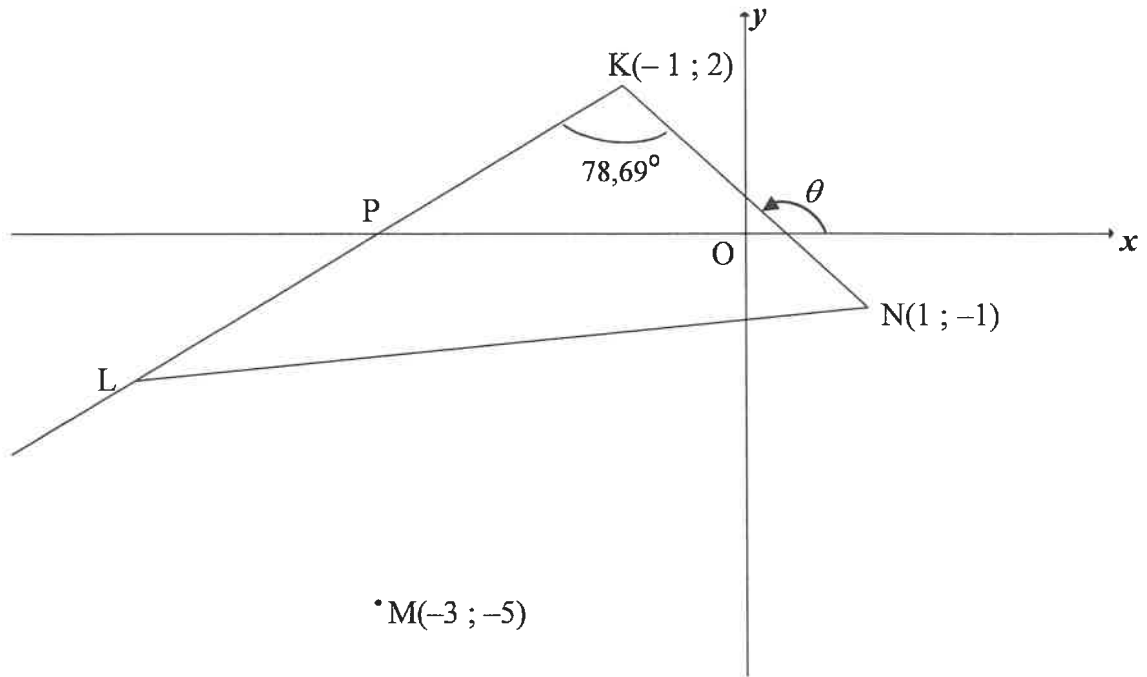
**QUESTION/VRAAG 2**



|       | <b>Solution/Oplissing</b> | <b>Marks<br/>Punte</b> |
|-------|---------------------------|------------------------|
| 2.1   |                           | (1)                    |
| 2.2.1 |                           | (1)                    |
| 2.2.2 |                           | (1)                    |

|     | <b>Solution/<i>Oplossing</i></b> | <b>Marks<br/><i>Punte</i></b> |
|-----|----------------------------------|-------------------------------|
| 2.3 |                                  | (1)                           |
|     |                                  |                               |
|     |                                  |                               |
|     |                                  |                               |
|     |                                  |                               |
| 2.4 |                                  | (1)                           |
|     |                                  |                               |
|     |                                  |                               |
|     |                                  |                               |
|     |                                  |                               |
|     |                                  | <b>[5]</b>                    |

**QUESTION/VRAAG 3**



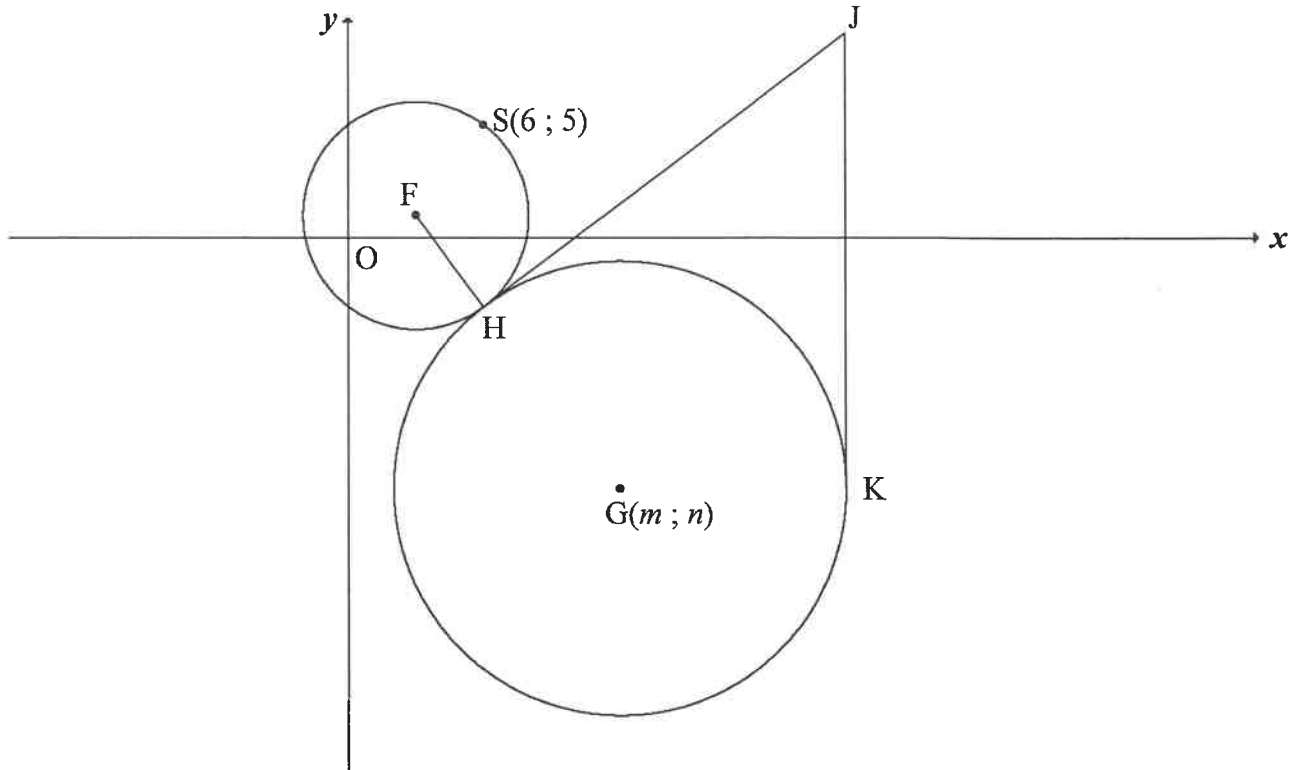
|       | <b>Solution/Oplissing</b> | <b>Marks<br/>Punte</b> |
|-------|---------------------------|------------------------|
| 3.1.1 |                           | (2)                    |
| 3.1.2 |                           | (2)                    |



|       | <b>Solution/<i>Oplossing</i></b> | <b>Marks<br/><i>Punte</i></b> |
|-------|----------------------------------|-------------------------------|
| 3.2   |                                  | (2)                           |
| 3.3   |                                  | (2)                           |
| 3.4   |                                  | (2)                           |
| 3.5.1 |                                  | (5)                           |

|       | <b>Solution/<i>Oplossing</i></b> | <b>Marks<br/><i>Punte</i></b> |
|-------|----------------------------------|-------------------------------|
| 3.5.2 |                                  |                               |
| 3.6   |                                  | (3)                           |
|       |                                  | (4)                           |
|       |                                  | <b>[22]</b>                   |

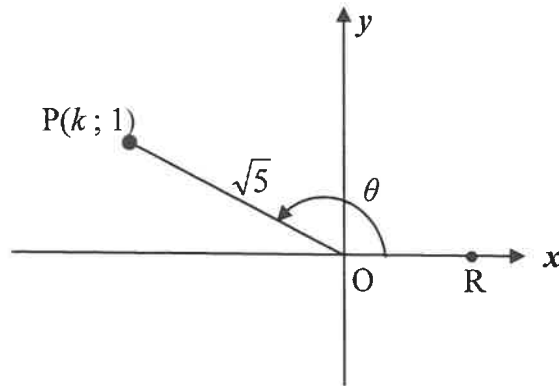
**QUESTION/VRAAG 4**



|     | <b>Solution/Oplissing</b> | <b>Marks<br/>Punte</b> |
|-----|---------------------------|------------------------|
| 4.1 |                           | (2)                    |
|     |                           |                        |
| 4.2 |                           | (2)                    |
|     |                           |                        |
| 4.3 |                           | (1)                    |
|     |                           |                        |
| 4.4 |                           | (1)                    |
|     |                           |                        |

|       | <b>Solution/Oplissing</b> | <b>Marks<br/>Punte</b> |
|-------|---------------------------|------------------------|
| 4.5.1 |                           | (4)                    |
| 4.5.2 |                           | (1)                    |
| 4.5.3 |                           | (7)                    |
|       |                           | <b>[18]</b>            |

**QUESTION/VRAAG 5**

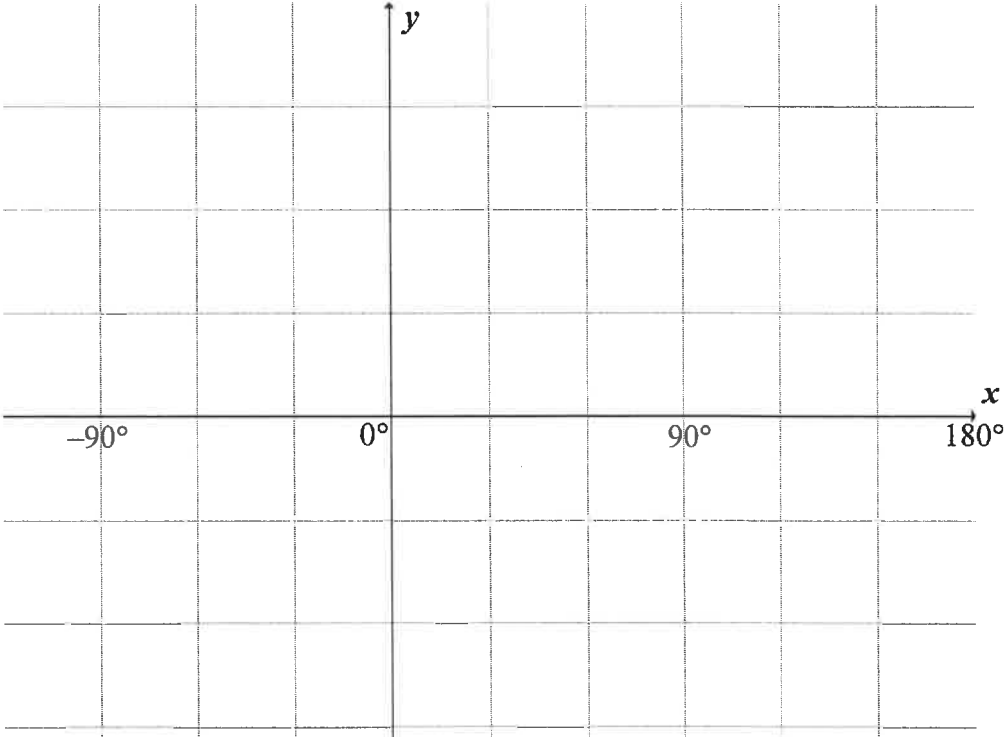


|          | <b>Solution/Oplissing</b> | <b>Marks<br/>Punte</b> |
|----------|---------------------------|------------------------|
| 5.1.1    |                           |                        |
|          |                           |                        |
|          |                           |                        |
|          |                           |                        |
|          |                           |                        |
|          |                           | (2)                    |
| 5.1.2(a) |                           |                        |
|          |                           |                        |
|          |                           |                        |
|          |                           |                        |
|          |                           | (1)                    |
| 5.1.2(b) |                           |                        |
|          |                           |                        |
|          |                           |                        |
|          |                           |                        |
|          |                           |                        |
|          |                           | (2)                    |





