



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NASIONALE
SENIOR SERTIFIKAAT**

GRAAD 11

WISKUNDE V2

NOVEMBER 2018

PUNTE: 150

TYD: 3 uur

Hierdie vraestel bestaan uit 16 bladsye en 'n 24 bladsy-antwoordeboek.

INSTRUKSIES EN INLIGTING

Lees die volgende instruksies aandagtig deur voordat jy die vrae beantwoord.

1. Hierdie vraestel bestaan uit 11 vrae.
2. Beantwoord AL die vrae in die SPESIALE ANTWOORDEBOEK wat verskaf word.
3. Dui ALLE berekeninge, diagramme, grafieke, ens. wat jy gebruik het om die antwoorde te bepaal, duidelik aan.
4. Volpunte sal NIE noodwendig aan slegs antwoorde toegeken word NIE.
5. Indien nodig, rond antwoorde tot TWEE desimale plekke af, tensy anders aangedui.
6. Diagramme is NIE noodwendig volgens skaal geteken NIE.
7. Jy mag 'n goedgekeurde wetenskaplike sakrekenaar (nieprogrammeerbaar en niegrafies) gebruik, tensy anders aangedui.
8. Skryf netjies en leesbaar.

VRAAG 1

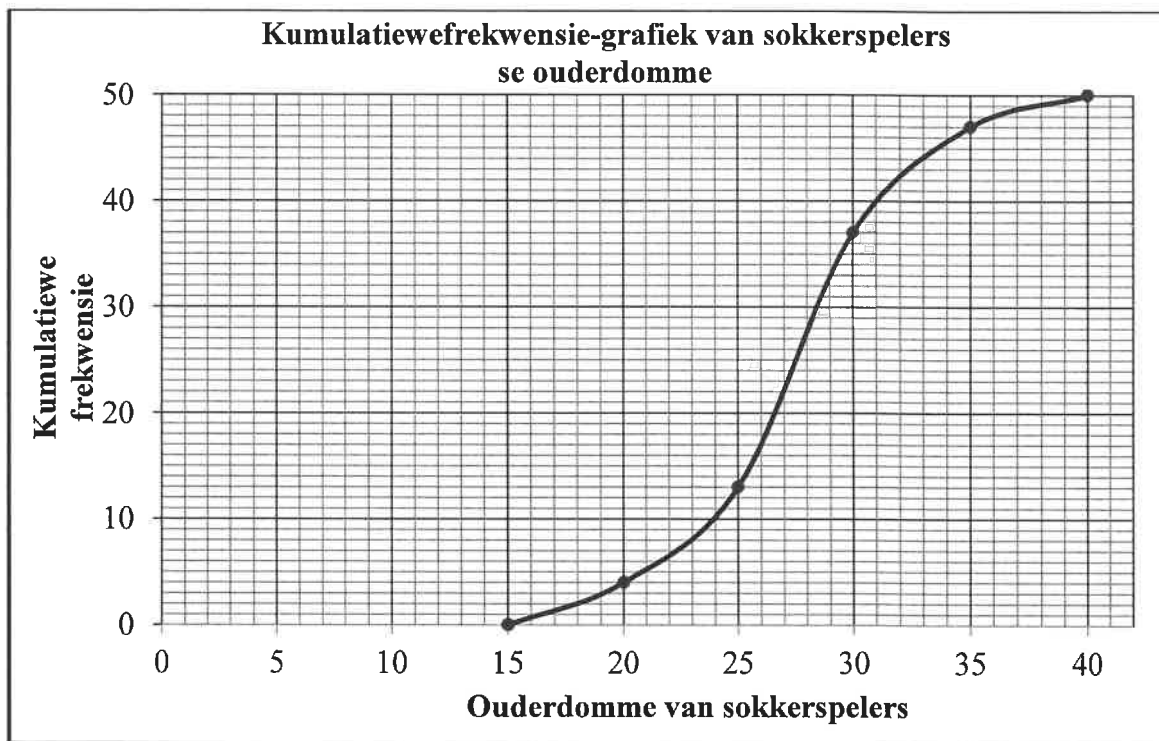
'n Skool het 'n sportdag gehou. Een van die items op die program was 'n hinderniswedloop. Spanne van 10 ouers en leerders het aan hierdie wedloop deelgeneem. Die tabel hieronder toon die tyd, in minute, wat dit elke lid van 'n bepaalde span geneem het om die wedloop te voltooi.

4	12	13	16	17	18	20	22	22	25
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- 1.1 Hoe lank, in minute, het dit die vinnigste lid van die span geneem om die wedloop te voltooi? (1)
- 1.2 Bepaal hierdie span se gemiddelde tyd. (2)
- 1.3 Bereken die standaardafwyking van die data. (1)
- 1.4 Hoeveel lede van die span het die hinderniswedloop buite twee standaardafwykings van die gemiddeld voltooi? (3)
- 1.5 Dit het 'n ander span 'n totale tyd van $x+5$ minute geneem om die wedloop te voltooi. Bereken die waarde van x as die algehele gemiddeld van die twee spanne saam 18 minute was. (3)
- [10]**

VRAAG 2

2.1 'n Opname is van die ouderdomme van sokkerspelers by 'n sokkertoernooi gemaak. Die uitslae word op die kumulatiewefrekwensie-grafiek (ogief) hieronder getoon.



2.1.1 Hoeveel spelers het aan die sokkertoernooi deelgeneem? (1)

2.1.2 Bepaal die getal spelers tussen die ouderdomme van 24 en 31 jaar. (2)

2.1.3 Voltooi die frekwensiekolom van die tabel hieronder in die ANTWOORDEBOEK.

KLASINTERVAL	FREKWENSIE	KUMULATIEWE FREKWENSIE
$15 \leq x < 20$		4
$20 \leq x < 25$		13
$25 \leq x < 30$		37
$30 \leq x < 35$		47
$35 \leq x < 40$		50

(3)

2.1.4 Gebruik die rooster wat in die ANTWOORDEBOEK verskaf word om 'n frekwensievelhoek vir die data te teken. (4)

- 2.2 Twee graad 11-Wiskundeklasse het dieselfde getal leerders. Die vyf-getal-opsommings van die punte wat hierdie klasse vir 'n toets behaal het, word hieronder getoon.

KLAS A (30 ; 48 ; 65 ; 82 ; 90)

KLAS B (50 ; 58 ; 65 ; 75 ; 90)

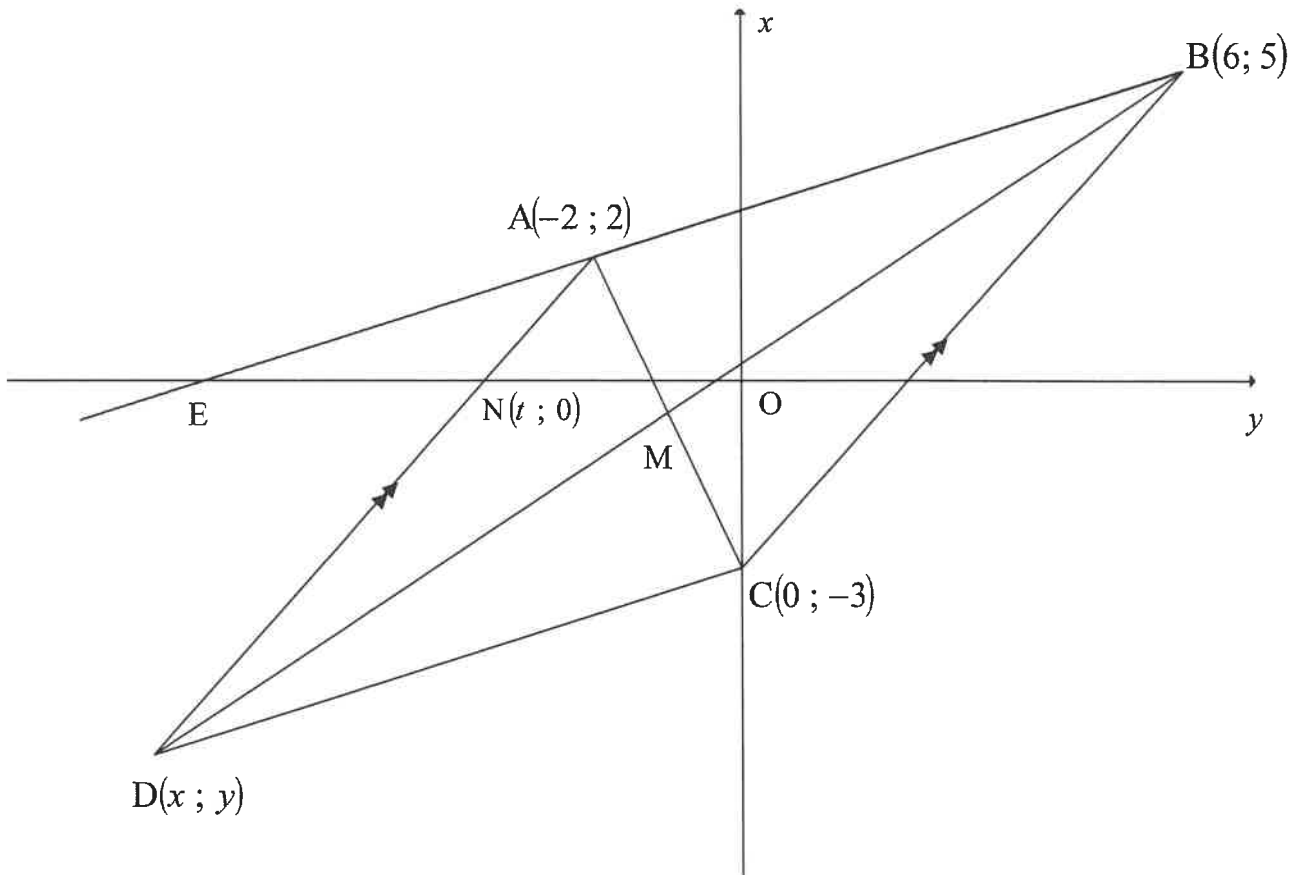
Die ouers van leerders in KLAS A en KLAS B neem waar dat albei klasse dieselfde mediaan en dieselfde maksimum punt het en beweer dus dat daar geen prestasieverskil tussen hierdie klasse is nie.

Stem jy met hierdie bewering saam? Gebruik ten minste TWEE verskillende argumente om jou antwoord te regverdig.

(3)
[13]

VRAAG 3

In die diagram is $A(-2 ; 2)$, $B(6 ; 5)$, $C(0 ; -3)$ en $D(x ; y)$ die hoekpunte van 'n vierhoek met $AD \parallel BC$. BA verleng het 'n x -afsnit by E . BD en AC sny by M . $N(t ; 0)$ is 'n punt op AD .



