



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

JUNE 2018

MATHEMATICS P2

MARKS: **150**

TIME: **3 hours**



This question paper consists of 14 pages, including a 1 page information sheet and a special answer book.

INSTRUCTIONS AND INFORMATION

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams graphs, et cetera which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary round off your answers to TWO decimal places, unless stated otherwise.
6. Diagrams are not necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

A class of 15 learners was given a test out of 100. The marks obtained by the learners are as follows:

34 58 26 44 28 29 36 49 54 43 45 59 37 29 48

- 1.1 Calculate the mean mark for these learners. (2)
 - 1.2 Calculate the standard deviation. (2)
 - 1.3 How many learners got marks that are within one standard deviation of the mean? (3)
 - 1.4 Calculate the semi-IQR. (3)
- [10]**

QUESTION 2

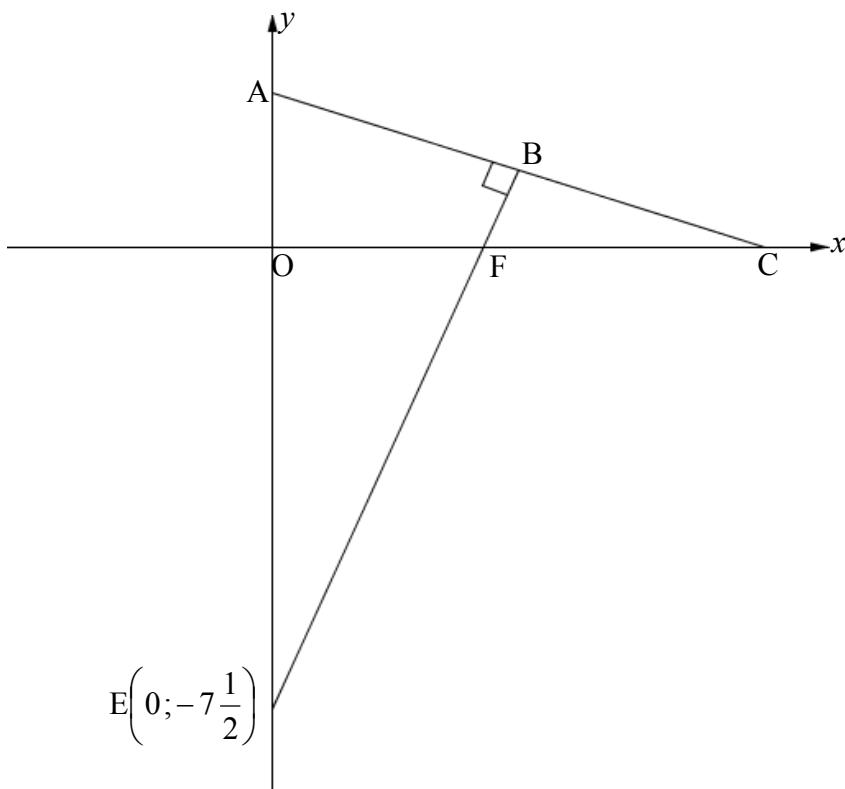
A group of 64 learners wrote an English essay and the time taken to complete the task was recorded as follows:

Time (in minutes)	Frequency	Cumulative frequency
$5 \leq t < 10$	3	
$10 \leq t < 15$	5	
$15 \leq t < 20$	y	
$20 \leq t < 25$	16	
$25 \leq t < 30$	15	
$30 \leq t < 35$	17	
$35 \leq t < 40$	y	

- 2.1 Calculate the value of y . (2)
 - 2.2 Complete the cumulative frequency table. (2)
 - 2.3 Draw an ogive (cumulative frequency curve) to represent the data on the grid provided in the ANSWER BOOK. (3)
 - 2.4 Use your graph to estimate the number of learners who completed the task after 33 minutes. (3)
- [10]**

QUESTION 3

In the diagram below, the straight line AC is drawn having A and C, y- and x- intercepts respectively. The equation of AC is $x + py = p$, $p > 0$. It is also given that $OC = 4OA$. The straight line EB is drawn. B is the point on AC such that $EB \perp AC$. EB cuts the y-axis at $E\left(0; -7\frac{1}{2}\right)$ and x-axis at F.