**QUESTION/VRAAG 6**

|     | <b>Solution/Oplissing</b>  | <b>Marks<br/>Punte</b> |
|-----|--|------------------------|
| 6.1 |  | (1)                    |
| 6.2 |  | (3)                    |
| 6.3 |  | (4)                    |



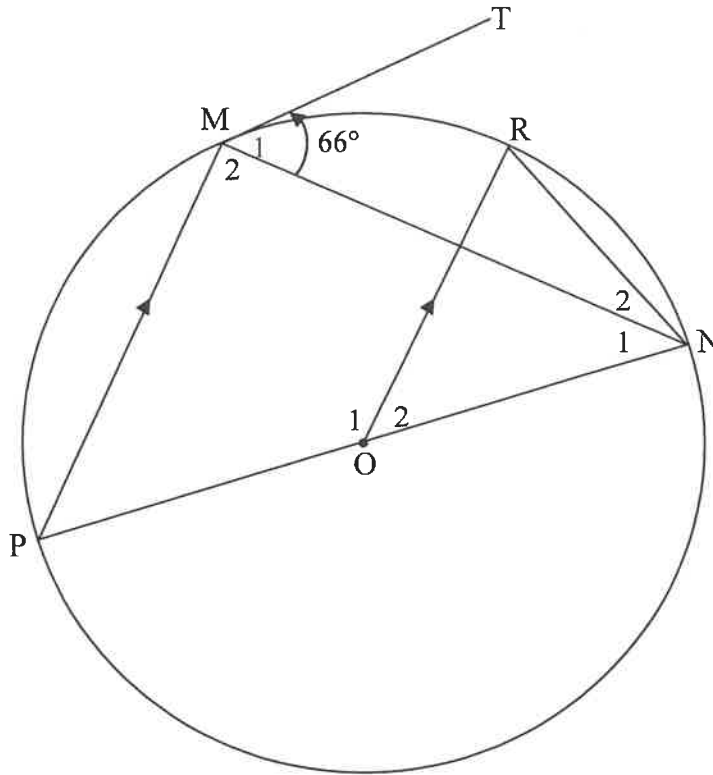
|     | <b>Solution/<i>Oplossing</i></b> | <b>Marks<br/><i>Punte</i></b> |
|-----|----------------------------------|-------------------------------|
| 6.4 |                                  | (3)                           |
| 6.5 |                                  | (2)                           |
|     |                                  | <b>[13]</b>                   |





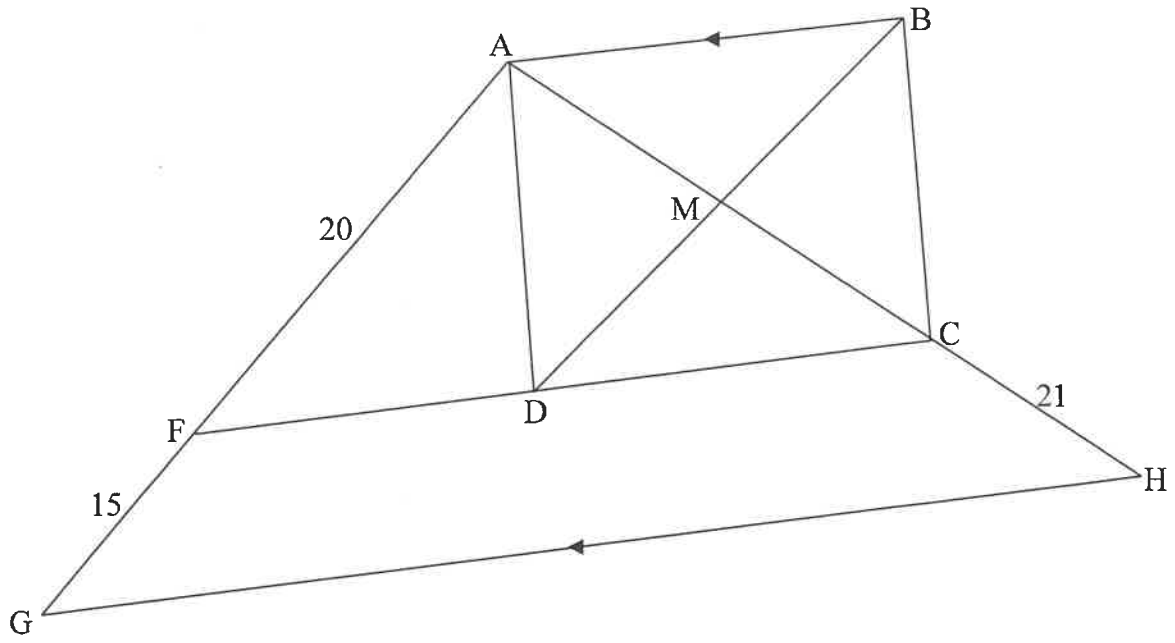
**QUESTION/VRAAG 8**

8.1



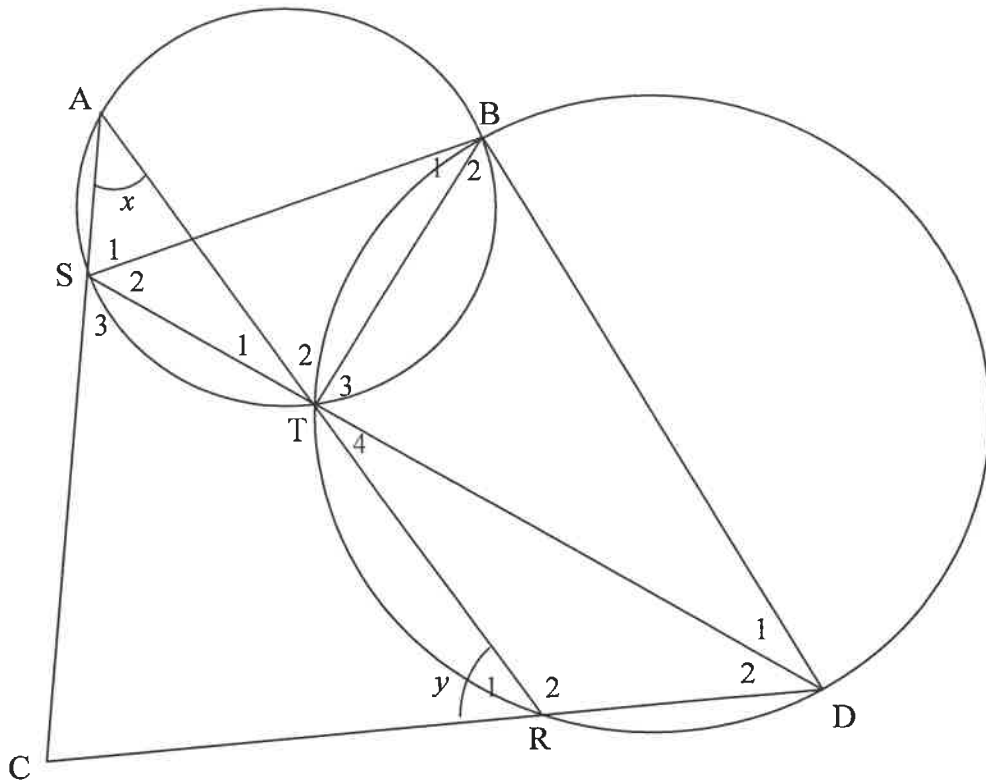
|       | <b>Solution/Oplissing</b> | <b>Marks<br/>Punte</b> |
|-------|---------------------------|------------------------|
| 8.1.1 |                           | (2)                    |
|       |                           |                        |
|       |                           |                        |
|       |                           |                        |
|       |                           |                        |
| 8.1.2 |                           | (2)                    |
|       |                           |                        |
|       |                           |                        |
|       |                           |                        |
|       |                           |                        |

|       | <b>Solution/<i>Oplossing</i></b> | <b>Marks<br/><i>Punte</i></b> |
|-------|----------------------------------|-------------------------------|
| 8.1.3 |                                  | (1)                           |
|       |                                  |                               |
|       |                                  |                               |
|       |                                  |                               |
|       |                                  |                               |
| 8.1.4 |                                  | (2)                           |
|       |                                  |                               |
|       |                                  |                               |
|       |                                  |                               |
|       |                                  |                               |
| 8.1.5 |                                  | (3)                           |
|       |                                  |                               |
|       |                                  |                               |
|       |                                  |                               |
|       |                                  |                               |
|       |                                  |                               |
|       |                                  |                               |
|       |                                  |                               |
|       |                                  |                               |
|       |                                  |                               |
|       |                                  |                               |



|       | <b>Solution/Oplissing</b> | <b>Marks<br/>Punte</b> |
|-------|---------------------------|------------------------|
| 8.2.1 |                           |                        |
|       |                           |                        |
|       |                           |                        |
|       |                           |                        |
|       |                           | (1)                    |
| 8.2.2 |                           |                        |
|       |                           |                        |
|       |                           |                        |
|       |                           |                        |
|       |                           |                        |
|       |                           |                        |
|       |                           |                        |
|       |                           |                        |
|       |                           |                        |
|       |                           | (5)                    |
|       |                           | [16]                   |





|          | <b>Solution/Oplissing</b> | <b>Marks<br/>Punte</b> |
|----------|---------------------------|------------------------|
| 9.2.1(a) |                           | (2)                    |
|          |                           |                        |
|          |                           |                        |
|          |                           |                        |
| 9.2.1(b) |                           | (2)                    |
|          |                           |                        |
|          |                           |                        |
|          |                           |                        |



|       | <b>Solution/<i>Oplossing</i></b> | <b>Marks<br/><i>Punte</i></b> |
|-------|----------------------------------|-------------------------------|
| 9.2.2 |                                  | (3)                           |
| 9.2.3 |                                  | (4)                           |
|       |                                  | <b>[16]</b>                   |



|        | <b>Solution/Oplissing</b> | <b>Marks<br/>Punte</b> |
|--------|---------------------------|------------------------|
| 10.1.2 |                           | (3)                    |
| 10.2.1 |                           | (6)                    |
| 10.2.2 |                           | (2)                    |
|        |                           | <b>[16]</b>            |