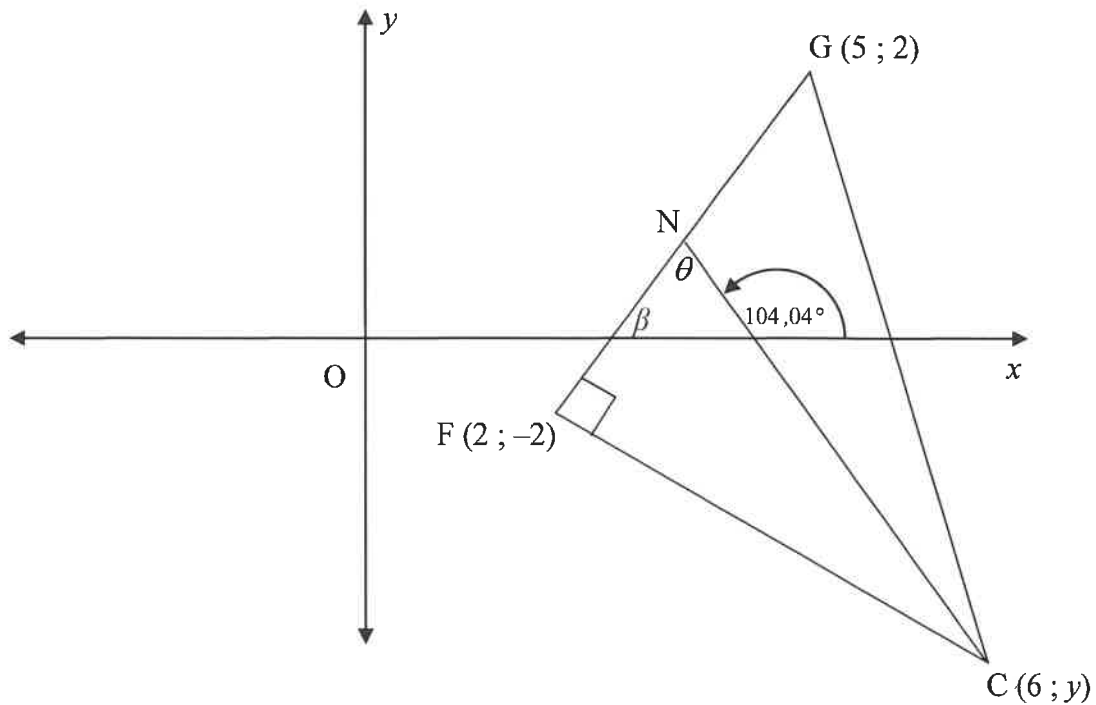
- 3.1 Bereken die gradiënt van BC . (2)
- 3.2 Bepaal die vergelyking van AD . (3)
- 3.3 Bepaal die waarde van t . (2)
- 3.4 Bereken die lengte van AN . (2)
- 3.5 Indien DC deur $y = \frac{3}{8}x - 3$ gedefinieer word, bepaal die koördinate van D . (4)
- 3.6 Bewys dat $ABCD$ 'n parallelogram is. (3)
- 3.7 Bereken die koördinate van M . (3)
- [19]**

VRAAG 4

In die diagram is $F(2; -2)$, $G(5; 2)$ en $C(6; y)$ die hoekpunte van $\triangle FGC$. $FG \perp FC$.

N is 'n punt op FG sodanig dat die inklinasie van NC , $104,04^\circ$ is.

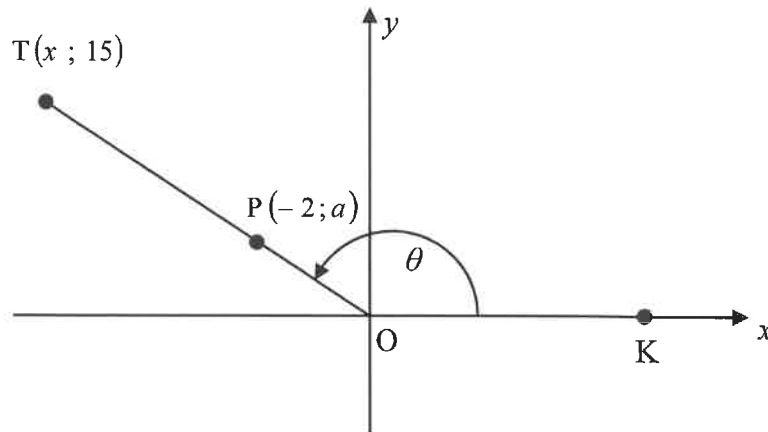
Die inklinasiehoek van FG is β en $\widehat{FNC} = \theta$.



- 4.1 Bereken die gradiënt van FG . (2)
- 4.2 Bereken die waarde van y . (3)
- 4.3 Bereken die grootte van θ . (3)
- 4.4 Bereken die lengte van NC . (4)
- [12]**

VRAAG 5

- 5.1 In die diagram hieronder is $T(x; 15)$ 'n punt op die Kartesiese vlak sodanig dat $OT = 17$ eenhede. $P(-2; a)$ lê op OT . K is 'n punt op die positiewe x -as en $\hat{TOK} = \theta$.

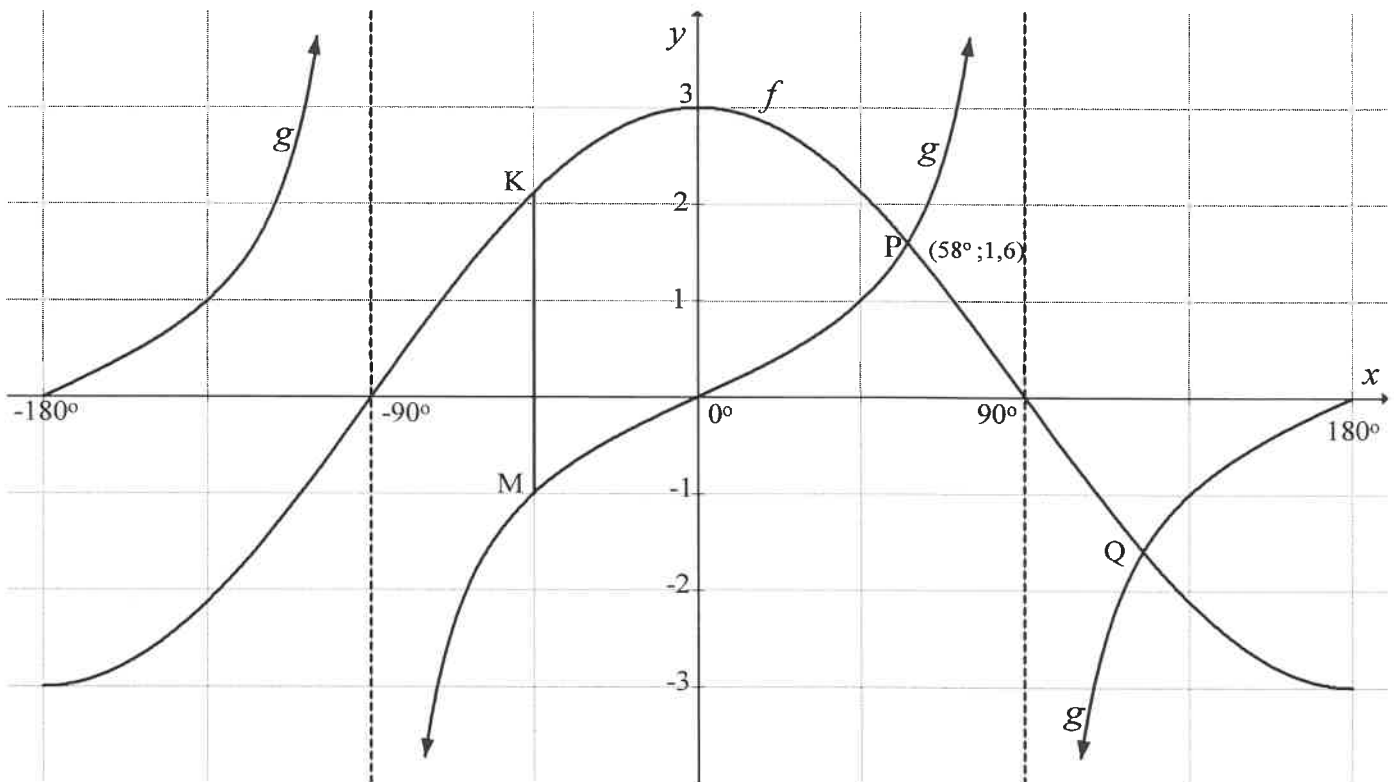


Bepaal, met behulp van die diagram, die volgende:

- 5.1.1 Die waarde van x (2)
- 5.1.2 $\tan \theta$ (1)
- 5.1.3 $\cos(180^\circ - \theta)$ (2)
- 5.1.4 $\sin^2 \theta$ (2)
- 5.1.5 Die waarde van a . (3)
- 5.2 Vereenvoudig SONDER die gebruik van 'n sakrekenaar:
- $$\frac{\sin 120^\circ \cdot \cos 210^\circ \cdot \tan 315^\circ \cdot \cos 27^\circ}{\sin 63^\circ \cdot \cos 540^\circ} \quad (7)$$
- 5.3 Bewys die identiteit:
- $$\frac{1}{\cos \theta} - \frac{\cos \theta}{1 + \sin \theta} = \tan \theta \quad (5)$$
- 5.4 Bepaal die algemene oplossing van $3 \sin x = 2 \tan x$ (6)
- [28]**

VRAAG 6

Die grafieke van die funksies $f(x) = a \cos b\theta$ en $g(x) = c \tan \theta$ vir $x \in [-180^\circ; 180^\circ]$ is hieronder getrek. Die grafieke sny by $P(58^\circ; 1,6)$ en Q .

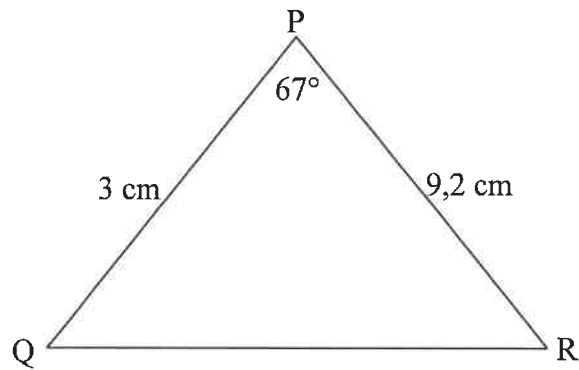


- 6.1 Skryf die waardeversameling van f neer. (2)
- 6.2 Indien $M(-45^\circ; -1)$ op g lê, bepaal die waarde van c . (1)
- 6.3 Skryf die waardes van a en b neer. (2)
- 6.4 Bepaal die koördinate van Q . (2)
- 6.5 K lê op f sodanig dat KM parallel aan die y -as is. Bereken die lengte van KM . (2)
- 6.6 Indien die assestelsel 45° na links geskuif word en die grafieke bly vas, skryf die vergelyking neer van die grafiek wat nou deur grafiek f verteenwoordig word. (2)

[11]

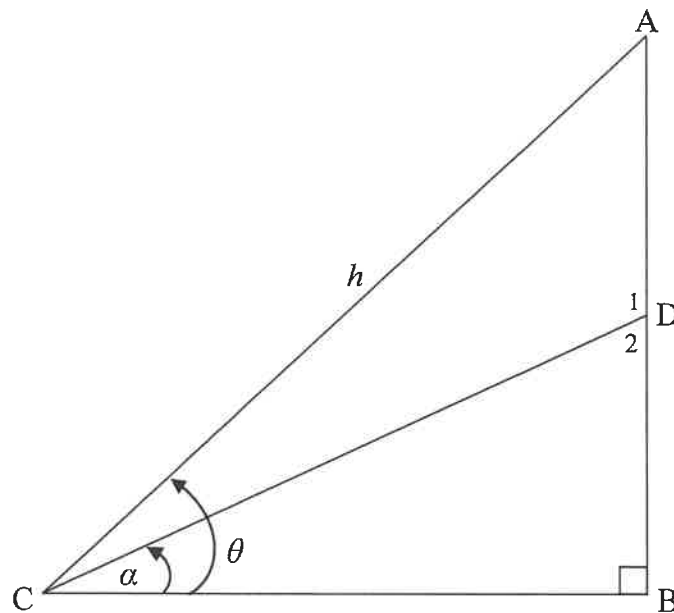
VRAAG 7

- 7.1 In die diagram is $\hat{P} = 67^\circ$, $PQ = 3$ cm en $PR = 9,2$ cm.
Bepaal die lengte van QR .



(3)

- 7.2 In die diagram hieronder is $\hat{DCB} = \alpha$, $AC = h$ eenhede en $\hat{ACB} = \theta$.