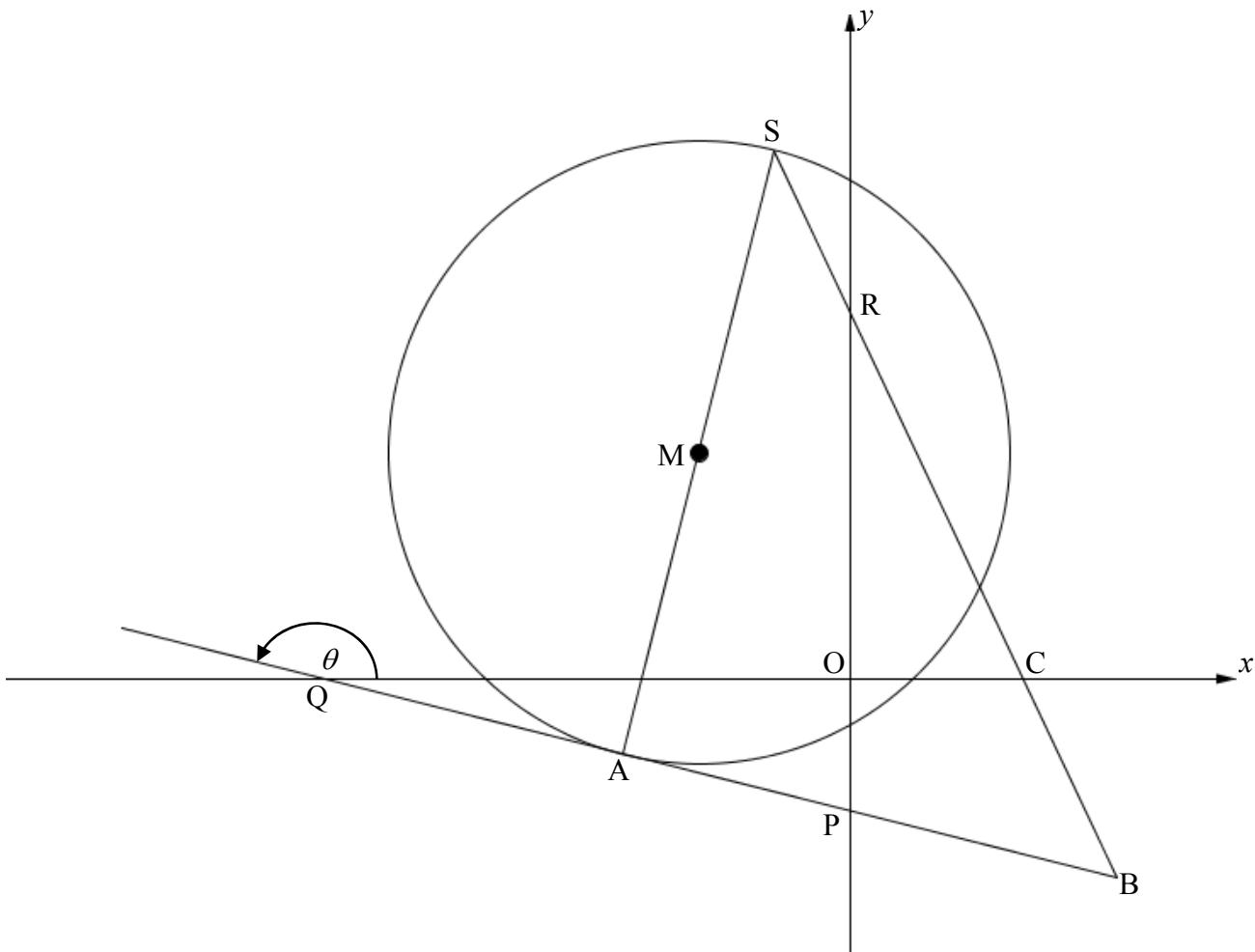
- 3.1 Calculate the coordinates of A. (2)
 - 3.2 Calculate the value of p . (4)
 - 3.3 Determine the equation of EB in the form $y = mx + c$ (2)
 - 3.4 Calculate the coordinates of B. (4)
 - 3.5 Calculate the coordinates of F. (2)
 - 3.6 Calculate area of quadrilateral AOFB. (5)
 - 3.7 Write down the length of the radius of the circle passing through F, B and C. (1)
 - 3.8 Hence, write down the equation of the circle passing through F, B and C if the coordinates of the midpoint of FC are $\left(\frac{47}{16}; 0\right)$. (2)
- [22]

QUESTION 4

In the diagram below, the equation of the circle with centre M is $x^2 + y^2 + 6x - 8y - 1 = 0$.

AS is the diameter of the circle. The equation of the tangent to the circle at A is $y = -\frac{1}{5}x + k$

Line SRCB is drawn.



- 4.1 Determine the coordinates of M. (3)
 - 4.2 Write down the length of the radius of the circle. (Leave your answer in simple surd form.) (1)
 - 4.3 Determine the equation of the diameter AS. (3)
 - 4.4 Determine the coordinates of A. (6)
 - 4.5 Hence or otherwise, calculate the value of k . (2)
 - 4.6 If it is further given that $\hat{O}RC = 11^\circ$, prove that RCPQ is a cyclic quadrilateral. (Round off your answer to the nearest degree.) (4)
- [19]

QUESTION 5

5.1 Given that $2 \sin 27^\circ = t$, express each of the following in terms of t :

5.1.1 $\sin 54^\circ$ (4)

5.1.2 $\tan 513^\circ \cdot \cos 27^\circ$ (3)

5.1.3 $\cos 87^\circ$ (4)