# basic education

Department:  
Basic Education  
**REPUBLIC OF SOUTH AFRICA**

**NATIONAL  
SENIOR CERTIFICATE/  
NASIONALE  
SENIOR SERTIFIKAAT**

**GRADE/GRAAD 12**

**MATHEMATICS P2/WISKUNDE V2  
NOVEMBER 2018  
MARKING GUIDELINES/NASIENRIGLYNE**

**MARKS/PUNTE: 150**

**These marking guidelines consist of 23 pages.  
*Hierdie nasienriglyne bestaan uit 23 bladsye.***

**NOTE:**

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

**NOTA:**

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

| <b>GEOMETRY • MEETKUNDE</b> |   |
|-----------------------------|---|
| <b>S</b>                    | <b>A mark for a correct statement<br/>(A statement mark is independent of a reason)</b>                       |
|                             | <i>'n Punt vir 'n korrekte bewering<br/>( 'n Punt vir 'n bewering is onafhanklik van die rede)</i>            |
| <b>R</b>                    | <b>A mark for the correct reason<br/>(A reason mark may only be awarded if the statement is correct)</b>      |
|                             | <i>'n Punt vir 'n korrekte rede<br/>( 'n Punt word slegs vir die rede toegeken as die bewering korrek is)</i> |
| <b>S/R</b>                  | <b>Award a mark if statement AND reason are both correct</b>  |
|                             | <i>Ken 'n punt toe as die bewering EN rede beide korrek is</i>  |

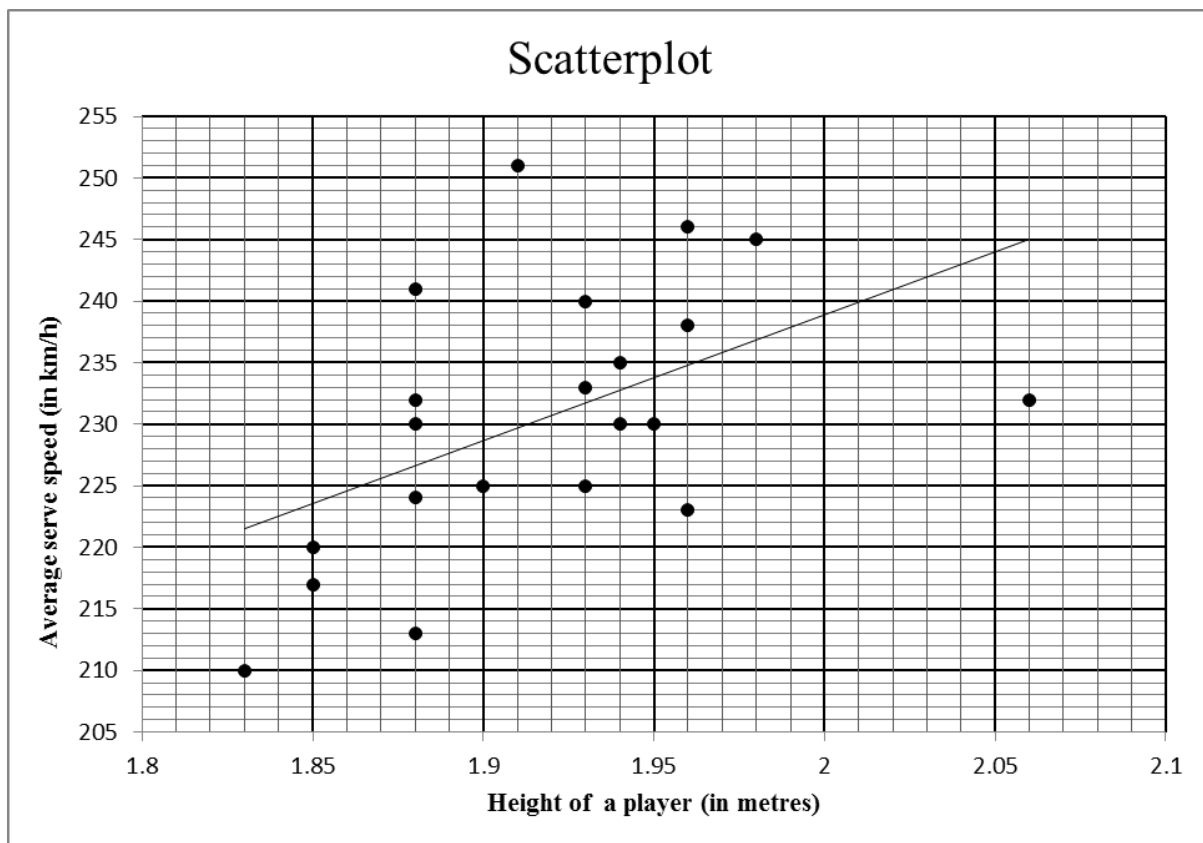
## QUESTION/VRAAG 1

|       |   |  |     |
|-------|---|--|-----|
| 1.1.1 | 140 items   | answer                                     | (1) |
| 1.1.2 | Modal class/modale klas: $20 < x \leq 30$ minutes<br><b>OR/OF</b><br>$20 \leq x < 30$ minutes                         | answer                                     | (1) |
|       |   | answer                                     | (1) |
| 1.1.3 | Number of minutes taken = 20 minutes  | answer                                     | (1) |
| 1.1.4 | 140 – 126 [Accept: 124 to 128]<br>14 orders (12 to 16)  | 126<br>answer                              | (2) |
|       | Answer only: Full marks   |  |     |
| 1.1.5 | 75 <sup>th</sup> percentile is at 105 items<br>=37 minutes [accept 36 – 38 minutes]                                   | 105<br>answer                              | (2) |
|       | Answer only: Full marks   |  |     |
| 1.1.6 | Lower quartile is at 35 items<br>=21,5 min [accept 21 – 23 min]<br>IQR = 37 – 21,5<br>= 15,5 min [accept 13 – 17 min] | lower quartile (Q <sub>1</sub> )<br>answer | (2) |
|       | Answer only: Full marks   |  |     |



|     |     |     |     |     |
|-----|-----|-----|-----|-----|
| 35  | 70  | 75  | 80  | 80  |
| 90  | 100 | 100 | 105 | 105 |
| 110 | 110 | 115 | 120 | 125 |

|             |   |                         |                |     |
|-------------|---|-------------------------|----------------|-----|
| 1.2.1(a)    | $\bar{x} = \frac{1420}{15}$<br>= R94,666.. = R94,67   | Answer only: Full marks | 1420<br>answer | (2) |
| 1.2.1(b)    | $\sigma = R22,691... = R22,69$  |                         | answer         | (2) |
| 1.2.2(a)    | They both collected the <b>same (equal) amount</b> in tips, i.e. R1 420 over the 15-day period.<br><br><i>Hulle albei het dieselfde bedrag met footjies ontvang, nl. R1 420 oor die 15 dae-tydperk</i>  |                         | answer         | (1) |
| 1.2.2(b)    | Mary's standard deviation is smaller than Reggie's which suggests that there was <b>greater variation in the amount of tips that Reggie collected</b> each day compared to the number of tips that Mary collected each day.<br><br><i>Marie se standaardafwyking is kleiner as Reggie s'n wat beteken dat daar groter variasie/verspreiding in die footjies was wat Reggie elke dag ontvang het in vergelyking met die getal footjies wat Marie elke dag ontvang het.</i> |                         | explanation    | (1) |
| <b>[15]</b> |   |                         |                |     |

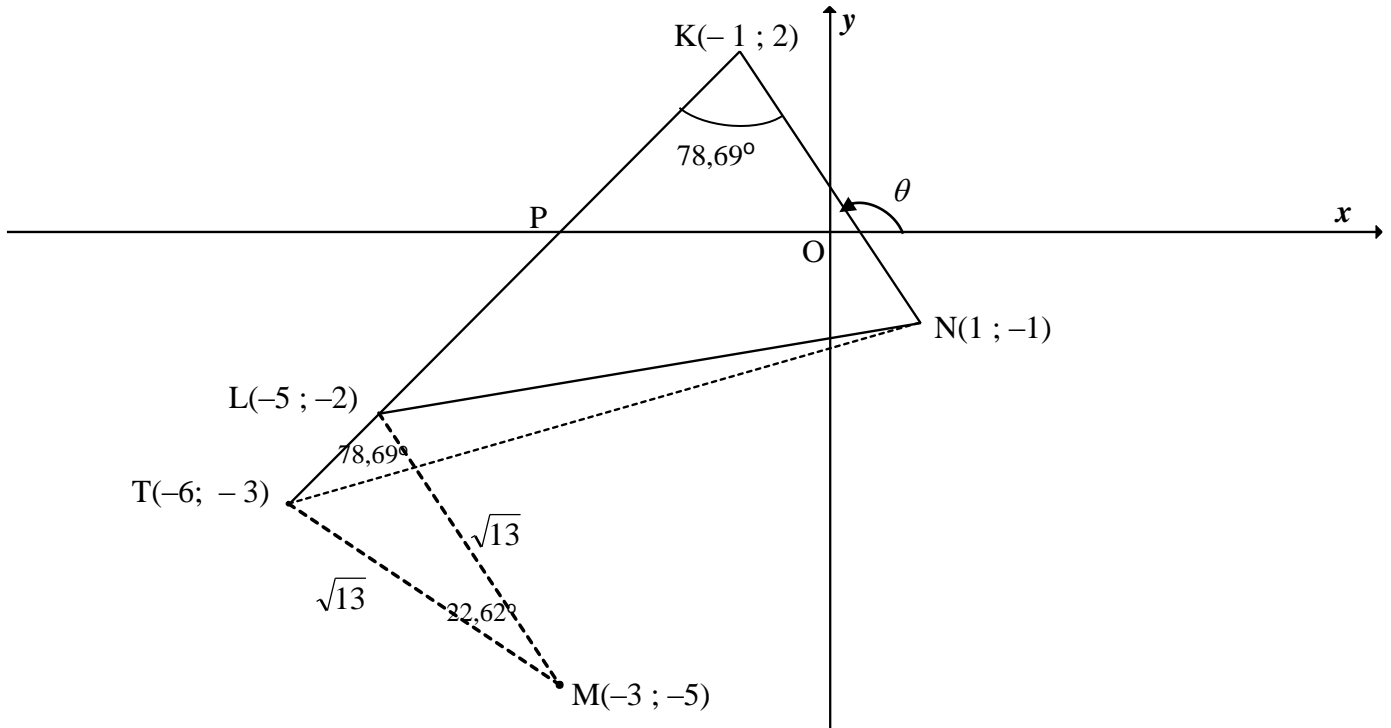
QUESTION/VRAAG 2



|       |   |            |
|-------|---|------------|
| 2.1   | 251 km/h  | answer (1) |
| 2.2.1 | $r = 0,52$ OR C   | answer (1) |
| 2.2.2 | The points are <b>fairly scattered</b> and the least squares regression line is increasing.<br><br><i>Die punte is redelik verspreid en die kleinstekwadrate-regressielyn neem toe.</i>   | reason (1) |
| 2.3   | There is a weak positive relation hence the height could have an influence<br><br><i>Daar is 'n swak positiewe verband, tog kan die lengte 'n invloed hê.</i>   | answer (1) |
|       | <b>OR/OF</b><br>There is no conclusive evidence that the height of a player will influence his/her tennis serve speed.<br><br><i>Daar is geen duidelike bewys dat die lengte van die speler sy/haar afslaanspoed kan beïnvloed nie.</i> | answer (1) |
|       | <b>OR/OF</b><br>There is no conclusive evidence that a taller person will serve faster than a shorter person.<br><br><i>Daar is geen duidelike bewys dat 'n langer speler vinniger sal afslaan as 'n korter een nie.</i>                | answer (1) |

|            |  |   |
|------------|--|---|
| 2.4        | <p>For (0 ; 27,07), it means that the player has a height of 0 m but can serve at a speed of 27,07 km/h.<br/> <b>It is impossible for a person to have a height of 0 m.</b></p> <p><i>(0 ; 27,07) beteken dat 'n speler 'n lengte van 0 m kan hê en teen 'n spoed van 27,07 km/h kan afslaan. Dit is onmoontlik om 'n lengte van 0 m te hê.</i></p> <p><b>OR/OF</b></p> <p>This means that the <b>player does not exist and therefore cannot serve and have a serve speed.</b></p> <p><i>Dit beteken dat die speler nie bestaan nie en daarom nie kan afslaan en 'n afslaanspoed hê nie.</i></p> | <p> explanation (1)</p> <p> explanation (1)</p> |
| <b>[5]</b> |  |   |