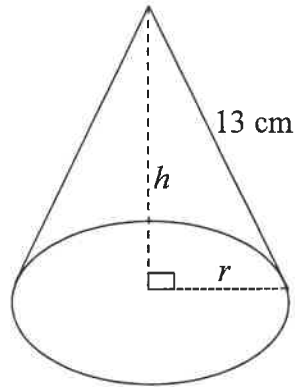


- 7.2.1 Bepaal die grootte van \hat{ACD} in terme van θ en α . (1)
- 7.2.2 Bewys dat $AD = \frac{h \sin(\theta - \alpha)}{\cos \alpha}$ (4)
- 7.2.3 Bepaal die lengte van AD as $h = 17$ eenhede, $\theta = 58^\circ$ en $\alpha = 23^\circ$. (2)
- 7.2.4 Bereken die oppervlakte van $\triangle ADC$. (3)

[13]

VRAAG 8

Die diagram hieronder toon 'n keël met 'n loodregte hoogte van h cm, 'n radius van r cm en 'n skuinshoogte van 13 cm.



$$\text{Volume van keël} = \frac{1}{3} \pi r^2 h$$

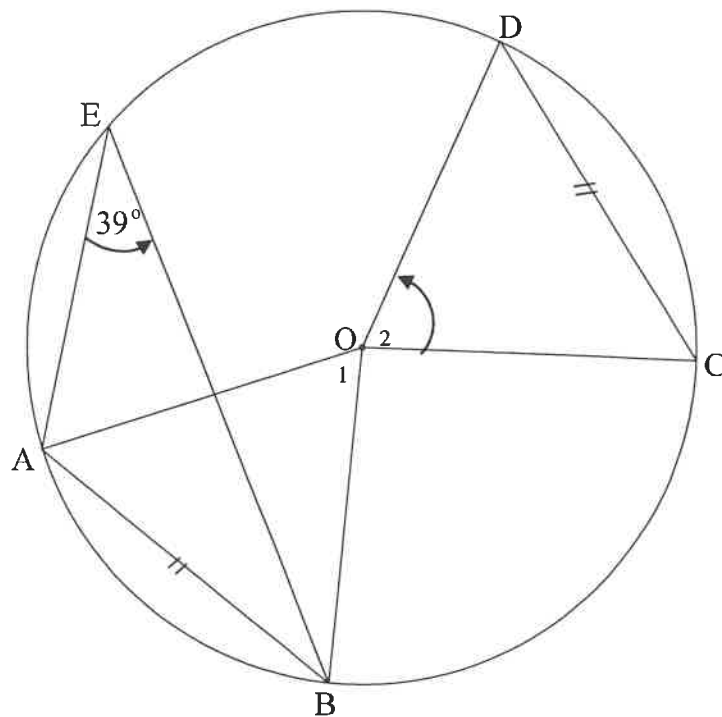
$$\text{Totale buiteoppervlakte van die keël} \\ = \pi r^2 + \pi r s$$

- 8.1 Bewys dat die volume van die keël deur $V = \frac{169\pi h - \pi h^3}{3}$ gegee word. (4)
- 8.2 Indien $h = 12$ cm, bepaal die totale buiteoppervlakte van die keël. (3)
- [7]

Gee redes vir jou bewerings en berekeninge in VRAAG 9, 10 en 11.

VRAAG 9

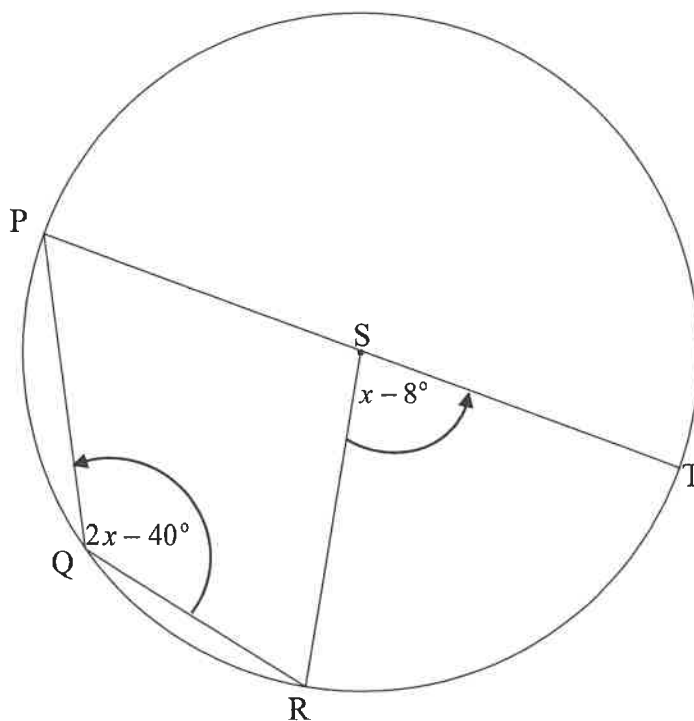
9.1 In die figuur is O die middelpunt van die sirkel. A , B , C , D en E lê op die sirkel sodanig dat koord AB en koord DC gelyk in lengte is en $\hat{AEB} = 39^\circ$.



9.1.1 Bepaal die grootte van \hat{O}_1 . (2)

9.1.2 Bepaal die grootte van \hat{O}_2 . (2)

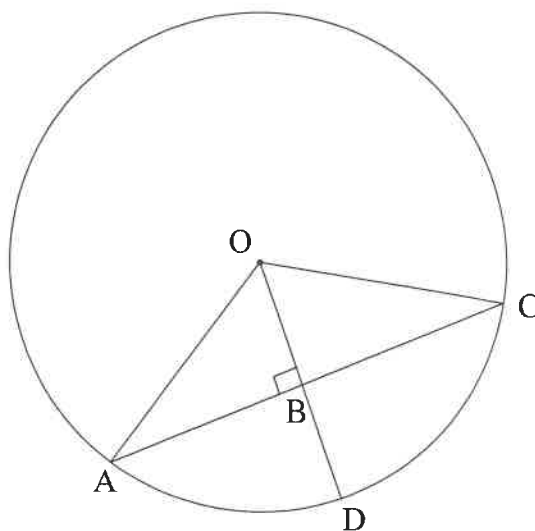
- 9.2 In die diagram is S die middelpunt van sirkel $PQRT$. PT is 'n middellyn.
 $\widehat{RST} = x - 8^\circ$ en $\widehat{PQR} = 2x - 40^\circ$.



Bepaal die waarde van x .

(4)

- 9.3 In die diagram is O die middelpunt van die sirkel. Koord AC is loodreg op radius OD by B . $OB = 2x$ eenhede en $AC = 8x$ eenhede.

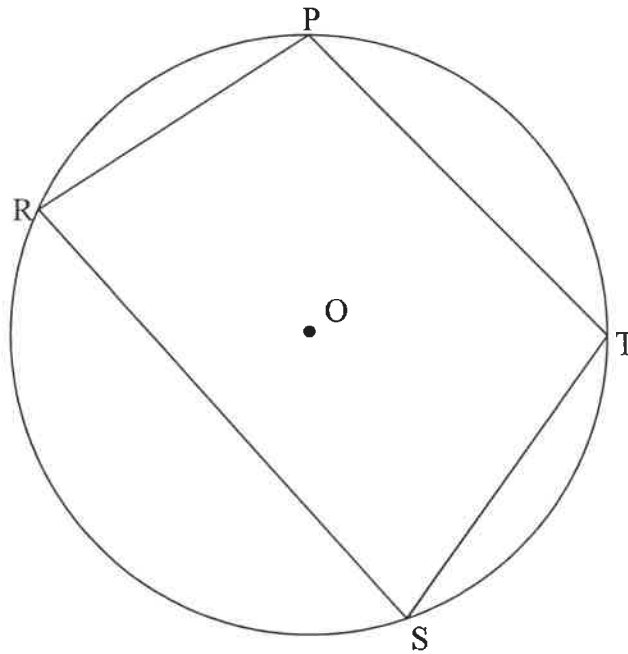


Toon dat die lengte van BD $2x(\sqrt{5}-1)$ eenhede is.

(5)
[13]

VRAAG 10

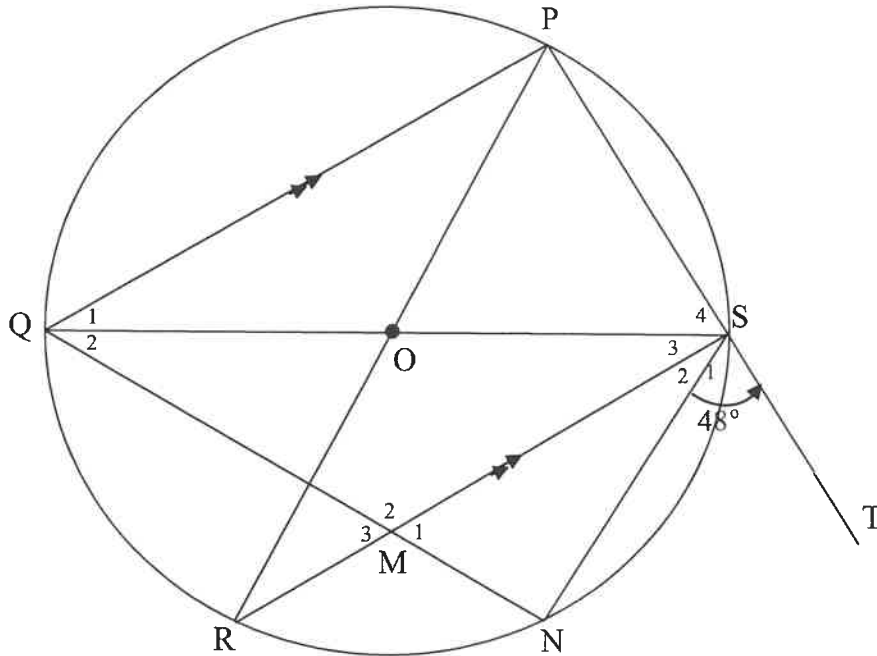
- 10.1 In die diagram hieronder is O die middelpunt van die sirkel en $PTSR$ is 'n koordevierhoek.



Bewys die stelling wat beweer dat $\hat{P} + \hat{S} = 180^\circ$.