5.2 Simplify, without using a calculator:

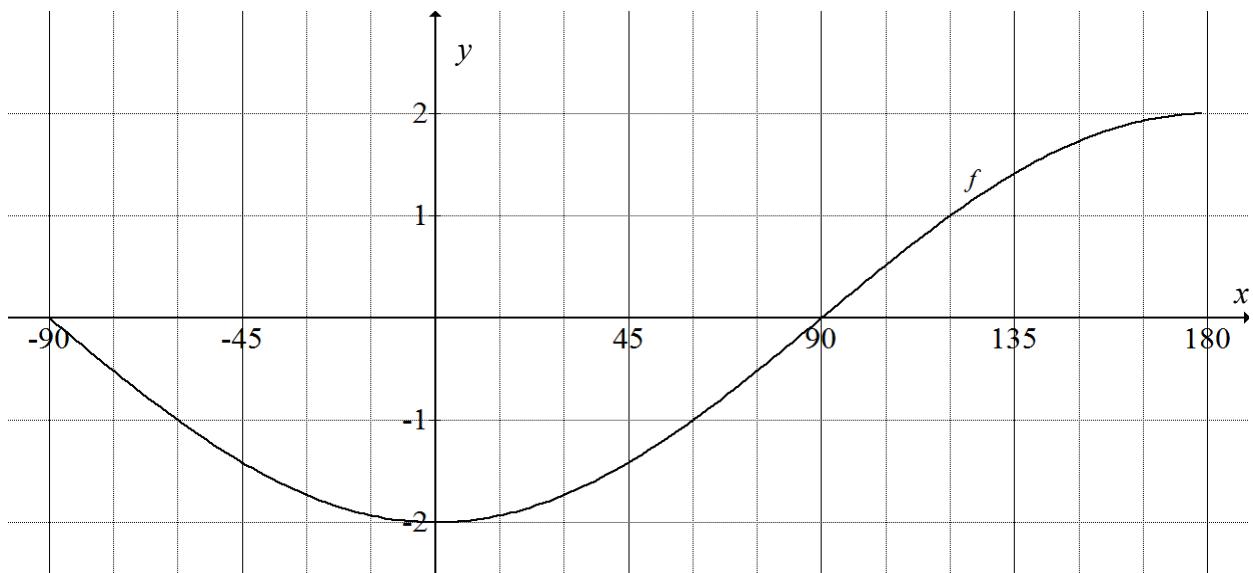
$$\frac{\sin(-2\alpha)\cos(90^\circ + \alpha)}{\sin(-\alpha + 360^\circ)\cos(-\alpha - 180^\circ)} \quad (6)$$

5.3 Determine the general solution of the equation: $9 \sin^2 x - 4 \cos^2 x = 0$ (5)

[22]

QUESTION 6

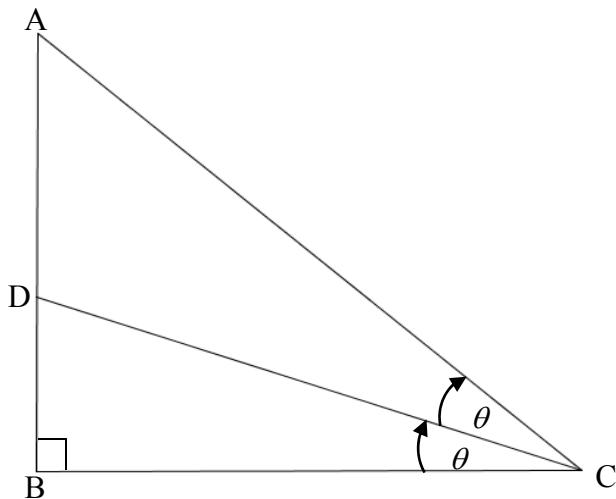
In the diagram below, the graph of $f(x) = -2 \cos x$ is drawn for the interval $-90^\circ \leq x \leq 180^\circ$



- 6.1 Write down the amplitude of f . (1)
 - 6.2 Write down the range of $f(x) + 3$. (2)
 - 6.3 On the same system of axes draw the graph of $g(x) = \sin(x + 30^\circ)$. (3)
 - 6.4 Determine the values of x in the interval $-90^\circ \leq x \leq 90^\circ$ for which $x \cdot g(x) < 0$? (2)
 - 6.5 Write the equation of h , where h is formed by shifting g , 60° to the left and 2 units downwards. (Leave your answer in simplified form.) (3)
- [11]

QUESTION 7

In the diagram below, ABC is a right angled-triangle with $\hat{B} = 90^\circ$. The straight line CD bisects \hat{ACB} and cuts AB at D. $\hat{DCB} = \theta$.



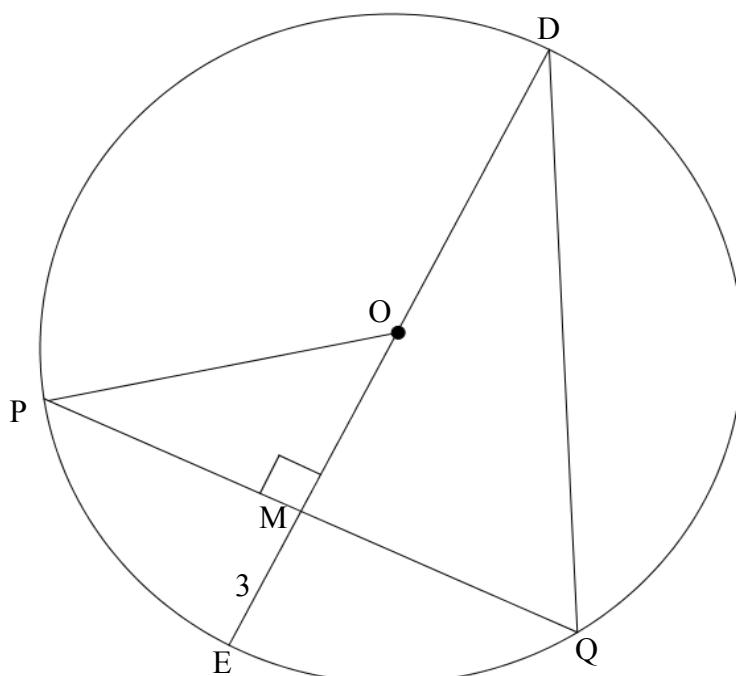
- 7.1 Write down the size of \hat{A} in terms of θ . (1)
- 7.2 Write down the ratio of $\sin \theta$. (1)
- 7.3 If it is further given that $\frac{DB}{AD} = \frac{1}{2}$ hence, or otherwise, show that $2\cos 2\theta - 1 = 0$. (5)
[7]