**QUESTION/VRAAG 3**



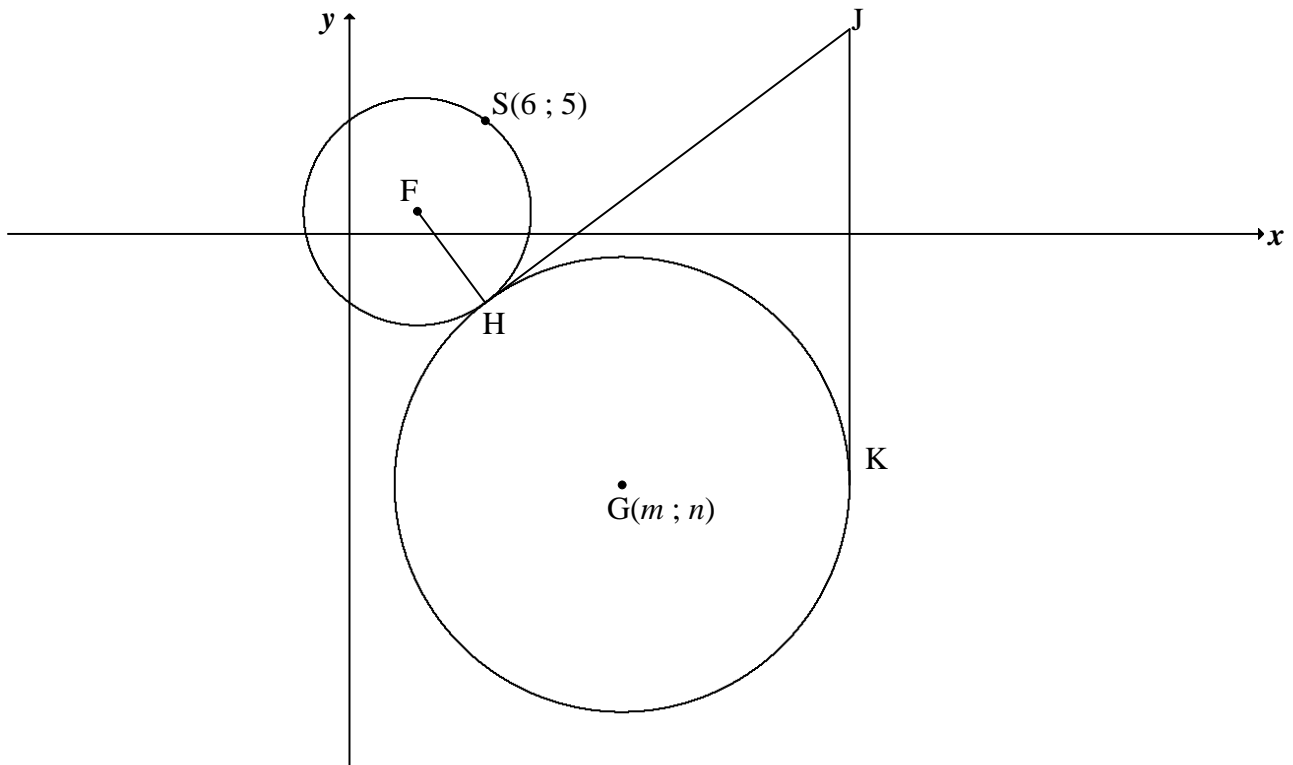
|       |  |   |
|-------|--|---|
| 3.1.1 | $m_{KN} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{KN} = \frac{2 - (-1)}{-1 - 1}$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin: 5px 0;">Answer only: Full marks</div> $= -\frac{3}{2}$       | <p>🚩 correct substitution</p> <p>🚩 answer</p> <p style="text-align: right;">(2)</p>   |
| 3.1.2 | $\tan \theta = m_{KN} = -\frac{3}{2}$ $\theta = 180^\circ - 56,31^\circ$ $\theta = 123,69^\circ$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin: 5px 0;">Answer only: Full marks</div> | <p>🚩 <math>\tan \theta = m_{KN} = -\frac{3}{2}</math></p> <p>🚩 answer</p> <p style="text-align: right;">(2)</p>   |
| 3.2   | $\text{Inclination } KL = 123,69^\circ - 78,69^\circ = 45^\circ \text{ [ext } \angle \Delta]$ $\tan 45^\circ = m_{KL} = 1$   | <p>🚩 S</p> <p>🚩 <math>\tan 45^\circ = m_{KL} = 1</math></p> <p style="text-align: right;">(2)</p>   |
| 3.3   | $y = x + c$ $2 = -1 + c$ $c = 3$ $y = x + 3$ <p><b>OR/OF</b></p> $y - y_1 = 1(x - x_1)$ $y - 2 = 1(x - (-1))$ $y = x + 3$  | <p>🚩 substitute <math>(-1; 2)</math> and <math>m</math></p> <p>🚩 equation</p> <p style="text-align: right;">(2)</p> <p>🚩 substitute <math>(-1; 2)</math> and <math>m</math></p> <p>🚩 equation</p> <p style="text-align: right;">(2)</p> |

|       |   |   |
|-------|---|---|
| 3.4   | $KN = \sqrt{(1+1)^2 + (-1-2)^2}$ $KN = \sqrt{13} \text{ or } 3,61$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 100px;">Answer only: Full marks</div>   | <p>🚩 substitute K and N into distance formula</p> <p>🚩 answer (2)</p>   |
| 3.5.1 | $(x+3)^2 + (y+5)^2 = 13 \quad \dots(1)$ <p>L is a point on KL</p> $y = x + 3 \quad \dots(2)$ <p>(2) in (1):</p> $(x+3)^2 + (x+3+5)^2 = 13$ $x^2 + 6x + 9 + x^2 + 16x + 64 = 13$ $2x^2 + 22x + 60 = 0$ $x^2 + 11x + 30 = 0$ $(x+5)(x+6) = 0$ $x = -5 \text{ or } x = -6$ $y = -2 \text{ or } y = -3$ <p>L(-5 ; -2) or (-6 ; -3)</p> <p><b>OR/OF</b></p> $(x+3)^2 + (y+5)^2 = 13 \quad \dots(1)$ <p>L is a point on KL</p> $y = x + 3 \quad \therefore x = y - 3 \quad \dots(2)$ <p>(2) in (1):</p> $(y-3+3)^2 + (y+5)^2 = 13$ $y^2 + y^2 + 10y + 25 = 13$ $2y^2 + 10y + 12 = 0$ $y^2 + 5y + 6 = 0$ $(y+2)(y+3) = 0$ $y = -2 \text{ or } y = -3$ $x = -5 \text{ or } x = -6$ <p>L(-5 ; -2) or (-6 ; -3)</p> | <p>🚩 equation (1)</p> <p>🚩 substituting eq (2)</p> <p>🚩 standard form</p> <p>🚩 x-values</p> <p>🚩 y-values (5)</p> <p>🚩 equation (1)</p> <p>🚩 substituting eq (2)</p> <p>🚩 standard form</p> <p>🚩 y-values (both)</p> <p>🚩 x-values (both) (5)</p> |
| 3.5.2 | <p>Midpoint of KM: (-2 ; -1,5)</p> $\therefore \frac{x_L + 1}{2} = -2 \text{ and } \frac{y_L - 1}{2} = -\frac{3}{2}$ $\therefore L(-5 ; -2)$ <p><b>OR/OF</b></p> $m_{KN} = m_{LM}$ $\frac{y - (-5)}{x - (-3)} = -\frac{3}{2}$ $2(x+3+5) = -3(x+3)$ $2x+16 = -3x-9$ $5x = -25$ $x = -5$ $\therefore L(-5 ; -2)$ <div style="border: 1px solid black; padding: 2px; display: inline-block; margin-left: 100px;">Answer only: Full marks</div>   | <p>🚩 midpoint of KM</p> <p>🚩 x value 🚩 y value (3)</p> <p>🚩 <math>m_{LM} = m_{KN}</math></p> <p>🚩 x value</p> <p>🚩 y value (3)</p>  |















|             |  |   |
|-------------|--|---|
|             | <p><b>OR/OF</b><br/> <math>N \rightarrow M:</math> <math>(x; y) \rightarrow (x - 4; y - 4)</math><br/> <math>\therefore L(-1 - 4; 2 - 4)</math><br/> <math>\therefore L(-5; -2)</math></p> <p><math>N \rightarrow K:</math> <math>(x; y) \rightarrow (x - 2; y + 3)</math><br/> <math>\therefore L(-3 - 2; -5 + 3)</math><br/> <math>\therefore L(-5; -2)</math></p>   | <p>☞ transformation</p> <p>☞ x value ☞ y value</p> <p>(3)</p>   |
| 3.6         | <p><math>T(-6; -3)</math> (from Question 3.5.1)<br/> <math>KT = \sqrt{(-1 - (-6))^2 + (2 - (-3))^2}</math><br/> <math>= \sqrt{50}</math><br/> <math>KN = \sqrt{13}</math> (CA from 3.4)<br/> Area of <math>\Delta KTN = \frac{1}{2} KT \cdot KN \sin \hat{LKN}</math><br/> <math>= \frac{1}{2} \sqrt{50} \cdot \sqrt{13} \sin 78,69^\circ</math><br/> <math>= 12,50</math> square units</p> <p><b>OR/OF</b></p> <p>In <math>\Delta KLM:</math><br/> <math>\frac{TL}{\sin 22,62^\circ} = \frac{\sqrt{13}}{\sin 78,69^\circ}</math><br/> <math>TL = 1,414..</math></p> <p><math>KL = \sqrt{(-1 - (-5))^2 + (2 - (-2))^2}</math><br/> <math>= \sqrt{32}</math><br/> <math>\therefore KT = 7,0708...</math><br/> Area of <math>\Delta KTN = \frac{1}{2} KT \cdot KN \sin \hat{LKN}</math><br/> <math>= \frac{1}{2} (7,0708) \cdot \sqrt{13} \sin 78,69^\circ</math><br/> <math>= 12,50</math> square units</p> | <p>☞ coordinates of T</p> <p>☞ length of KT</p> <p>☞ substitution into area rule</p> <p>☞ answer</p> <p>(4)</p> <p>☞ length of TL</p> <p>☞ length of KT</p> <p>☞ substitution into area rule</p> <p>☞ answer</p> <p>(4)</p> |
| <b>[22]</b> |  |   |