(5)

- 10.2 In die figuur is QS en PR middellyne van die sirkel met middelpunt O sodat $PQ \parallel SR$. PS is verleng na T. N is 'n punt op die sirkel sodat $\hat{Q}_1 = \hat{Q}_2$. SN is getrek. RS sny QN by M. $\hat{S}_1 = 48^\circ$



10.2.1 Bepaal, met redes, die grootte van:

- (a) \hat{Q}_1 (3)
- (b) \hat{R} (2)
- (c) \hat{M}_1 (2)

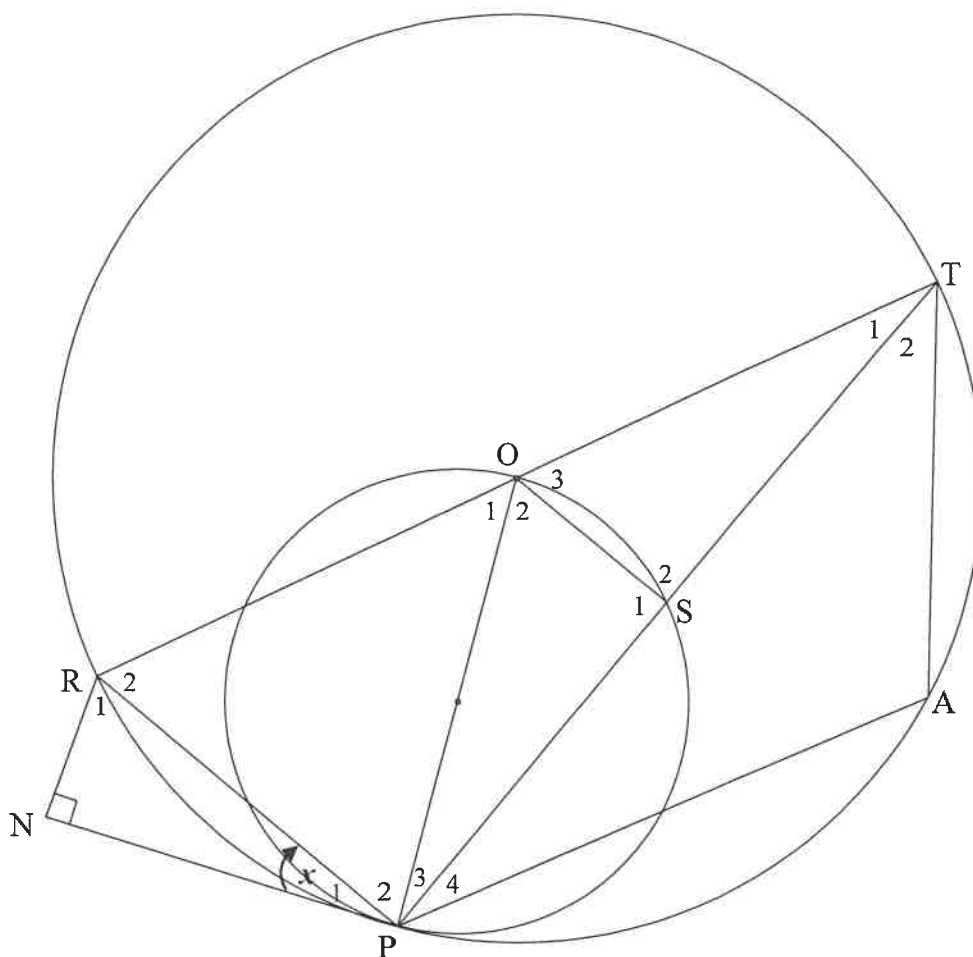
10.2.2 Bewys dat ST 'n raaklyn aan die sirkel is wat deur M, N en S gaan. (2)

[14]

VRAAG 11

O is die middelpunt van die groter sirkel RTAP. OP is die middellyn van die kleiner sirkel PSO. NP is 'n raaklyn aan beide sirkels by P. $RN \perp NP$.

Laat $\hat{P}_1 = x$.



11.1 Bewys dat PR vir \hat{ORN} halveer. (5)

11.2 Bewys dat $\hat{ROS} = \hat{PAT}$. (5)

[10]

TOTAAL: 150



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GRADE/*GRAAD* 11

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2018

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE*: 150

**This marking guideline consists of 28 pages.
*Hierdie nasienriglyne bestaan uit 28 bladsye.***

NOTE:

- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking memorandum.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

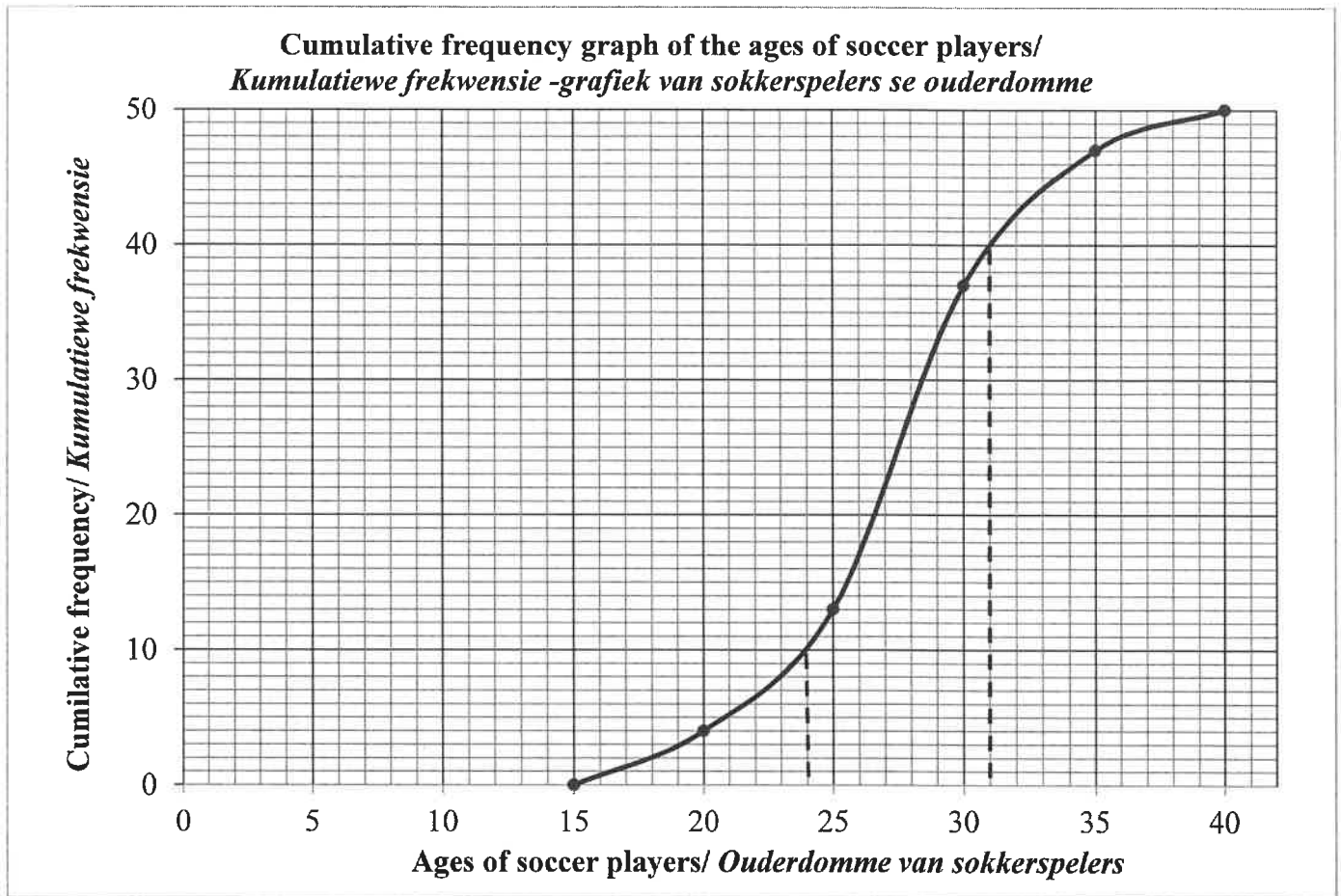
- *As 'n kandidaat 'n vraag TWEE keer beantwoord het, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord deurgehaal en nie oorgedoen het nie, sien die deurgehaalde antwoord na.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die memorandum van toepassing.*
- *Dit is onaanvaarbaar om waardes/antwoorde te veronderstel om 'n probleem op te los.*

QUESTION/VRAAG 1

4	12	13	16	17	18	20	22	22	25
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1.1	4 minutes/ <i>minute</i>	✓ answer/ <i>antwoord</i> (1)
1.2	Mean/ <i>gemiddeld</i> = $\frac{169}{10} = 16,9$	✓ 169 ✓ answer/ <i>antwoord</i> (2)
1.3	Standard deviation/ <i>Standardafwyking</i> = 5,79	✓ answer/ <i>antwoord</i> (1)
1.4	$(16,9 - 2 \times 5,79; 16,9 + 2 \times 5,79)$ $(5,32 ; 28,48)$ ∴ 1 member of the team completed the obstacle race outside of 2 standard deviations of the mean./ <i>1 lid van die span het die hundernisbaan buite twee standardafwykings van die gemiddeld voltooi.</i>	✓ $\bar{x} - 2\sigma$ ✓ $\bar{x} + 2\sigma$ ✓ answer/ <i>antwoord</i> (3)
1.5	$\frac{169 + x + 5}{20} = 18$ $x = 18 \times 20 - 174$ $x = 186$	✓ $169 + x + 5$ ✓ dividing by 20/ <i>deel deur 20</i> ✓ answer/ <i>antwoord</i> (3)
		[10]