Give reasons for your statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

- 8.1 Complete:
A line drawn from the centre of the circle to the midpoint of the chord is ... (1)

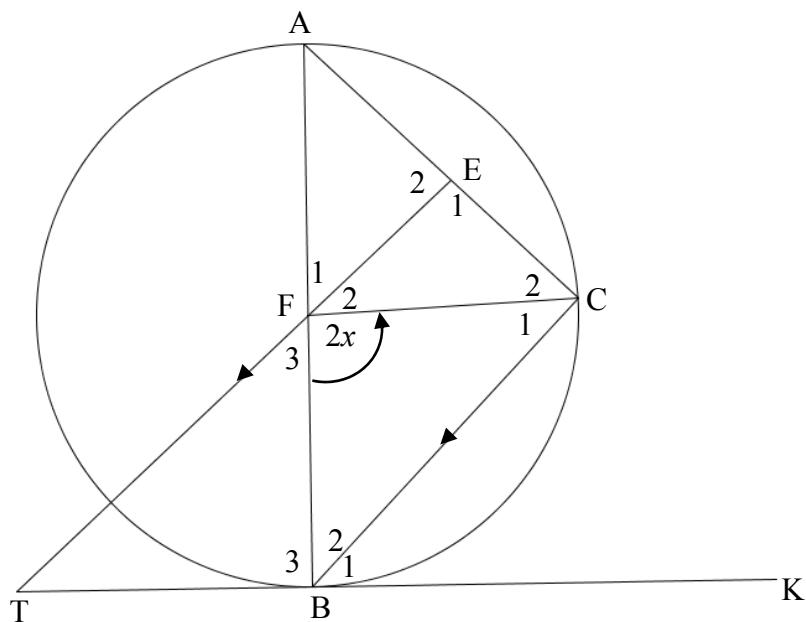
- 8.2 In the diagram drawn below, O is the centre of the circle. PQ is a chord and DE is the diameter. $PQ = 12$ units, $DM = 2x$ and $ME = 3$ units.



- 8.2.1 Write down the length of MO in terms of x . (1)
8.2.2 Calculate the value of x . (4)
8.2.3 Hence, calculate the length of DQ. (Leave your answer in simplest surd form.) (3)
[9]

QUESTION 9

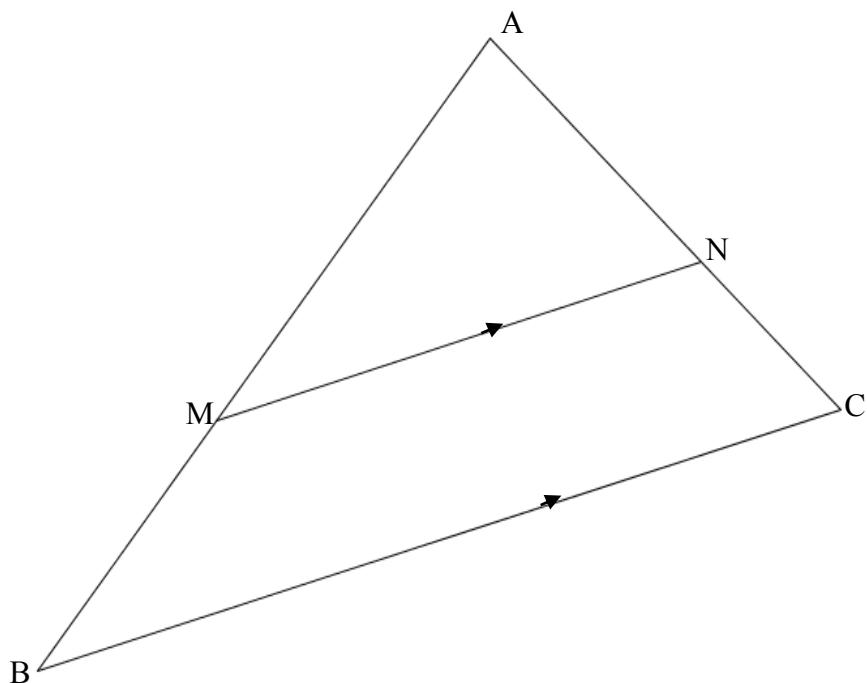
In the diagram below, F is the centre of the circle. TBK is a tangent to the circle at B. E and F are points on AC and AB respectively. EF is produced to T. $BC \parallel TE$ and $\hat{BFC} = 2x$.