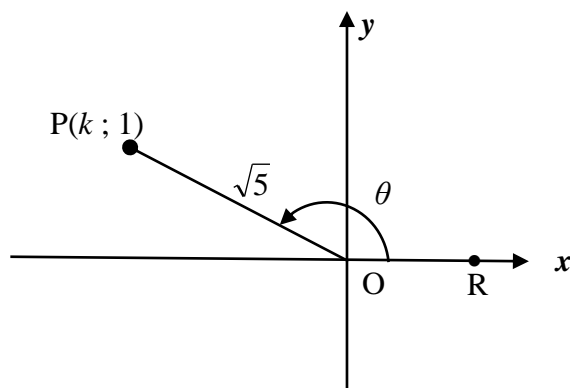
**QUESTION/VRAAG 4**



|       |   |  |
|-------|---|--|
| 4.1   | $F(3;1)$  | ✎ x value ✎ y value<br>(2)                 |
| 4.2   | $FS = \sqrt{(6-3)^2 + (5-1)^2}$<br>$FS = 5$   | ✎ substitution of F & S<br>✎ answer<br>(2) |
| 4.3   | $FH(FS) : HG = 1 : 2$<br>$\therefore HG = 2 FH$<br>$= 10$   | ✎ $HG = 10$<br>(1)                         |
| 4.4   | Tangents from common/same point /<br><i>Raaklyne vanaf gemeenskaplike of dieselfde punt</i>   | ✎ answer<br>(1)                            |
| 4.5.1 | $\hat{F}HJ = 90^\circ$<br>$FJ^2 = 20^2 + 5^2$<br>$FJ = \sqrt{425}$ or $5\sqrt{17}$ or 20,62<br>[tan $\perp$ radius / <i>rkl <math>\perp</math> radius</i> ]<br>[Pyth theorem/ <i>stelling</i> ] | ✎ S ✎ R<br>✎ S<br>✎ answer<br>(4)          |
| 4.5.2 | $(x - m)^2 + (y - n)^2 = 100$   | ✎ answer<br>(1)                            |

|              |   |   |   |
|--------------|---|---|---|
| <p>4.5.3</p> | <p>K(22; n)<br/> GK = HG = 10<br/> FH = FS = 5<br/> <math>m = 22 - 10</math><br/> <math>m = 12</math><br/> F, H and G are collinear<br/> <i>F, H en G is saamlynig</i><br/> <math>FG^2 = (12 - 3)^2 + (n - 1)^2</math><br/> <math>15^2 = 81 + (n - 1)^2</math><br/> <math>(n - 1)^2 = 144</math><br/> <math>n - 1 = \pm 12</math><br/> <math>n \neq 13</math> or <math>n = -11</math><br/> <math>\therefore G(12; -11)</math></p> <p><b>OR/OF</b></p> <p>K(22; n)<br/> GK = HG = 10<br/> FH = FS = 5<br/> <math>m = 22 - 10</math><br/> <math>m = 12</math><br/> Let J(22 ; y):<br/> <math>FJ^2 = (22 - 3)^2 + (y - 1)^2</math><br/> <math>425 = 361 + y^2 - 2y + 1</math><br/> <math>0 = y^2 - 2y - 63</math><br/> <math>0 = (y - 9)(y + 7)</math><br/> <math>\therefore y = 9</math> or/of <math>y \neq -7</math><br/> <math>\therefore n = 9 - 20 = -11</math><br/> <math>\therefore G(12; -11)</math></p> | <p>[radius <math>\perp</math> tangent]<br/> [radii]<br/> [radii]</p> <p>[HJ is a common tangent]<br/> <i>[HJ is 'n gemeenskaplike raaklyn]</i></p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <math>n^2 - 2n - 143 = 0</math><br/> <math>(n + 11)(n - 13) = 0</math><br/> <math>n = -11</math> or <math>n \neq 13</math> </div> <p>[radius <math>\perp</math> tangent]<br/> [radii]<br/> [radii]</p> | <p> K(22; n)</p> <p> value of <math>m</math></p> <p> subst. of F and G in distance formula<br/>  <math>FG = 15</math><br/>  simplification/standard form<br/>  value of <math>n</math><br/>  coordinates of G (7)</p> <p> K(22; n)</p> <p> value of <math>m</math></p> <p> subst. of F and J in distance formula<br/>  <math>FJ = \sqrt{425}</math><br/>  standard form</p> <p> value of <math>n</math><br/>  coordinates of G (7)</p> |
| <b>[18]</b>  |   |   |   |

QUESTION/VRAAG 5




|          |  |  |
|----------|--|--|
| 5.1.1    | $k^2 = (\sqrt{5})^2 - 1^2$ $= 4$ $k = -2$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 100px;">Answer only: full marks</div>   | <ul style="list-style-type: none"> <li>🏹 substitution into theorem of Pythagoras</li> <li>🏹 answer</li> </ul> <p style="text-align: right;">(2)</p>  |
| 5.1.2(a) | $\tan \theta = -\frac{1}{2}$   | <ul style="list-style-type: none"> <li>🏹 answer</li> </ul> <p style="text-align: right;">(1)</p>   |
| 5.1.2(b) | $\cos(180^\circ + \theta) = -\cos \theta$ $= \frac{2}{\sqrt{5}}$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 100px;">Answer only: full marks</div>  | <ul style="list-style-type: none"> <li>🏹 reduction</li> <li>🏹 answer</li> </ul> <p style="text-align: right;">(2)</p>  |
| 5.1.2(c) | $\sin(\theta + 60^\circ) = \frac{a+b}{\sqrt{20}}$ $\text{LHS} = \sin \theta \cos 60^\circ + \cos \theta \sin 60^\circ$ $= \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{2}\right) + \left(-\frac{2}{\sqrt{5}}\right)\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{1-2\sqrt{3}}{2\sqrt{5}}$ $= \frac{1-2\sqrt{3}}{\sqrt{20}}$ | <ul style="list-style-type: none"> <li>🏹 expansion</li> <li>🏹 subst of <math>\sin \theta</math></li> <li>🏹 subst of <math>\cos \theta</math></li> <li>🏹 both special <math>\angle</math>s</li> <li>🏹 <math>\frac{1-2\sqrt{3}}{2\sqrt{5}}</math></li> </ul> <p style="text-align: right;">(5)</p> |
| 5.1.3    | $\tan \theta = -\frac{1}{2}$ $\therefore \theta = 180^\circ - 26,57^\circ$ $\therefore \theta = 153,43^\circ$ $\tan(2\theta - 40^\circ) = \tan[(2 \times 153,43^\circ) - 40^\circ]$ $= \tan 266,87^\circ$ $= 18,3$   | <ul style="list-style-type: none"> <li>🏹 <math>\theta</math></li> <li>🏹 substitution</li> <li>🏹 answer</li> </ul> <p style="text-align: right;">(3)</p>  |


5.2


$$\begin{aligned}
 \text{LHS} &= \frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} & \text{RHS} &= 2 \tan 2x \\
 &= \frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)} \\
 &= \frac{\cos^2 x + 2 \sin x \cos x + \sin^2 x - \cos^2 x + 2 \sin x \cos x - \sin^2 x}{\cos^2 x - \sin^2 x} \\
 &= \frac{2(2 \sin x \cos x)}{\cos^2 x - \sin^2 x} \\
 &= \frac{2 \sin 2x}{\cos 2x} \\
 &= 2 \tan 2x \\
 &= \text{RHS}
 \end{aligned}$$

 single fraction

 expansion

 simplification (both)

 double  $\angle$  identity


 double  $\angle$  identity

(5)


**OR/OF**


$$\begin{aligned}
 \text{LHS} &= \frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} & \text{RHS} &= 2 \tan 2x \\
 &= \frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x - \sin x)(\cos x + \sin x)} \\
 &= \frac{(\cos x + \sin x + \cos x - \sin x)(\cos x + \sin x - \cos x + \sin x)}{\cos^2 x - \sin^2 x} \\
 &= \frac{(2 \cos x)(2 \sin x)}{\cos^2 x - \sin^2 x} \\
 &= \frac{2(2 \sin x \cos x)}{\cos^2 x - \sin^2 x} \\
 &= \frac{2 \sin 2x}{\cos 2x} \\
 &= 2 \tan 2x \\
 &= \text{RHS}
 \end{aligned}$$

 single fraction

 difference of two squares

 simplification (both)

 double  $\angle$  identity


 double  $\angle$  identity


(5)


**OR/OF**


$$\begin{aligned}
 \text{RHS} &= 2 \tan 2x \\
 &= \frac{2 \sin 2x}{\cos 2x} \\
 &= \frac{2(2 \sin x \cdot \cos x)}{\cos^2 x - \sin^2 x} \\
 &= \frac{4 \sin x \cdot \cos x}{\cos^2 x - \sin^2 x} \\
 &= \frac{1 + 2 \sin x \cdot \cos x - (1 - 2 \sin x \cdot \cos x)}{\cos^2 x - \sin^2 x} \\
 &= \frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{(\cos x + \sin x)(\cos x - \sin x)} \\
 &= \frac{(\cos x + \sin x)^2}{(\cos x + \sin x)(\cos x - \sin x)} - \frac{(\cos x - \sin x)^2}{(\cos x + \sin x)(\cos x - \sin x)} \\
 &= \frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = \text{LHS}
 \end{aligned}$$

 double  $\angle$  identity

 double  $\angle$  identity

 identity & method

 factorising numerator and denominator

 writing as 2 terms

(5)

5.3

$$\sum_{A=38^{\circ}}^{52^{\circ}} \cos^2 A$$

$$= \cos^2 38^{\circ} + \cos^2 39^{\circ} + \cos^2 40^{\circ} + \dots + \cos^2 51^{\circ} + \cos^2 52^{\circ}$$

$$= \sin^2 52^{\circ} + \sin^2 51^{\circ} + \sin^2 50^{\circ} + \dots + \cos^2 51^{\circ} + \cos^2 52^{\circ}$$

$$= 7(1) + \cos^2 45^{\circ}$$

$$= 7 + \left(\frac{\sqrt{2}}{2}\right)^2 \quad \text{or} \quad = 7 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 7\frac{1}{2}$$