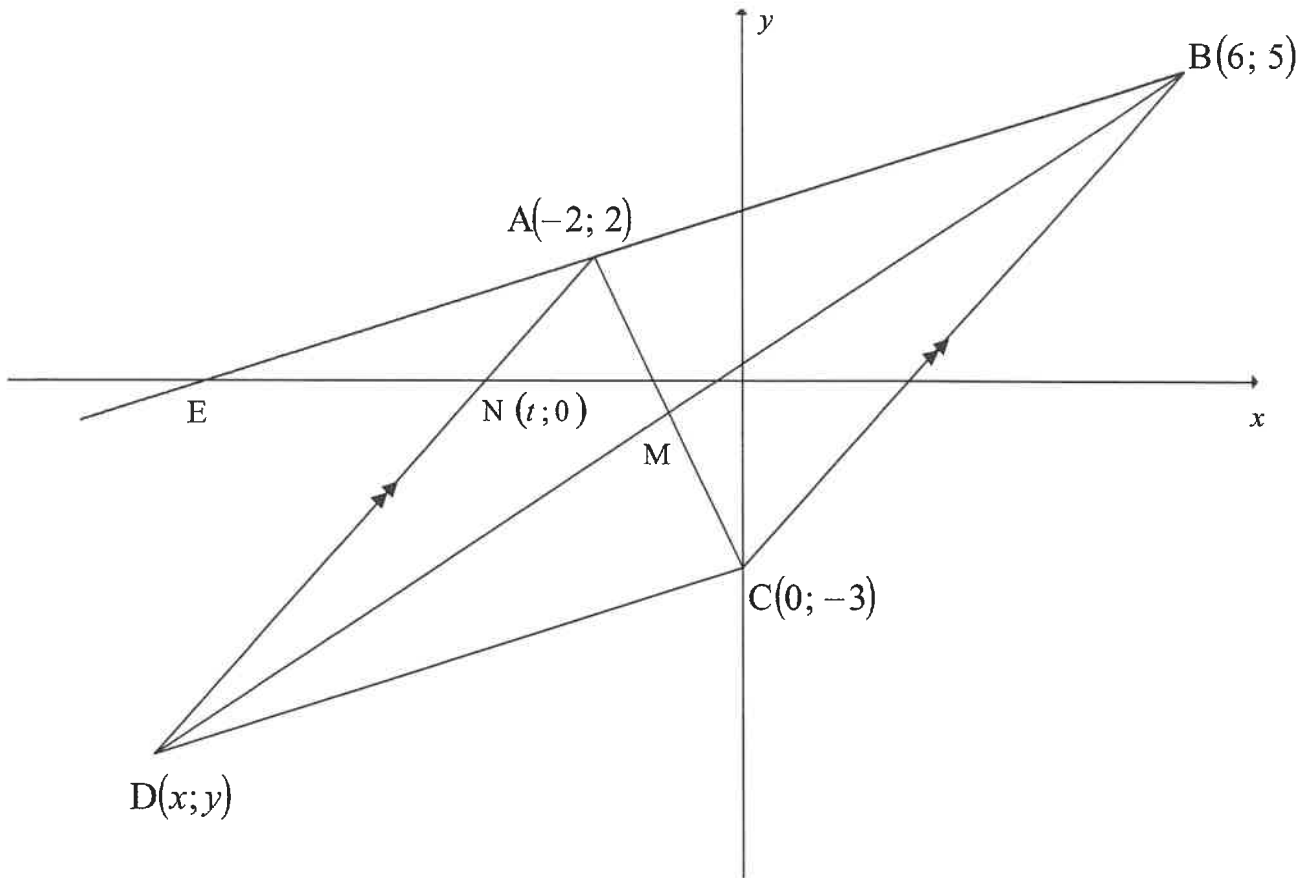
QUESTION/VRAAG 2



2.1.1	50 players/ <i>spelers</i>	✓ answer/ <i>antwoord</i> (1)																		
2.1.2	40 – 10 = 30 players/ <i>spelers</i>	✓ 40 and/ <i>en</i> 10 ✓ answer/ <i>antwoord</i> (2)																		
2.1.3	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="text-align: center;">Class interval/ <i>Klas interval</i></th> <th style="text-align: center;">Frequency/ <i>Frekwensie</i></th> <th style="text-align: center;">Cumulative frequency <i>Kumulatiewe frekwensie</i></th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">$15 \leq x < 20$</td> <td style="text-align: center;">4</td> <td style="text-align: center;">4</td> </tr> <tr> <td style="text-align: center;">$20 \leq x < 25$</td> <td style="text-align: center;">9</td> <td style="text-align: center;">13</td> </tr> <tr> <td style="text-align: center;">$25 \leq x < 30$</td> <td style="text-align: center;">24</td> <td style="text-align: center;">37</td> </tr> <tr> <td style="text-align: center;">$30 \leq x < 35$</td> <td style="text-align: center;">10</td> <td style="text-align: center;">47</td> </tr> <tr> <td style="text-align: center;">$35 \leq x < 40$</td> <td style="text-align: center;">3</td> <td style="text-align: center;">50</td> </tr> </tbody> </table>	Class interval/ <i>Klas interval</i>	Frequency/ <i>Frekwensie</i>	Cumulative frequency <i>Kumulatiewe frekwensie</i>	$15 \leq x < 20$	4	4	$20 \leq x < 25$	9	13	$25 \leq x < 30$	24	37	$30 \leq x < 35$	10	47	$35 \leq x < 40$	3	50	✓ two correct values/ <i>twee korrekte waardes</i> ✓ three correct values/ <i>drie korrekte waardes</i> ✓ all correct values/ <i>al die waardes korrek</i> (3)
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	<p style="text-align: center;">Frequency polygon of the ages of soccer players/ <i>Frekwensie- veelhoek van sokkerspelers se oudermmme</i></p>	<ul style="list-style-type: none"> ✓ using midpoints / gebruik middelpunte ✓ plotting the points correctly/ korrekte punte geplot ✓ points joined by straight line/ punte verbind met 'n reguitlyn ✓ grounding at/ geanker by (12,5;0) and/ en (42,5 ; 0) <p style="text-align: right;">(4)</p>																		
<p>2.2</p>	<p>The claim is not valid. / <i>Die bewering is nie geldig nie</i></p> <p>Range of class/ <i>Omvang van klas A = 60</i> Range of class/ <i>Omvang van klas B = 40</i></p> <p>The range of class A is bigger than the range of class B. Therefore the marks of class A are more spread out than the class B./ <i>Die omvang van klas A is groter as die omvang van klas B. Dus is die punte in klas A meer verspreid as klas B</i></p> <p>At least 25% of class A have lower marks than any learner in class B./ <i>ten minste 25% van klas A het laer punte as enige leerder in klas B.</i></p> <p>Class A performed worse at the bottom end. / <i>Klas A het slegter gevorder aan die onderste groep</i></p>	<ul style="list-style-type: none"> ✓ claim not valid/ bewering nie geldig nie ✓ comment on the overall spread/ kommentaar oor die algehele verspreiding ✓ comparison of the lower marks/ vergelyk laer punte <p style="text-align: right;">(3)</p> <p style="text-align: right;">[13]</p>																		

QUESTION/VRAAG 3



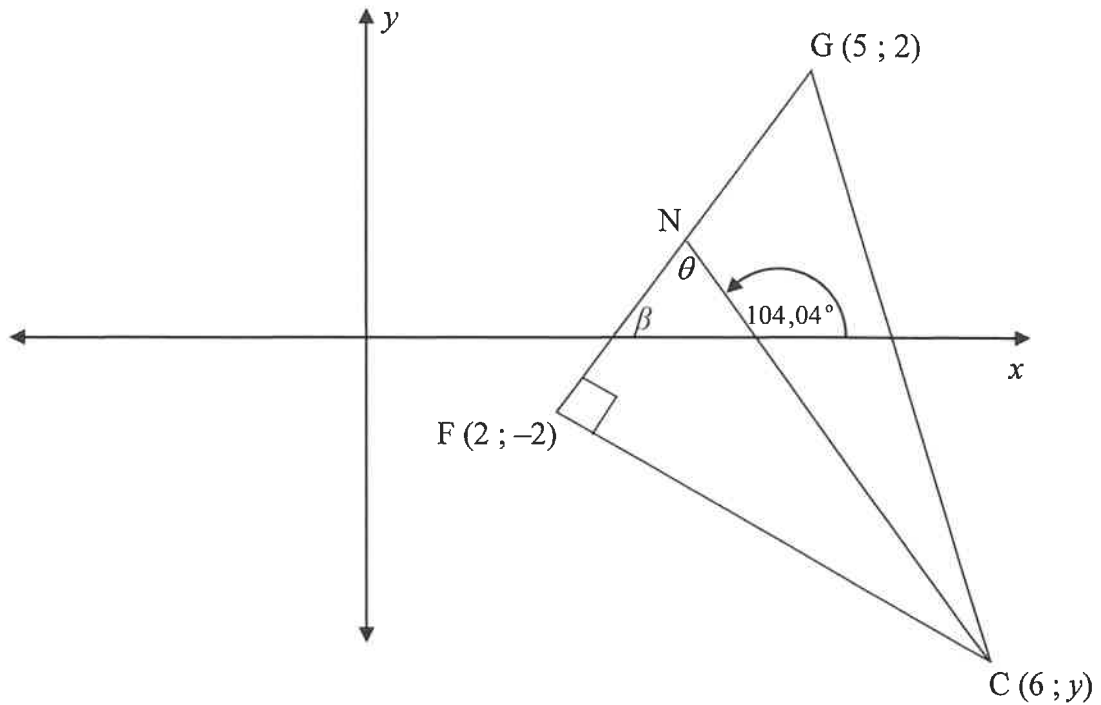
<p>3.1</p>	<p>B(6;5) C(0;-3)</p> $m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{-3 - 5}{0 - 6}$ $= \frac{-8}{-6}$ $= \frac{4}{3}$ <p>OR/ OF</p> $m_{BC} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{5 - (-3)}{6 - 0}$ $= \frac{8}{6}$ $= \frac{4}{3}$	<p>✓ subst into correct grad.form / <i>verv in grad form.</i></p> <p>✓ answer/ <i>antwoord</i> (2)</p>
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3.2	$m_{AD} = m_{BC} = \frac{4}{3} \text{ (AD} \parallel \text{BC)}$ $y = \frac{4}{3}x + c$ $2 = \frac{4}{3}(-2) + c$ $\frac{14}{3} = c$ $\therefore y = \frac{4}{3}x + \frac{14}{3}$ <p>OR/OF</p> $m_{AD} = \frac{4}{3} \text{ (AD} \parallel \text{BC)}$ $y - 2 = \frac{4}{3}(x - (-2))$ $y = \frac{4}{3}x + \frac{14}{3}$ $\therefore y = \frac{4}{3}x + \frac{14}{3}$	$\checkmark m_{AD} = \frac{4}{3}$ <p>\checkmark subst of m and point $(-2; 2)$ / <i>verv. m en punt $(-2; 2)$</i></p> <p>\checkmark answer/ <i>antwoord</i> (3)</p> $\checkmark m_{AD} = \frac{4}{3}$ <p>\checkmark subst of m and point $(-2; 2)$ / <i>verv. m en punt $(-2; 2)$</i></p> <p>\checkmark answer/ <i>antwoord</i> (3)</p>
3.3	$y = \frac{4}{3}x + \frac{14}{3}$ $0 = \frac{4}{3}t + \frac{14}{3}$ $\frac{-14}{3} = \frac{4}{3}t$ $t = \frac{-14}{4} = \frac{-7}{2}$	<p>\checkmark subst/ <i>verv. $y=0$</i></p> <p>\checkmark answer/ <i>antwoord</i> (2)</p>
3.4	$AN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{\left((-2) - \left(-\frac{7}{2}\right)\right)^2 + (2 - 0)^2}$ $= \sqrt{\frac{25}{4}}$ $= \frac{5}{2}$	<p>\checkmark subst. in distance formula/ <i>verv. in afstand formule</i></p> <p>\checkmark answer/ <i>antwoord</i> (2)</p>

<p>3.5</p>	$\frac{3}{8}x - 3 = \frac{4}{3}x + \frac{14}{3}$ $\frac{23}{24}x = -\frac{23}{3}$ $x = -8$ $y = \frac{4}{3}(-8) + \frac{14}{3}$ $= -6$ $D(-8; -6)$	<p>✓ equating/ <i>vergelyk</i></p> <p>✓ simplification/ <i>vereenv.</i></p> <p>✓ x- value/ <i>waarde</i></p> <p>✓ y- value/ <i>waarde</i></p> <p>(4)</p>
<p>3.6</p>	$m_{AB} = \frac{5-2}{6-(-2)} = \frac{3}{8}$ <p>$m_{AB} = m_{DC}$ $\therefore AB \parallel DC$ but/maar $AD \parallel BC$ $\therefore ABCD$ is a parallelogram [opp sides are \parallel / <i>teenoorst sye is</i> \parallel]</p> <p>OR/OF</p> $AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{((-2) - (-8))^2 + (2 - 6)^2}$ $= \sqrt{100}$ $= 10$ $BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(6 - 0)^2 + (5 - (-3))^2}$ $= \sqrt{100}$ $= 10$ <p>$\therefore AD = BC$ but/maar $AD \parallel BC$ $\therefore ABCD$ is a parallelogram [2 opp sides are $=$ and \parallel / <i>teenoorst sye is</i> $=$ en \parallel]</p> <p>OR/OF</p>	<p>✓ $m_{AB} = \frac{3}{8}$</p> <p>✓ $AB \parallel DC$</p> <p>✓ reason/ <i>rede</i></p> <p>(3)</p> <p>✓ length of AD/ <i>lengte van AD</i></p> <p>✓ length of BC/ <i>lengte van BC</i></p> <p>✓ reason/ <i>rede</i></p> <p>(3)</p>

	<p>M is the midpoint of AC <i>M is die middelpunt van AC</i></p> $M\left(\frac{(-2)+0}{2}; \frac{2+(-3)}{2}\right)$ $M\left(-1; -\frac{1}{2}\right)$ <p>M is the midpoint of BD <i>M is die middelpunt van BD</i></p> $M\left(\frac{(-8)+6}{2}; \frac{(-6)+5}{2}\right)$ $M\left(-1; -\frac{1}{2}\right)$ <p>\therefore ABCD is a parallelogram [diagonals bisect each other] [hoeklyne halveer mekaar]</p>	<p>✓ midpoint of AC/ <i>middelpunt van AC</i></p> <p>✓ midpoint of BD/ <i>middelpunt van AC</i></p> <p>✓ reason/ <i>rede</i></p> <p style="text-align: right;">(3)</p>
3.7	<p>M is the midpoint of AC [diagonals bisect] <i>M is die middelpunt van AC [hoeklyne halveer]</i></p> $M\left(\frac{(-2)+0}{2}; \frac{2+(-3)}{2}\right)$ $M\left(-1; -\frac{1}{2}\right)$	<p>✓ Substitution into the correct formula/ <i>Verv. in korrekte form.</i></p> <p>✓ x- value / <i>waarde</i> ✓ y- value / <i>waarde</i></p> <p style="text-align: right;">(3)</p> <p style="text-align: right;">[19]</p>