- 9.1 Name with reasons, four angles each equal to x . (8)
9.2 Prove that ATBE is a cyclic quadrilateral. (2)
[10]

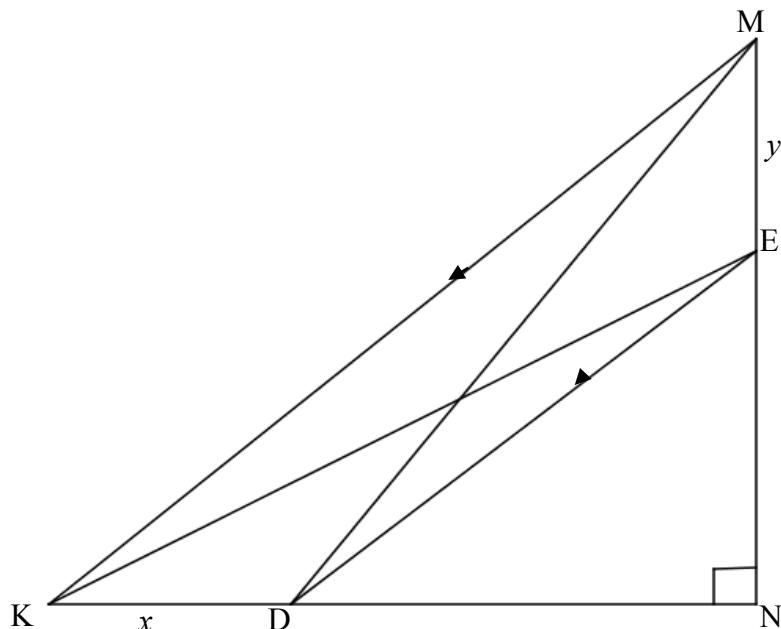
QUESTION 10

10.1 $\triangle ABC$ is drawn such that MN is parallel to BC .



Prove that $\frac{AM}{MB} = \frac{AN}{NC}$ (5)

- 10.2 In $\triangle MNK$; $\hat{N} = 90^\circ$ and D is a point on KN and E is a point on MN such that $DE \parallel KM$.
 $ND : DK = 2 : 1$, $ME = y$ and $KD = x$.

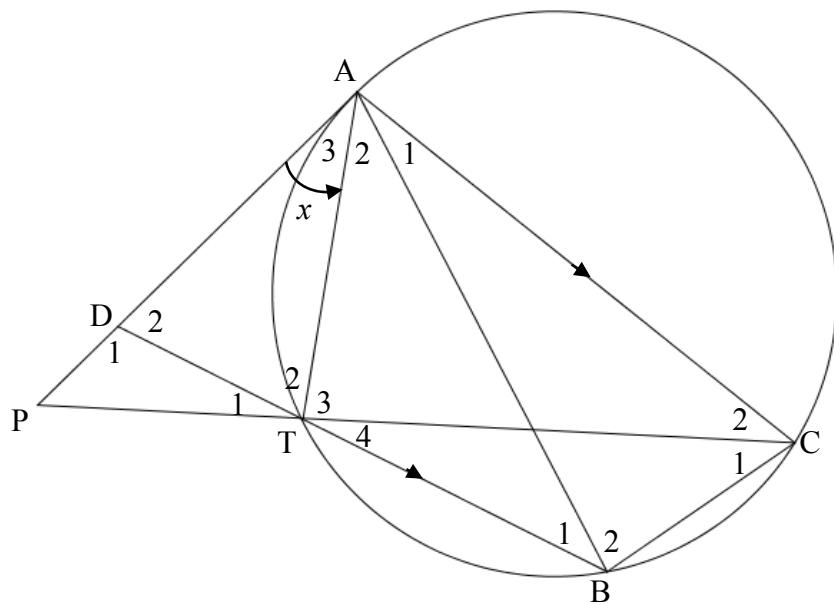


10.2.1 Determine the length of KM^2 in terms of x and y . (4)

10.2.2 Show that $\frac{DM^2 + KE^2}{KM^2} = \frac{13}{9}$ (4)
[13]

QUESTION 11

In the diagram, PA is a tangent to the circle ACBT at A. CT and AD are produced to meet at P. BT is produced to cut PA at D. AC, CB, AB and AT are joined. $AC \parallel BD$.



- 11.1 Prove that $\Delta ABC \parallel \Delta ADT$. (6)
- 11.2 Prove that PT is a tangent to the circle ADT at T. (4)
- 11.3 Prove that $\Delta APT \parallel \Delta TPD$ (3)
- 11.4 If $AD = \frac{2}{3}AP$, show that $AP^2 = 3PT^2$. (4)
[17]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni)$$

$$A = P(1-ni)$$

$$A = P(1-i)^n$$

$$A = P(1+i)^n$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r-1}; r \neq 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{area } \Delta ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2 \sin^2 \alpha \\ 2 \cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$



**NATIONAL
SENIOR CERTIFICATE/
*NASIONALE
SENIOR SERTIFIKAAT***

GRADE/GRAAD 12

JUNE/JUNIE 2018

**MATHEMATICS P2/WISKUNDE V2
MARKING GUIDELINE/NASIENRIGLYN**

MARKS /PUNTE: 150

This marking guideline consists of 20 pages./
Hierdie nasienriglyn bestaan uit 20 bladsye.