**OR/OF**

$$\sum_{A=38^{\circ}}^{52^{\circ}} \cos^2 A$$

$$= \cos^2 38^{\circ} + \cos^2 39^{\circ} + \cos^2 40^{\circ} + \dots + \cos^2 51^{\circ} + \cos^2 52^{\circ}$$


$$= (\cos^2 38^{\circ} + \sin^2 52^{\circ}) + (\cos^2 39^{\circ} + \sin^2 51^{\circ}) \dots + \cos^2 45^{\circ}$$



$$= 7(1) + \cos^2 45^{\circ}$$


$$= 7 + \left(\frac{\sqrt{2}}{2}\right)^2 \quad \text{or} \quad = 7 + \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 7\frac{1}{2}$$


 expansion

 co ratio



  $\cos^2 45^{\circ}$ 
  $7 \times$  identity


 answer

(5)

 expansion

 pairing

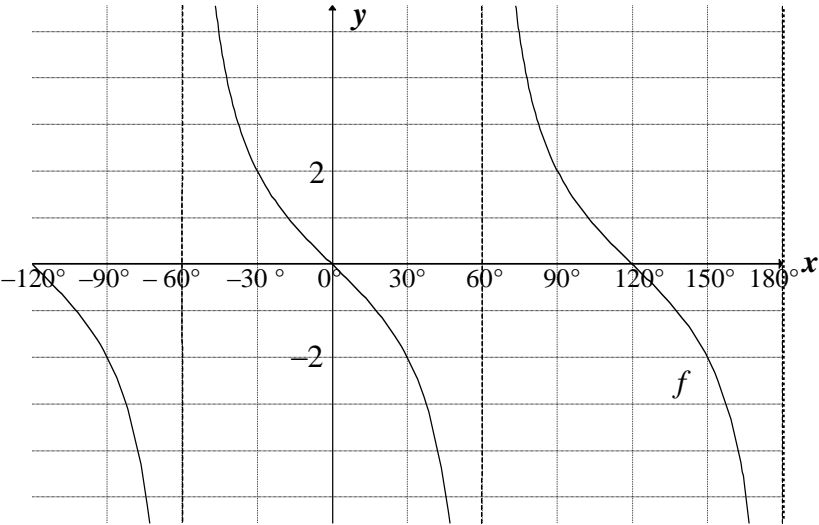
  $\cos^2 45^{\circ}$ 
  $7 \times$  identity

 answer

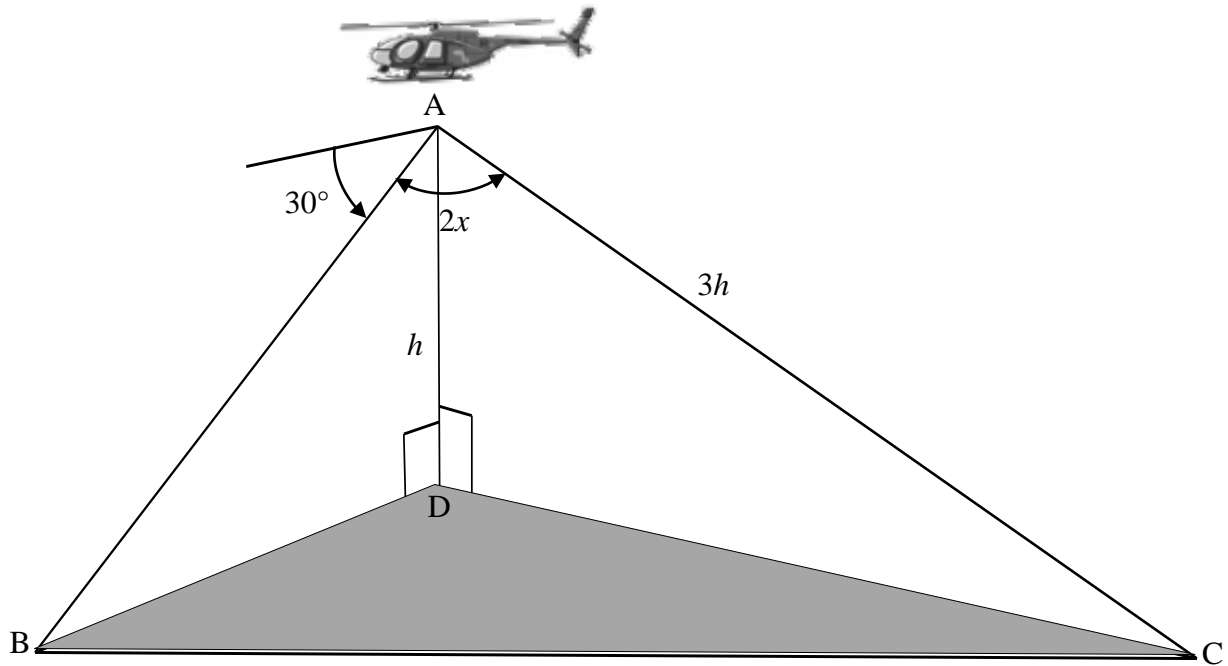
(5)

**[23]**

**QUESTION/VRAAG 6**

|                    |  |  |
|--------------------|--|--|
| 6.1                | Period = $120^\circ$   | <p>answer (1)</p>  |
| 6.2                | $2 = -2 \tan \frac{3}{2}x$ $\tan \left( \frac{3}{2}t \right) = -1$ $\frac{3}{2}t = 135^\circ + k.180^\circ \quad \text{OR/OF} \quad \frac{3}{2}t = -45^\circ + k.180^\circ$ $t = 90^\circ + k.120^\circ ; k \in Z \quad \quad \quad t = -30^\circ + k.120^\circ ; k \in Z$ <p><b>OR/OF</b></p> $2 = -2 \tan \frac{3}{2}x$ $\tan \left( \frac{3}{2}t \right) = -1$ $\frac{3}{2}t = 135^\circ + k.360^\circ \quad \text{or/of} \quad \frac{3}{2}t = 315^\circ + k.360^\circ$ $t = 90^\circ + k.240^\circ \quad \text{or/of} \quad t = 210^\circ + k.240^\circ ; k \in Z$ | <p>equating</p> <p>general solution of <math>\frac{3}{2}t</math></p> <p>general solution of <math>t</math> (3)</p> <p>equating</p> <p>general solution of <math>\frac{3}{2}t</math></p> <p>general solution of <math>t</math> (3)</p>  |
| 6.3                |    | <p>asymptotes:<br/><math>x = \pm 60^\circ ; x = 180^\circ</math></p> <p>x-intercepts <math>0^\circ ; \pm 120^\circ</math></p> <p>negative shape</p> <p><math>(90^\circ ; 2)</math> or<br/><math>(-30^\circ ; 2)</math> or<br/><math>(30^\circ ; -2)</math> or<br/><math>(-90^\circ ; -2)</math></p> <p>(4)</p> |
| 6.4                | $x \in (-60^\circ ; -30^\circ] \text{ or } (60^\circ ; 90^\circ]$ <p><b>OR/OF</b></p> $-60^\circ < x \leq -30^\circ \text{ or } 60^\circ < x \leq 90^\circ$  | <p>interval notation (3)</p> <p>interval notation (3)</p> <p>(3)</p>   |
| 6.5                | $g(x) = -2 \tan \left[ \frac{3}{2}(x + 40^\circ) \right] = f(x + 40^\circ)$ <p>Translation of <math>40^\circ</math> to the left / skuif met <math>40^\circ</math> links</p>  | <p>Translation of <math>40^\circ</math> to the left (2)</p>  |
| <p><b>[13]</b></p> |  |  |

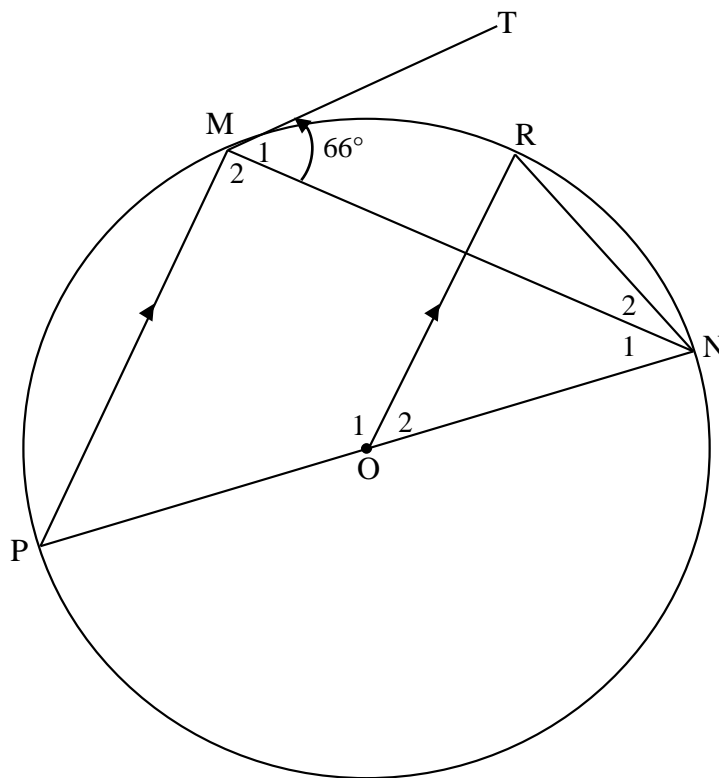
QUESTION/VRAAG 7



|            |   |  |
|------------|---|--|
| <p>7.1</p> | $\hat{A}BD = 30^\circ$ $\sin 30^\circ = \frac{h}{AB}$ $AB = \frac{h}{\sin 30^\circ} \quad \text{OR} \quad AB = \frac{h}{\frac{1}{2}} \quad \text{OR} \quad AB = 2h$ <p><b>OR/OF</b></p> $\hat{B}AD = 60^\circ$ $\cos 60^\circ = \frac{h}{AB}$ $AB = \frac{h}{\cos 60^\circ} \quad \text{OR} \quad AB = \frac{h}{\frac{1}{2}} \quad \text{OR} \quad AB = 2h$ | <p>🚁 <math>\hat{A}BD = 30^\circ</math></p> <p>🚁 answer (2)</p> <p>🚁 <math>\hat{B}AD = 60^\circ</math></p> <p>🚁 answer (2)</p>  |
| <p>7.2</p> | $BC^2 = AB^2 + AC^2 - 2AB \cdot AC \cos \hat{B}AC$ $= (2h)^2 + (3h)^2 - 2(2h)(3h) \cos 2x$ $= 13h^2 - 12h^2 (2 \cos^2 x - 1)$ $= 13h^2 - 24h^2 \cos^2 x + 12h^2$ $= 25h^2 - 24h^2 \cos^2 x$ $BC = h\sqrt{25 - 24 \cos^2 x}$   | <p>🚁 use of cosine rule in <math>\triangle ABC</math></p> <p>🚁 substitution</p> <p>🚁 double angle identity</p> <p>🚁 <math>25h^2 - 24h^2 \cos^2 x</math></p> <p>(4)</p> |

[6]