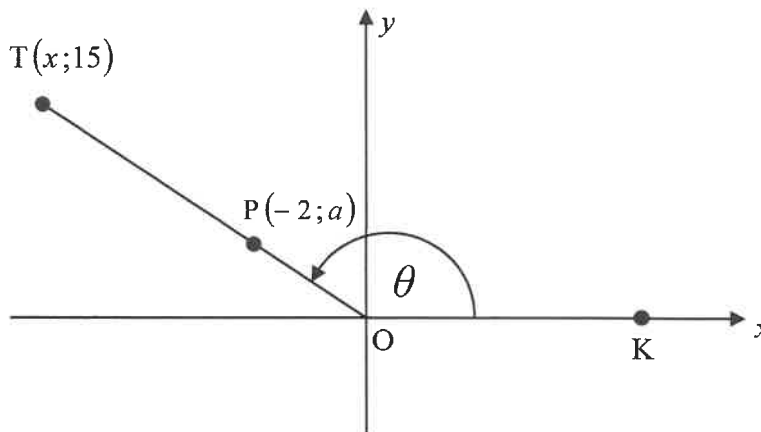
QUESTION/VRAAG 4



<p>4.1</p>	$m_{FG} = \frac{2 - (-2)}{5 - 2}$ $= \frac{4}{3}$	<p>✓ subst. into correct gradient form./ <i>vervang in gradiënt formule</i> ✓ answer (2)</p>
<p>4.2</p>	$m_{FC} = \frac{-3}{4} \quad (FC \perp FG)$ $\frac{y+2}{6-2} = \frac{-3}{4}$ $y = -5$ <p>OR/OF</p>	<p>✓ $m_{FC} = \frac{-3}{4}$ ✓ equating gradients/ <i>stel gradiënte gelyk</i> ✓ answer/ <i>antwoord</i> (3)</p>

	$m_{FC} \times m_{FG} = -1 \text{ (FC} \perp \text{FG)}$ $\frac{y+2}{6-2} \times \frac{4}{3} = -1$ $4(y+2) = -12$ $y+2 = -3$ $y = -5$	<p>✓ $m_{FC} \times m_{FG} = -1$</p> <p>✓ substitution/ <i>verv.</i></p> <p>✓ answer/ <i>antwoord</i></p> <p>(3)</p>
4.3	$\tan \beta = \frac{4}{3}$ $\beta = 53,13^\circ$ <p>$\theta = 104,04^\circ - 53,13^\circ$ [ext \angle of Δ/ <i>buite</i> \angle van Δ]</p> $\theta = 50,91^\circ$	<p>✓ $\tan \beta = \frac{4}{3}$</p> <p>✓ $\beta = 53,13^\circ$</p> <p>✓ answer/ <i>antwoord</i></p> <p>(3)</p>
4.4	$FC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(6-2)^2 + (-5-(-2))^2}$ $= \sqrt{16+9}$ $= 5$ $\sin \theta = \frac{FC}{NC}$ $\sin 50,91^\circ = \frac{5}{NC}$ $NC = \frac{5}{\sin 50,91^\circ}$ $= 6,44 \text{ unit}$	<p>✓ subst. into distance formula/ <i>verv. in afst. form.</i></p> <p>✓ length of FC / <i>lengte van FC</i></p> <p>✓ $\sin 50,91^\circ = \frac{5}{NC}$</p> <p>✓ answer/ <i>antwoord</i></p> <p>(4)</p> <p>[12]</p>

QUESTION/VRAAG 5



<p>5.1.1</p>	$x^2 + y^2 = r^2 \quad [\textit{Pythagoras}]$ $(x)^2 + (15)^2 = 17^2$ $x^2 = 64$ $x = -8 \quad (\textit{P is in quadrant 2/ is in kwadrant 2})$	<p>✓ subst in pyth/ verv in pyth</p> <p>✓ answer/ antwoord (2)</p>
<p>5.1.2</p>	$\tan \theta = \frac{15}{-8}$	<p>✓ answer/ antwoord (1)</p>
<p>5.1.3</p>	$\cos(180^\circ - \theta)$ $= -\cos \theta$ $= -\left(\frac{-8}{17}\right)$ $= \frac{8}{17}$	<p>✓ $-\cos \theta$</p> <p>✓ answer/ antwoord (2)</p>
<p>5.1.4</p>	$\sin^2 \theta$ $= \left(\frac{15}{17}\right)^2$ $= \frac{225}{289}$	<p>✓ substitution/ vervanging</p> <p>✓ answer/ antwoord (2)</p>

<p>5.1.5</p>	$\tan \theta = \frac{a}{-2} = \frac{15}{-8}$ $\frac{a}{-2} = \frac{15}{-8}$ $a = \frac{15}{4}$ <p>OR/OF</p> $m = \frac{15}{-8}$ $y = \frac{15}{-8}x$ $a = \frac{15}{-8}(-2)$ $a = \frac{15}{4}$	<p>✓ $\tan \theta = \frac{a}{-2}$</p> <p>✓ equating/ <i>stel gelyk</i></p> <p>✓ answer/ <i>antwoord</i> (3)</p> <p>✓ $y = \frac{15}{-8}x$</p> <p>✓ substitution of $P(-2; a)$/ <i>vervanging van</i> $P(-2; a)$</p> <p>✓ answer/ <i>antwoord</i> (3)</p>
<p>5.2</p>	$\text{LHS} = \frac{\sin 120^\circ \cdot \cos 210^\circ \cdot \tan 315^\circ \cdot \cos 27^\circ}{\cos 540^\circ \cdot \sin 63^\circ}$ $= \frac{\sin 60^\circ \cdot (-\cos 30^\circ) \cdot (-\tan 45^\circ) \cdot \sin 63^\circ}{\cos 180^\circ \cdot \sin 63^\circ}$ $= \frac{\frac{\sqrt{3}}{2} \cdot \frac{-\sqrt{3}}{2} \cdot (-1)}{-1}$ $= -\frac{3}{4}$	<p>✓ $\sin 60^\circ / \cos 30^\circ$</p> <p>✓ $-\cos 30^\circ$</p> <p>✓ $-\tan 45^\circ$</p> <p>✓ $\sin 63^\circ / \cos 27^\circ$</p> <p>✓ $\cos 180^\circ$</p> <p>✓ special angle ratios/ <i>spesiale hoeke verhoudings</i></p> <p>✓ answer/ <i>antwoord</i> (7)</p>

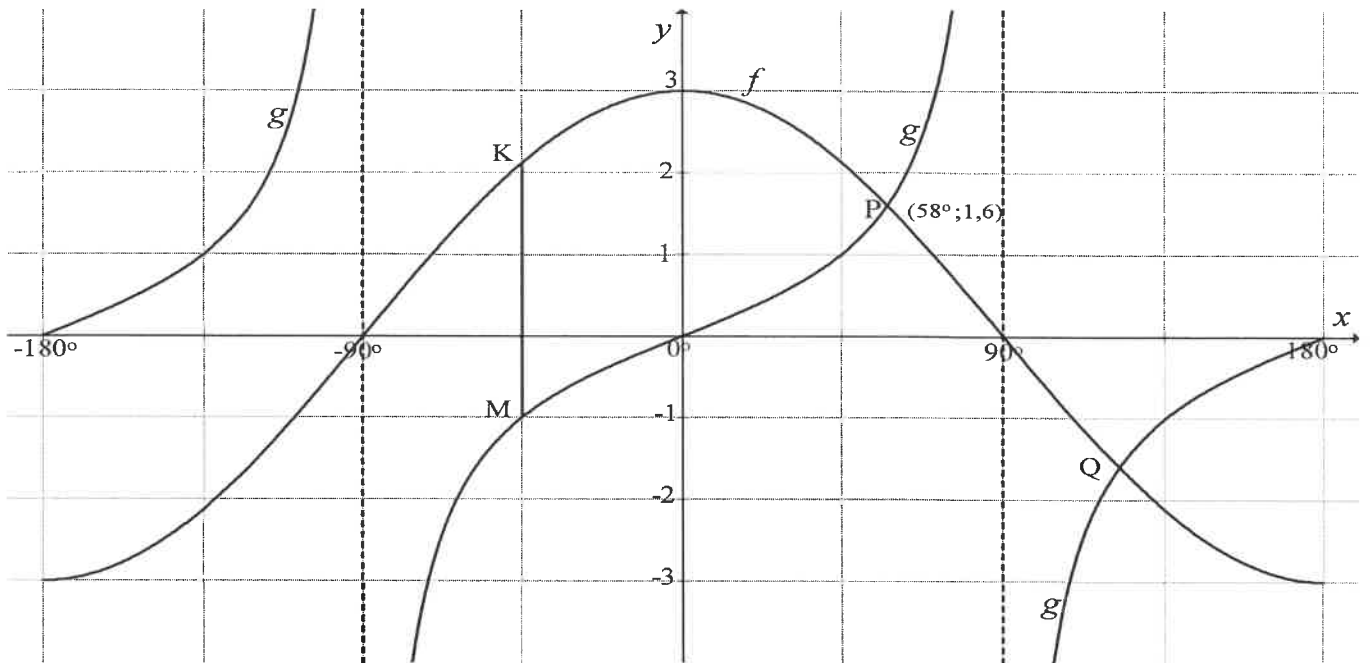
<p>5.3</p>	$\begin{aligned} \text{LHS} &= \frac{1}{\cos \theta} - \frac{\cos \theta}{1 + \sin \theta} \\ &= \frac{1 + \sin \theta - \cos^2 \theta}{\cos \theta(1 + \sin \theta)} \\ &= \frac{1 + \sin \theta - (1 - \sin^2 \theta)}{\cos \theta(1 + \sin \theta)} \\ &= \frac{\sin \theta(1 + \sin \theta)}{\cos \theta(1 + \sin \theta)} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta \\ &= \text{RHS} \end{aligned}$ <p>OR/OF</p> $\begin{aligned} \text{LHS} &= \frac{1}{\cos \theta} - \frac{\cos \theta}{1 + \sin \theta} \\ &= \frac{1 + \sin \theta - \cos^2 \theta}{\cos \theta(1 + \sin \theta)} \\ &= \frac{(1 - \cos^2 \theta) + \sin \theta}{\cos \theta(1 + \sin \theta)} \\ &= \frac{\sin^2 \theta + \sin \theta}{\cos \theta(1 + \sin \theta)} \\ &= \frac{\sin \theta(1 + \sin \theta)}{\cos \theta(1 + \sin \theta)} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta \\ &= \text{RHS} \end{aligned}$ <p>OR/OF</p>	<p>✓ common denominator/ <i>gemene noemer</i></p> <p>✓ $1 - \sin^2 \theta$</p> <p>✓ simplification/ <i>vereenv.</i></p> <p>✓ factors/ <i>faktore</i></p> <p>✓ $\frac{\sin \theta}{\cos \theta}$</p> <p>(5)</p> <p>✓ common denominator/ <i>gemene noemer</i></p> <p>✓ $1 - \cos^2 \theta$</p> <p>✓ simplification/ <i>vereenv.</i></p> <p>✓ factors/ <i>faktore</i></p> <p>✓ $\frac{\sin \theta}{\cos \theta}$</p> <p>(5)</p>
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	$\begin{aligned} \text{LHS} &= \frac{1}{\cos \theta} - \frac{\cos \theta}{1 + \sin \theta} \\ &= \frac{1 + \sin \theta - \cos^2 \theta}{\cos \theta(1 + \sin \theta)} \\ &= \frac{\sin^2 \theta + \cos^2 \theta + \sin \theta - \cos^2 \theta}{\cos \theta(1 + \sin \theta)} \\ &= \frac{\sin^2 \theta + \sin \theta}{\cos \theta(1 + \sin \theta)} \\ &= \frac{\sin \theta(1 + \sin \theta)}{\cos \theta(1 + \sin \theta)} \\ &= \frac{\sin \theta}{\cos \theta} \\ &= \tan \theta \\ &= \text{RHS} \end{aligned}$	<p>✓ common denominator/ <i>gemene noemer</i></p> <p>✓ $\sin^2 \theta + \cos^2 \theta$</p> <p>✓ simplification/ <i>vereenv.</i></p> <p>✓ factors/ <i>faktore</i></p> <p>✓ $\frac{\sin \theta}{\cos \theta}$</p> <p style="text-align: right;">(5)</p>
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<p>5.4</p>	$\begin{aligned} 3 \sin x &= 2 \tan x. \\ 3 \sin x &= 2 \times \frac{\sin x}{\cos x} \\ 3 \sin x \cos x &= 2 \sin x \\ 3 \sin x \cos x - 2 \sin x &= 0 \\ \sin x (3 \cos x - 2) &= 0 \\ \sin x &= 0 \\ x &= 360^\circ k, \quad k \in Z \\ \text{or} \\ x &= 180^\circ + 360^\circ k, \quad k \in Z \\ \text{or} \end{aligned}$	<p>✓ $\frac{\sin x}{\cos x}$</p> <p>✓ factors/ <i>faktore</i></p> <p>✓ both equations/ <i>beide vergelykings</i></p> <p>✓ both general solutions/ <i>beide algemene oplossings</i></p>
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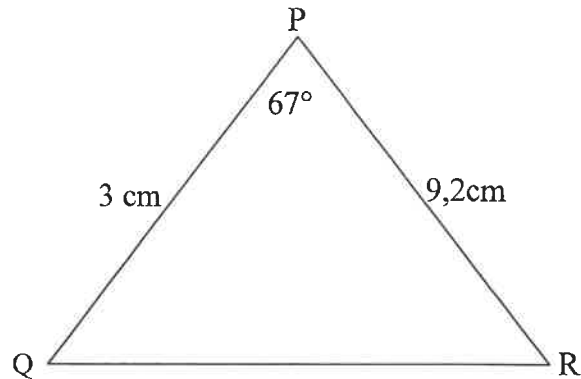
	$\cos x = \frac{2}{3}$ $x = 48,19^\circ + 360^\circ \cdot k, k \in Z$ <p>or</p> $x = 311,81^\circ + 360^\circ \cdot k, k \in Z$ <p>OR/ OF</p> $3 \sin x = 2 \tan x.$ $3 \sin x = 2 \times \frac{\sin x}{\cos x}$ $3 \sin x \cos x = 2 \sin x$ $3 \sin x \cos x - 2 \sin x = 0$ $\sin x (3 \cos x - 2) = 0$ $\sin x = 0$ $x = 180^\circ \cdot k, k \in Z$ $\cos x = \frac{2}{3}$ $x = \pm 48,19^\circ + 360^\circ \cdot k, k \in Z$	<p>✓ both general solutions/ <i>beide algemene oplossings</i> ✓ $k \in Z$</p> <p>(6)</p> <p>✓ $\frac{\sin x}{\cos x}$</p> <p>✓ factors/ <i>faktore</i></p> <p>✓ both equations/ <i>beide vergelykings</i> ✓ general solution/ <i>algemene oplossing</i> ✓ both general solutions/ <i>beide algemene oplossings</i> ✓ $k \in Z$</p> <p>(6)</p> <p>[28]</p>
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QUESTION/VRAAG 6