NOTE:

- If a candidate answers a question **TWICE**, only mark the **FIRST** attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in **ALL** aspects of the marking guideline.
- Assuming answers/ values in order to solve a problem is **NOT** acceptable.

NEEM KENNIS:

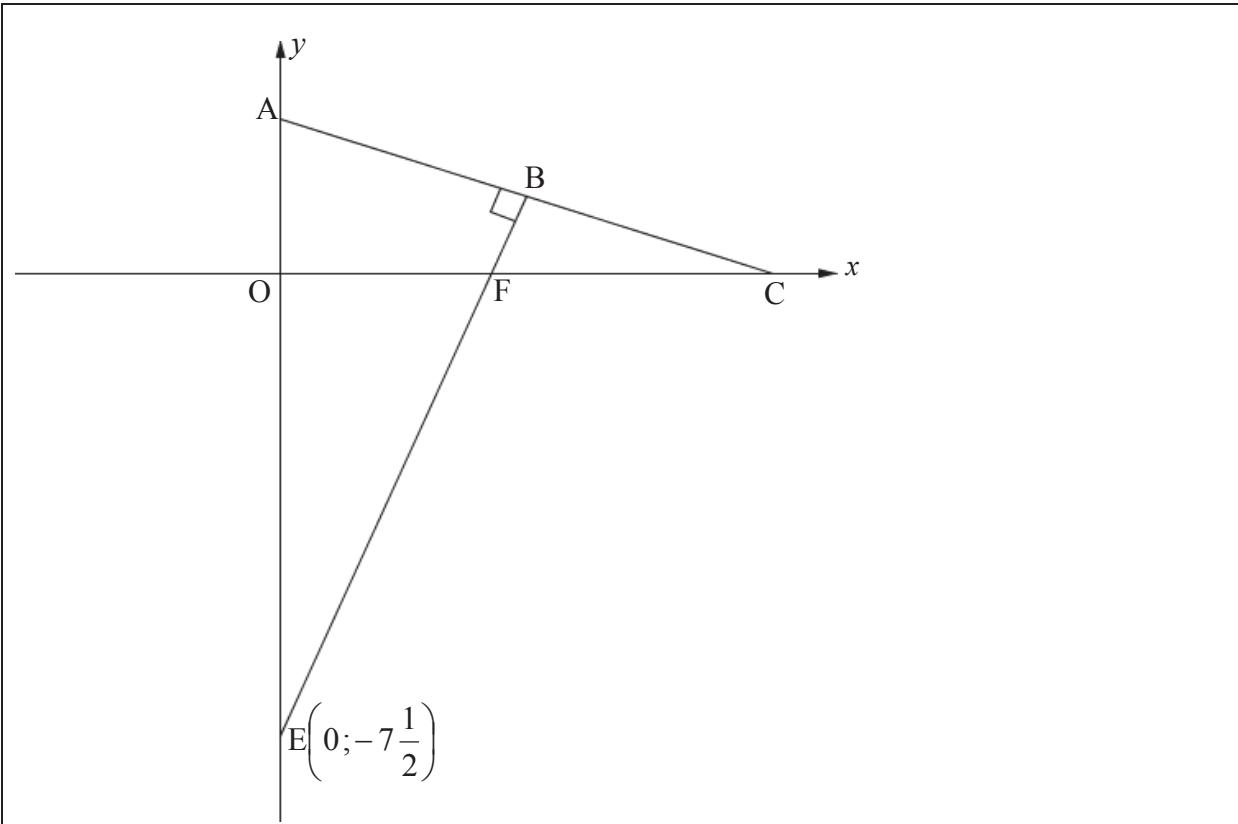
- As 'n kandidaat 'n vraag **TWEEKEER** beantwoord, merk slegs die **EERSTE** poging
- As 'n kandidaat 'n poging om 'n vraag te beantwoord deurgehaal het en nie weer probeer het nie, merk die deurgehaalde poging.
- Volgehoue akkuraatheid is van toepassing op **ALLE** aspekte van die nasienriglyn.
- Aanvaarding van antwoorde/waardes om sodoende probleme op te los is **NIE** aanvaarbaar **NIE**.