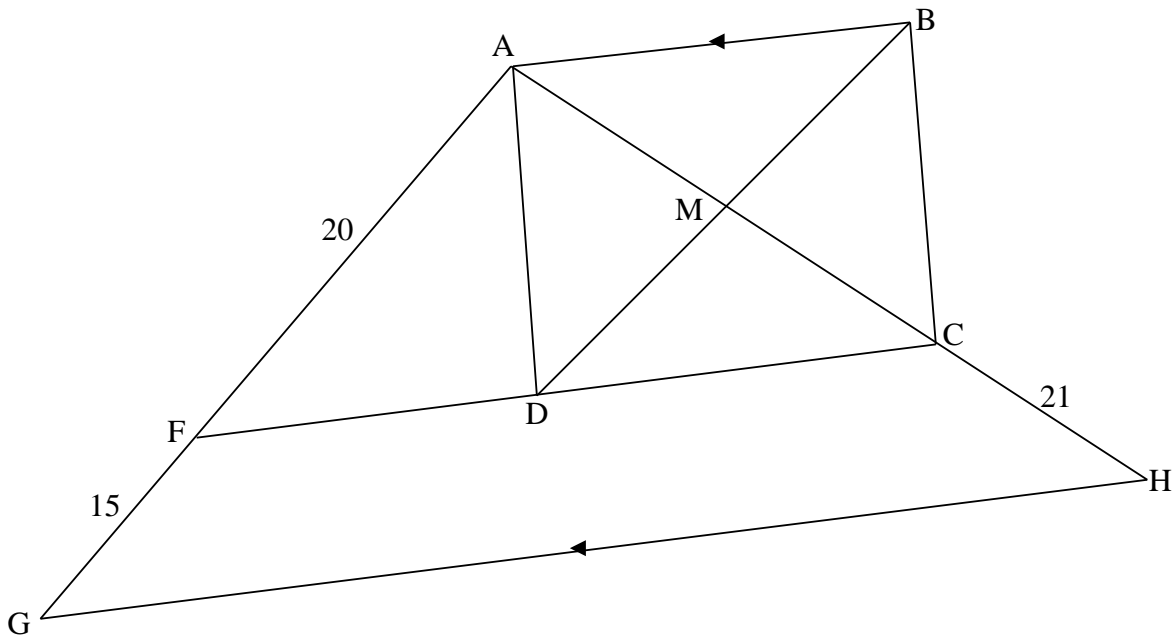
QUESTION/VRAAG 8



|       |  |  |                  |     |
|-------|--|--|------------------|-----|
| 8.1.1 | $\hat{P} = \hat{M}_1 = 66^\circ$   | [tan chord theorem/raaklyn koordst]  | ✓S ✓R            | (2) |
| 8.1.2 | $\hat{M}_2 = 90^\circ$   | [∠ in semi circle/∠ in halfsirkel]   | ✓S ✓R            | (2) |
| 8.1.3 | $\hat{N}_1 = 180^\circ - (90^\circ + 66^\circ)$<br>$= 24^\circ$  | [sum of ∠s of /som van ∠e ΔMNP]  | ✓S               | (1) |
| 8.1.4 | $\hat{O}_2 = \hat{P} = 66^\circ$   | [corres. ∠s;/ooreenk ∠e, PM    OR]   | ✓S ✓R            | (2) |
| 8.1.5 | $\hat{R} + \hat{N}_1 + \hat{N}_2 = 180^\circ - 66^\circ$<br>$= 114^\circ$<br>$\hat{R} = \hat{N}_1 + \hat{N}_2 = 57^\circ$<br>$\therefore \hat{N}_2 = 33^\circ$ | [sum of ∠s of /som van ∠e ΔRNO]  | ✓S<br>✓S/R<br>✓S | (3) |
|       | <b>OR/OF</b><br>$\hat{P}\hat{O}\hat{R} = 114^\circ$<br>$\hat{P}\hat{N}\hat{R} = 57^\circ$<br>$\therefore \hat{N}_2 = 33^\circ$                                 | [∠s on straight line/∠e op reguitlyn]<br>[∠ at centre = twice ∠ at circumference/<br>midpts∠ = 2 × omtreks∠] | ✓S<br>✓S/R<br>✓S | (3) |



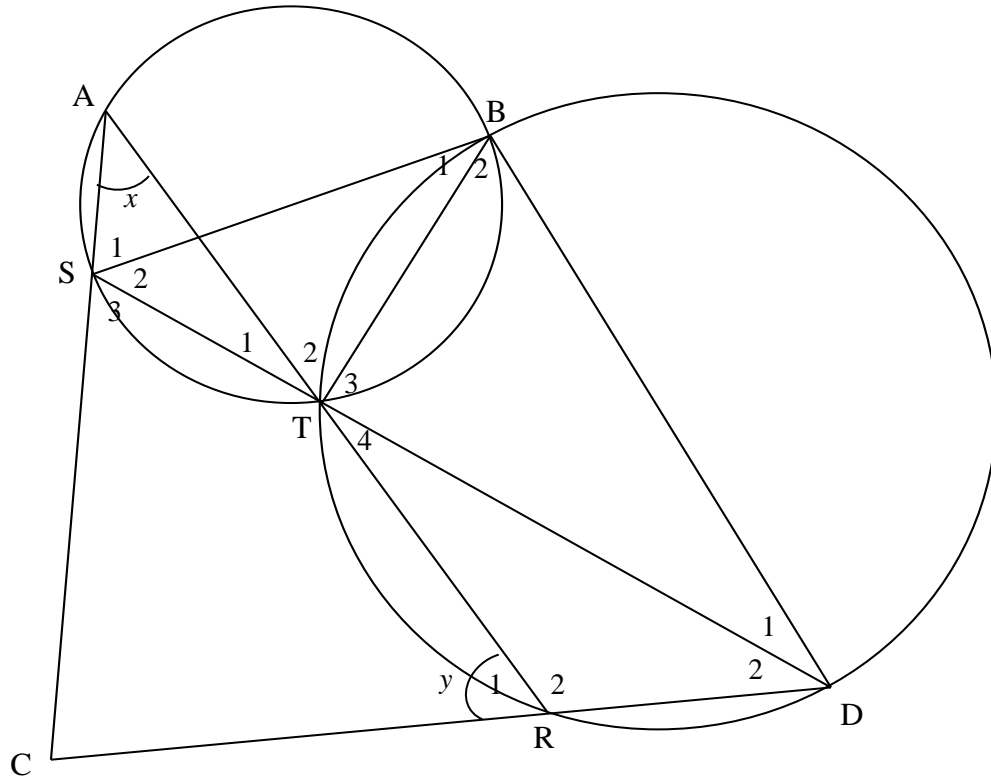
8.2











|             |  |   |
|-------------|--|---|
| 8.2.1       | $FC \parallel AB \parallel GH$ [opp sides of <b>rectangle</b> / <i>teenoorst sye v reghoek</i> ]   | $\checkmark$ R<br>(1)   |
| 8.2.2       | $\frac{AC}{CH} = \frac{AF}{FG}$ <p>[line <math>\parallel</math> one side of <math>\Delta</math>] <b>OR</b> [prop theorem; <math>FC \parallel GH</math>]<br/> <i>[lyn <math>\parallel</math> een sy van <math>\Delta</math>] OF [eweredighst; <math>FC \parallel GH</math>]</i></p> $\frac{AC}{21} = \frac{20}{15}$ $AC = \frac{20 \times 21}{15}$ $= 28$ $DB = AC = 28$ [diags of rectangle =/ <i>hoeklyne v reghoek</i> = ]<br>$DM = \frac{1}{2}DB = 14$ [diags of rectangle bisect/ <i>hoekl v reghoek halveer</i> ] | $\checkmark$ S $\checkmark$ R<br><br>$\checkmark$ AC<br>$\checkmark$ S<br>$\checkmark$ S<br>(5) |
| <b>[16]</b> |  |   |



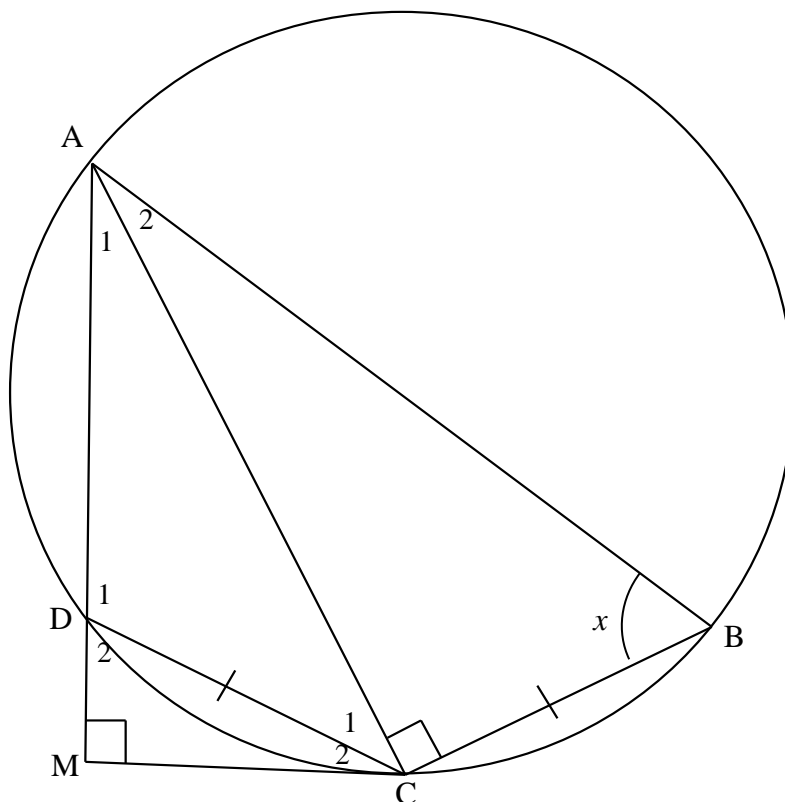
9.2



|          |   |             |     |
|----------|---|-------------|-----|
| 9.2.1(a) | $\hat{B}_1 = x$ [∠s in same seg/∠e in dieselfde segm]   | S  R        | (2) |
| 9.2.1(b) | $\hat{B}_2 = y$ [ext ∠ of cyclic quad/buite∠ koordevh]  | S  R        | (2) |
| 9.2.2    | $\hat{C} = 180^\circ - (x + y)$ [sum of ∠s of/som v ∠e, $\Delta ACR$ ]<br>$\hat{SBD} + \hat{C} = x + y + 180^\circ - (x + y)$<br>$\hat{SBD} + \hat{C} = 180^\circ$<br>SCDB is a cyclic quad [converse opp angles of cyclic quad]<br>[omgekeerde teenoorst ∠e koordevh]                | S<br>S<br>R | (3) |
|          | <b>OR/OF</b><br>$\hat{S}_1 = \hat{T}_2$ [∠s in same segment/∠e in dies. segment]<br>$\hat{T}_2 = \hat{D}_1 + \hat{D}_2 = \hat{BDR}$ [ext ∠ of cyc quad/buite∠ koordevh]<br>$\therefore \hat{S}_1 = \hat{BDR}$<br>$\therefore$ SCDB is cyc quad [ext ∠ of quad = opp ∠/buite∠ = tos ∠] | S<br>S<br>R | (3) |

|              |   |   |
|--------------|---|---|
| <p>9.2.3</p> | <p> <math>\hat{T}_4 = y - 30^\circ</math> [ext <math>\angle</math> of/buite <math>\angle \Delta</math> TDR]<br/> <math>\hat{T}_1 = y - 30^\circ</math> [vert opp <math>\angle</math>s =/regeerst <math>\angle</math>e =]<br/> <math>y - 30^\circ + x + 100^\circ = 180^\circ</math> [sum of <math>\angle</math>s of/som v <math>\angle</math>e, <math>\Delta</math> AST]<br/> <math>\therefore x + y = 110^\circ</math><br/> <math>\hat{S}\hat{B}\hat{D} = 110^\circ</math><br/> <math>\therefore</math> SD not diameter [line does not subtend <math>90^\circ \angle</math> ]<br/> <i>SD nie 'n middellyn [lyn onderspan nie <math>90^\circ \angle</math> ]</i><br/> <b>OR/OF</b><br/> <math>A\hat{S}\hat{T} = \hat{C} + \hat{D}_2</math> [ext <math>\angle</math> of/buite <math>\angle \Delta</math> SCD]<br/> <math>\hat{C} = 100^\circ - 30^\circ = 70^\circ</math><br/> <math>\hat{S}\hat{B}\hat{D} = 180^\circ - 70^\circ</math> [opp <math>\angle</math>s cyclic quad/ teenoorst <math>\angle</math>e kdvh]<br/> <math>= 110^\circ</math><br/> <math>\therefore</math> SD not diameter [line does not subtend <math>90^\circ \angle</math> ]<br/> <i>SD nie 'n middellyn [lyn onderspan nie <math>90^\circ \angle</math> ]</i> </p> | <p>  S<br/>  S<br/> <br/>  S<br/>  R<br/> <br/>  S<br/>  S<br/> <br/>  S<br/> <br/>  R </p> <p style="text-align: right;">(4)</p> |
|              |   | (4)   |
|              |   | <b>[16]</b>   |