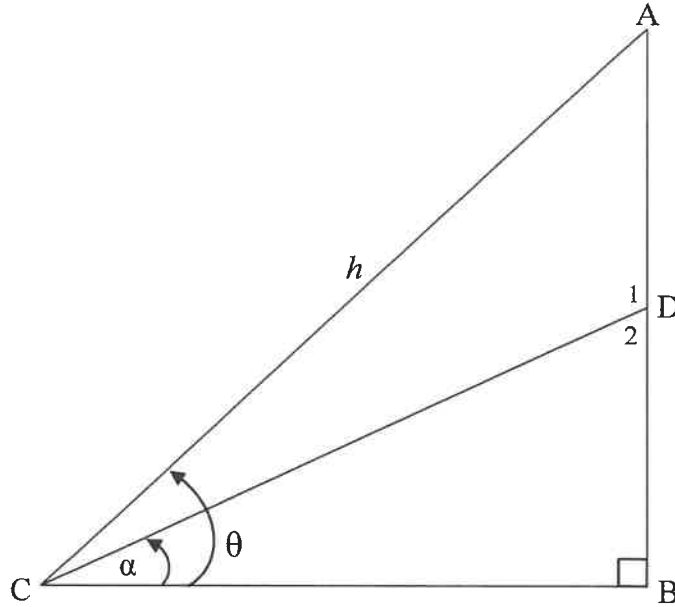


6.1	$-3 \leq y \leq 3$ or/ of $y \in [-3;3]$	✓ end points/ eindpunte ✓ notation/ notasie (2)
6.2	$c = 1$	✓ answer/ antwoord (1)
6.3	$a = 3, b = 1$	✓ $a = 3$ ✓ $b = 1$ (2)
6.4	$Q(122^\circ; -1,6)$	✓ x- value/ waarde ✓ y- value/ waarde (2)
6.5	$K(-45^\circ; \frac{3\sqrt{2}}{2})$ $M(-45^\circ; -1)$ $KM = \frac{3\sqrt{2}}{2} + 1$ $= \frac{3\sqrt{2} + 2}{2}$ $= 3,12$	✓ coordinates of/ koördinate van K ✓ length of/ lengte van KM (2)
6.6	$f(x) = 3 \cos(\theta - 45^\circ)$	✓ 3 ✓ -45° (2)
		[11]

QUESTION/VRAAG 7



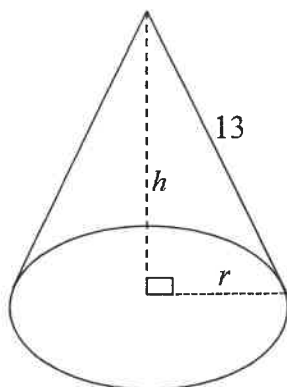
7.1	$QR^2 = PR^2 + PQ^2 - 2PR \cdot PQ \cos \hat{P}$ $QR^2 = (3)^2 + (9,2)^2 - 2(3)(9,2)\cos 67^\circ$ $QR = \sqrt{(3)^2 + (9,2)^2 - 2(3)(9,2)\cos 67^\circ}$ $QR = 8,49\text{cm}$	<p>✓ using cos rule/ gebruik cos reël</p> <p>✓ substitution/ vervanging</p> <p>✓ answer/ antwoord</p> <p>(3)</p>
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7.2.1	$\hat{A}CD = \theta - \alpha$	✓ answer/antw. (1)
7.2.2	$\hat{D}_1 = 90^\circ + \alpha$ $\frac{\sin(90^\circ + \alpha)}{h} = \frac{\sin(\theta - \alpha)}{AD}$ $\frac{\cos \alpha}{h} = \frac{\sin(\theta - \alpha)}{AD}$ $AD = \frac{h \sin(\theta - \alpha)}{\cos \alpha}$	✓ $\hat{D}_1 = 90^\circ + \alpha$ ✓ $\frac{\sin(90^\circ + \alpha)}{h}$ ✓ $\frac{\sin(\theta - \alpha)}{AD}$ ✓ $\sin(90^\circ + \alpha) = \cos \alpha$ (4)
7.2.3	$AD = \frac{17 \sin(58^\circ - 23^\circ)}{\cos 23^\circ}$ $AD = 10,59 \text{ units}$	✓ subst/ verv. ✓ answer/antw. (2)
7.2.4	$\text{Area of } \triangle ADC = \frac{1}{2} \times AD \times AC \times \sin \hat{A}$ $= \frac{1}{2} \times 10,59 \times 17 \times \sin 32^\circ$ $= 47,70 \text{ unit}^2$ <p>OR/ OF</p>	✓ correct area rule/ korrekte area reël ✓ 32° ✓ answer/antw. (3)

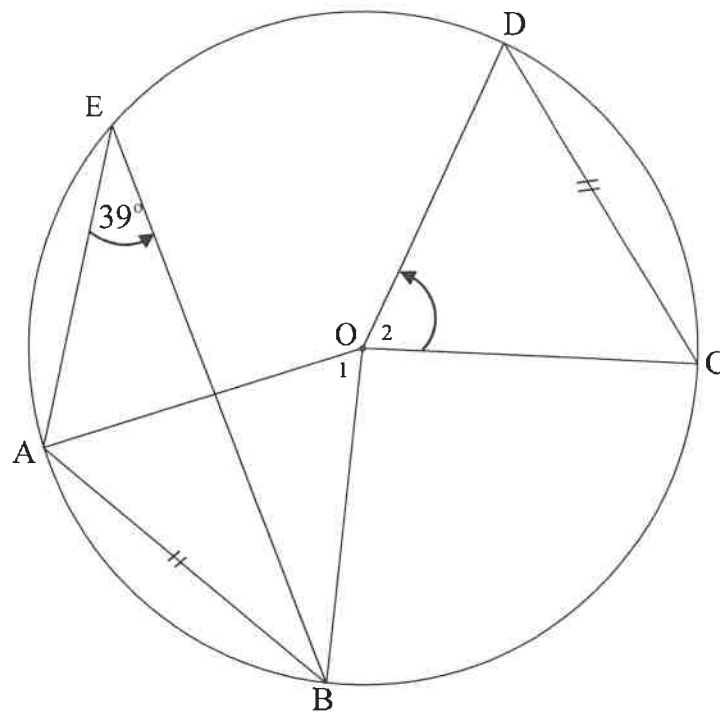
$\sin 58^\circ = \frac{AB}{17}$ $AB = 17 \sin 58^\circ$ $= 14,41682\dots$ $BD = 14,41682\dots - 10,59289\dots = 3,82393\dots$ $\sin 23^\circ = \frac{3,82393\dots}{CD}$ $CD = \frac{3,82393\dots}{\sin 23^\circ}$ $= 9,78660\dots$ $\text{Area of } \triangle ADC = \frac{1}{2} \times CD \times AC \times \sin 35^\circ$ $= \frac{1}{2} \times 9,78660\dots \times 17 \times \sin 35^\circ$ $= 47,71 \text{ unit}^2$	<p>✓ length of BD / lengte van BD</p> <p>✓ length of CD / lengte van CD</p> <p>✓ answer/antw. (3)</p> <p>[28]</p>
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QUESTION/VRAAG 8

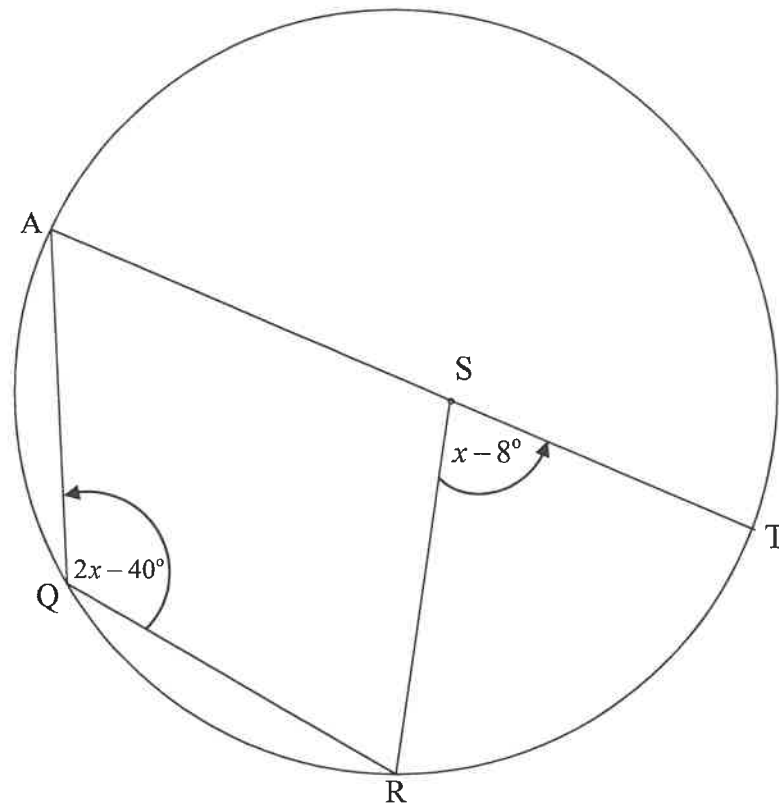


8.1	$r^2 = 13^2 - h^2 \text{ (Pythagoras)}$ $r^2 = 169 - h^2$ $V = \frac{1}{3} Ah$ $= \frac{1}{3} \pi r^2 h$ $= \frac{1}{3} \pi (169 - h^2) h$ $= \frac{169\pi h - \pi h^3}{3}$	<p>✓ using theorem of pythagoras/ gebruik stelling van pythagoras</p> <p>✓ $r^2 = 169 - h^2$</p> <p>✓ substitution/ vervanging</p> <p>✓ simplification/ vereenvoudig</p> <p>(4)</p>
8.2	$r = \sqrt{13^2 - 12^2} \text{ (Pythagoras)}$ $= 5$ <p>Total surface area/ buite oppervlakte = $\pi r^2 + \pi r s$</p> $= \pi(5^2) + \pi(5)(13)$ $= 90\pi$ $= 282,74 \text{ cm}^2$	<p>✓ value of/ waarde van r</p> <p>✓ subst. / verv.</p> <p>✓ answer/ antwoord</p> <p>(3)</p> <p>[7]</p>