GEOMETRY/MEETKUNDE	
S	<p>A mark for a correct statement. (A statement mark is independent of a reason.)</p> <p><i>'n Punt vir korrekte stelling/bewering. ('n Stelling/bewering punt is onafhanklik van die rede.)</i></p>
R	<p>A mark for a correct reason. (A reason mark may only be awarded if the statement is correct.)</p> <p><i>'n Punt vir korrekte rede. ('n Rede punt mag slegs toegeken word indien die stelling/bewering korrek is.)</i></p>
S/R	<p>Award a mark if statement and reason are both correct.</p> <p><i>Ken 'n punt toe indien die stelling/bewering sowel as die rede korrek is.</i></p>

QUESTION/VRAAG 1			
1.1	$\bar{x} = \frac{\sum x}{n}$ $= \frac{619}{15}$ $= 41,27$	Answer only full marks/ Slegs antwoord volpunte	✓ 619 ✓ 41,27 (2)
1.2	$\sigma = 10,63$		✓✓ Answer / antwoord (2)
1.3	$41,27 - 10,63 \leq x \leq 41,27 + 10,63$ $30,64 \leq x \leq 51,90$ $\therefore 8$ learners/leerders		✓ both c.v's / beide k.w's ✓ notation / notasie ✓ 8 learners/leerders (3)
1.4	$Q_1 = 29$ $Q_3 = 49$ $\therefore \text{Semi-IQR/IKW} = \frac{49 - 29}{2}$ $= 10$		✓ Q_1 ✓ Q_3 ✓ answer/antwoord (3)
			[10]

QUESTION/VRAAG 2																										
2.1	$56 + 2y = 64$ $2y = 8$ $y = 4$	Answer only 1 mark Slegs antwoord 1 punt	✓ correct equation korrekte vergelyking ✓ y-value/waarde (2)																							
2.2	<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th>Time (in minutes) <i>Tyd (in minute)</i></th> <th>Frequency / <i>Frekwensie</i></th> <th>Cumulative frequency/ <i>Kumulatiewe frekwensie</i></th> </tr> </thead> <tbody> <tr> <td>$5 \leq t < 10$</td> <td>3</td> <td>3</td> </tr> <tr> <td>$10 \leq t < 15$</td> <td>5</td> <td>8</td> </tr> <tr> <td>$15 \leq t < 20$</td> <td>4</td> <td>12</td> </tr> <tr> <td>$20 \leq t < 25$</td> <td>16</td> <td>28</td> </tr> <tr> <td>$25 \leq t < 30$</td> <td>15</td> <td>43</td> </tr> <tr> <td>$30 \leq t < 35$</td> <td>17</td> <td>60</td> </tr> <tr> <td>$35 \leq t < 40$</td> <td>4</td> <td>64</td> </tr> </tbody> </table>	Time (in minutes) <i>Tyd (in minute)</i>	Frequency / <i>Frekwensie</i>	Cumulative frequency/ <i>Kumulatiewe frekwensie</i>	$5 \leq t < 10$	3	3	$10 \leq t < 15$	5	8	$15 \leq t < 20$	4	12	$20 \leq t < 25$	16	28	$25 \leq t < 30$	15	43	$30 \leq t < 35$	17	60	$35 \leq t < 40$	4	64	✓ 8, 12 and/en 28 ✓ 43, 60 and/en 64 (2)
Time (in minutes) <i>Tyd (in minute)</i>	Frequency / <i>Frekwensie</i>	Cumulative frequency/ <i>Kumulatiewe frekwensie</i>																								
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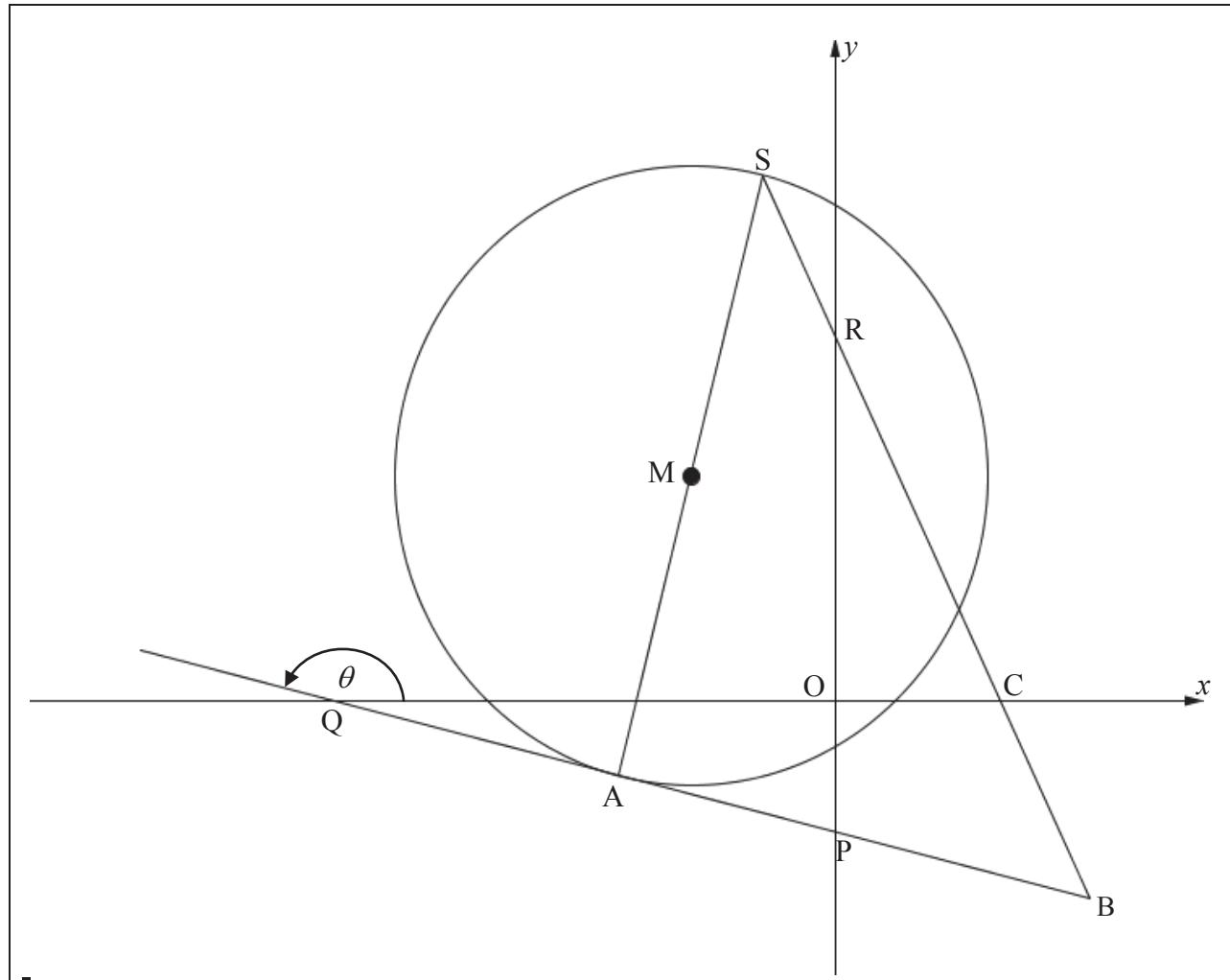
2.3	<p style="text-align: center;"><i>Ogive/Ogief</i></p> <table border="1"> <thead> <tr> <th>Time in minutes / Tyd in minute</th> <th>Cumulative frequency / Kumulatiewe frekwensie</th> </tr> </thead> <tbody> <tr><td>5</td><td>0</td></tr> <tr><td>10</td><td>3</td></tr> <tr><td>15</td><td>8</td></tr> <tr><td>20</td><td>12</td></tr> <tr><td>25</td><td>28</td></tr> <tr><td>30</td><td>43</td></tr> <tr><td>40</td><td>64</td></tr> </tbody> </table>	Time in minutes / Tyd in minute	Cumulative frequency / Kumulatiewe frekwensie	5	0	10	3	15	8	20	12	25	28	30	43	40	64	<ul style="list-style-type: none"> ✓ grounding/anker ✓ plotting against the upper limit/ afsteek teen die boonste limiet ✓ shape / vorm
Time in minutes / Tyd in minute	Cumulative frequency / Kumulatiewe frekwensie																	
5	0																	
10	3																	
15	8																	
20	12																	
25	28																	
30	43																	
40	64																	
2.4	<p>Number of learners / Aantal leerders $\approx 64 - 54 \approx 10$ Accept / Aanvaar [9–11]</p>	<ul style="list-style-type: none"> ✓ method/metode ✓ 54 ✓ answer/antwoord 																
		(3) [10]																