QUESTION/VRAAG 10



|               |  |   |
|---------------|--|---|
| <p>10.1.1</p> | <p><math>\hat{A}_2 = \hat{A}_1 = 90^\circ - x</math> [= chords subtend = <math>\angle</math>s<br/>= <i>kde onderspan</i> = <math>\angle e</math>]</p> <p><math>\hat{D}_2 = x</math> [exterior angle of cyclic quad/<i>buite <math>\angle</math> koordevh.</i>]</p> <p><math>\therefore \hat{C}_2 = 90^\circ - x</math> [sum of <math>\angle</math>s of/<i>som v <math>\angle e</math>, <math>\Delta DCM</math></i>]</p> <p><math>\therefore \hat{C}_2 = \hat{A}_1 = 90^\circ - x</math></p> <p><math>\therefore MC</math> is a tangent to the circle at <math>C</math> [converse: tan chord th]<br/><i>MC is 'n raaklyn by C [omgekeerde raakl koordst]</i></p> <p><b>OR/OF</b></p> <p><math>\hat{A}_2 = \hat{A}_1 = 90^\circ - x</math> [= chords subtend = <math>\angle</math>s/<br/>= <i>kde onderspan</i> = <math>\angle e</math>]</p> <p><math>\hat{C}_1 + \hat{C}_2 = x</math> [sum of <math>\angle</math>s of/<i>som v <math>\angle e</math>, <math>\Delta ACM</math></i>]</p> <p><math>\therefore \hat{C}_1 + \hat{C}_2 = \hat{B} = x</math></p> <p><math>\therefore MC</math> is a tangent to the circle at <math>C</math> [converse : tan chord th]<br/><i>MC is 'n raaklyn by C [omgekeerde raakl koordst]</i></p> <p><b>OR/OF</b></p> <p>In <math>\Delta AMC</math> and <math>\Delta ACB</math>:</p> <p><math>\hat{A}_2 = \hat{A}_1 = 90^\circ - x</math> [= chords subtend = <math>\angle</math>s/<br/>= <i>kde onderspan</i> = <math>\angle e</math>]</p> <p><math>\hat{AMC} = \hat{ACB} = 90^\circ</math> [given]</p> <p><math>\therefore \hat{C}_1 + \hat{C}_2 = \hat{B} = x</math></p> <p><math>\therefore MC</math> is a tangent to the circle at <math>C</math> [converse : tan chord th]<br/><i>MC is 'n raaklyn by C [omgekeerde raakl koordst]</i></p> | <p>✓ S  R</p> <p>✓ S/R</p> <p>✓ <math>\hat{C}_2 = 90^\circ - x</math></p> <p>✓ R</p> <p>(5)</p> <p>✓ S ✓ R</p> <p>✓✓ <math>\hat{C}_1 + \hat{C}_2 = x</math></p> <p>✓ R</p> <p>(5)</p> <p>✓ S ✓ R</p> <p>✓✓ <math>\hat{C}_1 + \hat{C}_2 = x</math></p> <p>✓ R</p> <p>(5)</p> |
|---------------|--|---|

|        |  |  |
|--------|--|--|
| 10.1.2 | <p>In <math>\triangle ACB</math> and/en <math>\triangle CMD</math></p> $\hat{B} = \hat{D}_2 = x \quad [\text{proved } \mathbf{OR} \text{ exterior } \angle \text{ of cyclic quad.}]$ <p style="text-align: center;"><i>[bewys OF buite <math>\angle</math> v koordevh]</i></p> $\hat{A}_2 = \hat{C}_2 = 90^\circ - x \quad [\text{proved } \mathbf{OR} \text{ sum of } \angle \text{s in } \triangle]$ <p style="text-align: center;"><i>[Bewys OF som v <math>\angle</math>e in <math>\triangle</math>]</i></p> $\triangle ACB \parallel \triangle CMD \quad [\angle, \angle, \angle]$ <p><b>OR/OF</b></p> <p>In <math>\triangle ACB</math> and/en <math>\triangle CMD</math></p> $\hat{B} = \hat{D}_2 = x \quad [\text{proved } \mathbf{OR} \text{ exterior } \angle \text{ of cyclic quad.}]$ <p style="text-align: center;"><i>[bewys OF buite <math>\angle</math> v koordevh]</i></p> $\hat{ACB} = \hat{AMC} = 90^\circ \quad [\text{given/gegee}]$ $\triangle ACB \parallel \triangle CMD \quad [\angle, \angle, \angle]$ <p><b>OR/OF</b></p> <p>In <math>\triangle ACB</math> and/en <math>\triangle CMD</math></p> $\hat{B} = \hat{D}_2 = x \quad [\text{proved } \mathbf{OR} \text{ exterior } \angle \text{ of cyclic quad}]$ <p style="text-align: center;"><i>[bewys OF buite <math>\angle</math> v koordevh]</i></p> $\hat{A}_2 = \hat{C}_2 = 90^\circ - x \quad [\text{proved } \mathbf{OR} \text{ sum of } \angle \text{s in } \triangle]$ <p style="text-align: center;"><i>[Bewys OF som v <math>\angle</math>e in <math>\triangle</math>]</i></p> $\hat{ACB} = \hat{AMC} = 90^\circ \quad [\text{given } \mathbf{OR} \text{ sum of } \angle \text{s in } \triangle]$ <p style="text-align: center;"><i>[gegee OF som v <math>\angle</math>e in <math>\triangle</math>]</i></p> $\triangle ACB \parallel \triangle CMD$ | <p>✓ S</p> <p>✓ S</p> <p>✓ R</p> <p>(3)</p> <p>✓ S</p> <p>✓ S</p> <p>✓ R</p> <p>(3)</p> <p>✓ S</p> <p>✓ S</p> <p>✓ S</p> <p>(3)</p>  |
| 10.2.1 | $\frac{BC}{MD} = \frac{AB}{DC} \quad [\triangle ACB \parallel \triangle CMD]$ $\frac{DC}{MD} = \frac{AB}{DC} \quad [BC = DC]$ $\therefore DC^2 = AB \times MD$ <p>In <math>\triangle AMC</math> and/en <math>\triangle CMD</math></p> <p><math>\hat{M}</math> is common/gemeen</p> $\hat{A}_1 = \hat{C}_2 \quad [\text{tan chord th /raaklyn koordst}]$ <p><b>OR/OF</b></p> $\hat{C}_1 + \hat{C}_2 = \hat{B} = \hat{D} = x \quad [\text{tan chord th /raaklyn koordst } \mathbf{OR/OF} \text{ exterior } \angle \text{ of cyclic quad/ buite } \angle \text{ v kdvh}]$ $\triangle AMC \parallel \triangle CMD \quad [\angle, \angle, \angle]$ $\frac{AM}{CM} = \frac{CM}{MD}$ $\therefore CM^2 = AM \times MD$ $\therefore \frac{CM^2}{DC^2} = \frac{AM \times MD}{AB \times MD}$ $= \frac{AM}{AB}$  | <p>✓ <math>\frac{BC}{MD} = \frac{AB}{DC}</math></p> <p>✓ <math>DC^2 = AB \times MD</math></p> <p>✓ S</p> <p>✓ S</p> <p>✓ <math>CM^2 = AM \times MD</math></p> <p>✓ <math>\frac{AM \times MD}{AB \times MD}</math></p> <p>(6)</p> |

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|        | <p><b>OR/OF</b></p> $\frac{AC}{MC} = \frac{AB}{DC} \quad [\Delta ACB \parallel \Delta CMD]$ $\therefore CM \times AB = AC \times DC$ <p>In <math>\Delta AMC</math> and/en <math>\Delta ACB</math><br/> <math>\hat{C} = \hat{M} = 90^\circ</math> [given]<br/> <math>\hat{A}_1 = \hat{A}_2</math> [proven]</p> <p><b>OR/OF</b><br/> <math>\hat{A}\hat{C}M = \hat{B} = x</math> [proven]<br/> <math>\Delta AMC \parallel \Delta ACB</math> [<math>\angle, \angle, \angle</math>]<br/> <math>\frac{AC}{AM} = \frac{BC}{MC}</math><br/> <math>\therefore AC \times MC = AM \times BC</math><br/> <math>\therefore AC = \frac{BC \cdot AM}{MC}</math></p> $CM \times AB = \frac{BC \cdot AM}{MC} \times DC$ $CM^2 = \frac{DC \cdot AM}{AB} \times DC \quad [BC = DC]$ $\frac{CM^2}{DC^2} = \frac{AM}{AB}$ | $\checkmark \frac{AC}{MC} = \frac{AB}{DC}$<br>$\checkmark S$<br>$\checkmark S$<br>$\checkmark AC \cdot MC = AM \cdot BC$<br>$\checkmark \text{equating}$<br>$\checkmark S$<br><p style="text-align: right;">(6)</p>                      |
| 10.2.2 | <p>In <math>\Delta DMC</math>:</p> $\frac{CM}{DC} = \sin x$ $\frac{CM^2}{DC^2} = \sin^2 x \quad \frac{AC}{AB} = \frac{CM}{DC}$ $\therefore \frac{AM}{AB} = \sin^2 x$ <p><b>OR/OF</b></p> <p>In <math>\Delta ABC</math>:</p> $\sin x = \frac{AC}{AB}$ <p>In <math>\Delta AMC</math>:</p> $\sin x = \frac{AM}{AC}$ $\sin x \cdot \sin x = \frac{AC}{AB} \times \frac{AM}{AC} = \frac{AM}{AB}$  | $\checkmark \text{trig ratio}$<br>$\checkmark \text{square both sides}$<br><p style="text-align: right;">(2)</p><br>$\checkmark 2 \text{ equations for } \sin x$<br>$\checkmark \text{product}$<br><p style="text-align: right;">(2)</p> |