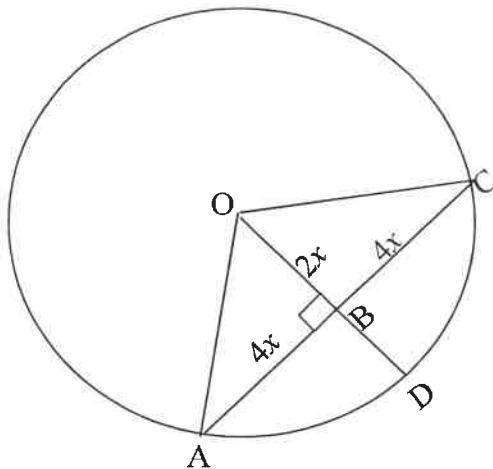
QUESTION/VRAAG 9



9.1.1	$\hat{O}_1 = 78^\circ$ [angle at centre = $2 \times \angle$ at circumference] [middelpuntshoek = $2 \times$ omtrekshoek]	✓ S ✓ R (2)
9.1.2	$\hat{O}_2 = 78^\circ$ [equal chords; equal \angle^s / gelyke koorde; gelyke hoeke]	✓ S ✓ R (2)

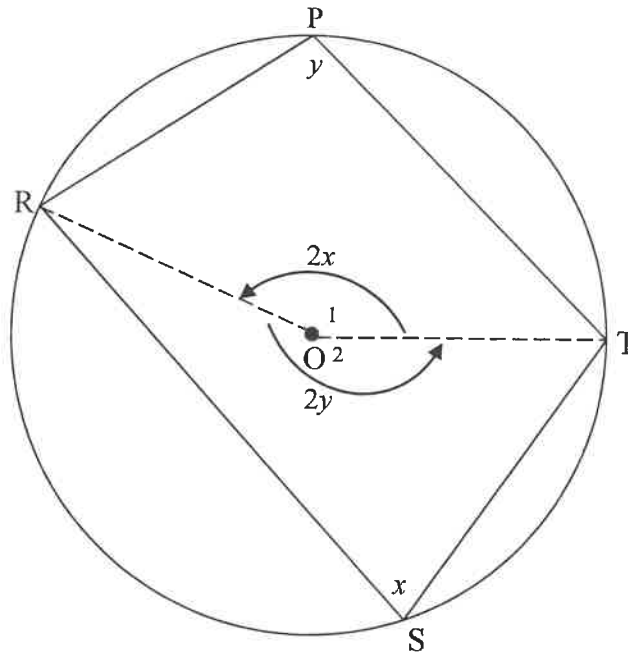


<p>9.2</p>	$x - 8^\circ + 180^\circ = 2(2x - 40^\circ)$ <p style="text-align: center;">[angle at centre = $2 \times \angle$ at circumference/ middelpunt shoek = $2 \times$ omtrekshoek]</p> $4x - 80^\circ = 172^\circ + x$ $3x = 252^\circ$ $x = 84^\circ$ <p>OR/OF</p> <p>Join T and R/ verbind T en R</p> $\hat{T} = 180^\circ - (2x - 40^\circ)$ <p style="text-align: center;">[opp \angle's of cyclic quad/ teenoorst. \angle^e van koordevierhoek]</p> $\hat{R} = \hat{T} = 220^\circ - 2x$ <p style="text-align: center;">[\angle^s opp. = sides / \angle^s teoor gelyke sye]</p> $x - 8^\circ + 220^\circ - 2x + 220^\circ - 2x = 180^\circ$ <p style="text-align: center;">[sum of int \angle^s of Δ] [som binne \angle^e van Δ]</p> $-3x = -252^\circ$ $x = 84^\circ$	<p>✓ S ✓ R</p> <p>✓ simplification/ vereenvoudiging</p> <p>✓ answer/ antwoord</p> <p style="text-align: right;">(4)</p> <p>✓ S ✓ R</p> <p>✓ S</p> <p>✓ answer/ antwoord</p> <p style="text-align: right;">(4)</p>
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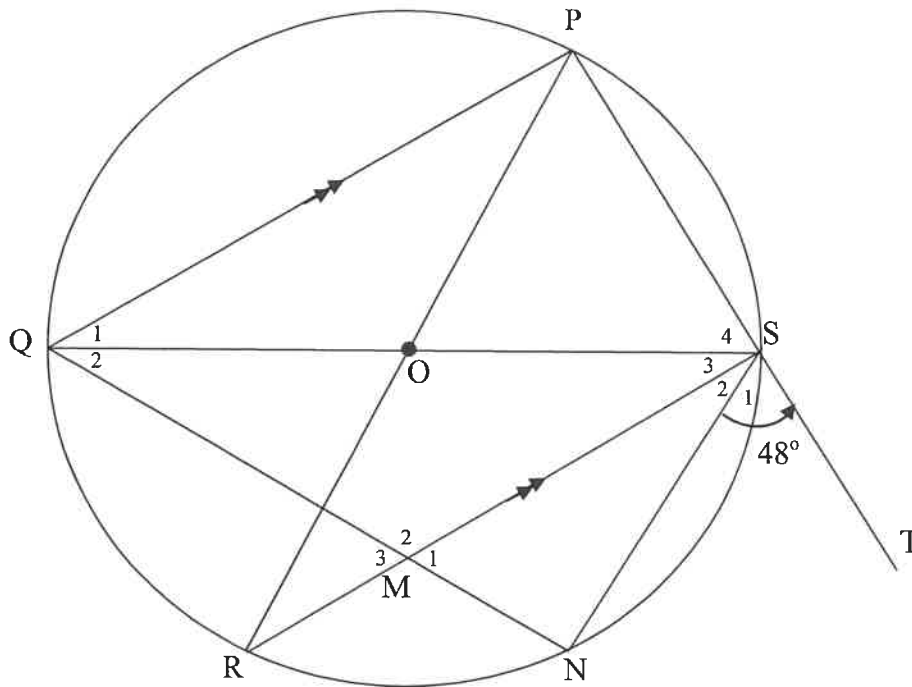


9.3	$AB = BC = 4x \quad \left[\begin{array}{l} \text{line from centre } \perp \text{ to chord /} \\ \text{lyn van middelpunt } \perp \text{ aan koord} \end{array} \right]$ $OA^2 = (4x)^2 + (2x)^2 \quad [\text{Pythagoras}]$ $OA = \sqrt{16x^2 + 4x^2}$ $= \sqrt{20x^2}$ $= 2\sqrt{5}x$ $OD = OA = 2\sqrt{5}x \quad [\text{radii}]$ $BD = 2\sqrt{5}x - 2x$ $= 2x(\sqrt{5} - 1)$	<p>✓ S ✓ R</p> <p>✓ Substitution/ vervanging</p> <p>✓ length of OA / lente van OA</p> <p>✓</p> $BD = 2\sqrt{5}x - 2x$ <p>(5)</p> <p>[13]</p>
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QUESTION/VRAAG 10



<p>10.1</p>	<p>Construction: Draw radii OR and OT <i>Konstruksie: teken raduise OR en OT</i></p> <p>Let/ laat: $\hat{S} = x$ and/en $\hat{P} = y$</p> <p>$\hat{O}_1 = 2\hat{S}$ [angle at centre = 2 times angle at circumference/ <i>middelpuntshoek = 2 keer omtrekshoek</i>]</p> <p>$\hat{O}_1 = 2x$</p> <p>Similarly/ <i>in die selfde manier</i>: $\hat{O}_2 = 2y$</p> <p>$2x + 2y = 360^\circ$ [<i>anlges around a pt / hoeke om 'n punt</i>] $x + y = 180^\circ$</p> <p>$\therefore \hat{S} + \hat{P} = 180^\circ$</p>	<p>✓ construction/ <i>konstruksie</i></p> <p>✓ S ✓ R</p> <p>✓ S</p> <p>✓ S/R</p> <p>(5)</p>
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<p>10.2.1(a)</p>	<p>$\hat{Q} = \hat{S}_1 = 48^\circ$ [ext \angle of cyclic quad/ buite \angle van 'n koodervierhoek] $\hat{Q}_1 = \hat{Q}_2 = 24^\circ$ [QS bisects/<i>halveer</i> P\hat{Q}N]</p>	<p>✓ S ✓R ✓ S (3)</p>
<p>10.2.1(b)</p>	<p>$\hat{R} = \hat{Q}_1 = 24^\circ$ [\angle^s in the same segment/<i>in dieselfde segment</i>]</p>	<p>✓ S ✓R (2)</p>
<p>10.2.1(c)</p>	<p>$\hat{M}_1 = \hat{Q} = 48^\circ$ [corresp/<i>ooreenkomst</i> \angle^s, PQ SR] OR/OF $\hat{S}_3 = \hat{Q}_1 = 24^\circ$ [alt \angle^s/<i>ooreenkomst</i> \angle^s, PQ SR] $\hat{M}_1 = 48^\circ$ [ext \angle of Δ/<i>buite \angle van Δ</i>]</p>	<p>✓ S ✓R (2) ✓ S / R ✓ answer/<i>antwoord</i> (2)</p>
<p>10.2.2</p>	<p>$\hat{M}_1 = \hat{S}_1 = 48^\circ$ \therefore ST is a tangent to circle MNS. [converse tan–chord theorem] \therefore ST is 'n raaklyn aan MNS [omgekrd raaklyn–koord st.]</p>	<p>✓ S ✓ R (2) [14]</p>

	<p> $\hat{RPT} = 90^\circ$ [\angle in the semi circle/ <i>in dieselfde segment</i>] $\hat{NPT} = 90^\circ + x$ $\hat{PAT} = \hat{NPT} = 90^\circ + x$ [tan – chord theorem / <i>raaklyn-koord st</i> $\hat{P}_3 = x$ [\angle^s opp. = sides / \angle^s <i>teoor gelyke sye</i>] $\hat{S}_1 = 90^\circ$ [Line from centre \perp to chord / <i>lyn van mdpt \perp aan koord</i> $\therefore \hat{O}_2 = 90^\circ - x$ [<i>sum of int \angle^s of Δ</i>] $\hat{ROS} = \hat{O}_1 + \hat{O}_2$ $= 2x + 90^\circ - x$ $= 90^\circ + x$ $\therefore \hat{ROS} = \hat{PAT}$ </p>	<p> \checkmarkS \checkmarkS \checkmarkR \checkmarkS/R \checkmarkS (5) </p>
		<p>[10]</p>

TOTAL/TOTAAL: 150