QUESTION/VRAAG 3

3.1	$x + py = p$ $y = -\frac{x}{p} + 1$ $\therefore A(0; 1)$	✓ y-subject of formula <i>y-onderwerp van formule</i> ✓ coordinates of A <i>koördinate van A</i> (2)
3.2	$OA = 1$ $\therefore OC = 4(1) = 4$ $\therefore C(4; 0)$ $m_{AC} = -\frac{1}{p} = \frac{1-0}{0-4}$ $p = 4$	✓ OC ✓ $-\frac{1}{p} = \frac{1-0}{0-4}$ ✓ simplification/vereenvoudiging ✓ <i>p</i> -value/waarde (4)
3.3	$m_{EB} = 4$ $y = 4x - 7\frac{1}{2}$	✓ $m_{EB} = 4$ ✓ equation/vergelyking (2)

3.4	$\begin{aligned} -\frac{x}{4} + 1 &= 4x - 7 \frac{1}{2} \\ -x + 4 &= 16x - 30 \\ 17x &= 34 \\ x &= 2 \\ y &= \frac{1}{2} \end{aligned}$	✓ equating / <i>gelykstel</i> ✓ simplification / <i>vereenvoudiging</i> ✓ <i>x</i> -value/waarde ✓ <i>y</i> -value/waarde (4)
3.5	$\begin{aligned} 4x - 7 \frac{1}{2} &= 0 \\ 4x &= \frac{15}{2} \\ x &= \frac{15}{8} \\ F\left(\frac{15}{8}; 0\right) \end{aligned}$	✓ <i>y</i> = 0 ✓ <i>x</i> -value/waarde (2)
3.6	$\begin{aligned} BF &= \sqrt{\left(\frac{15}{8} - 2\right)^2 + \left(0 - \frac{1}{2}\right)^2} \\ &= \frac{\sqrt{17}}{8} \\ \therefore \text{Area FOAB} &= \text{Area } \Delta AOC - \text{Area } \Delta BFC \\ &= \frac{1}{2} \times 4 \times 1 - \frac{1}{2} \left(\frac{\sqrt{17}}{2}\right) \frac{\sqrt{17}}{8} \\ &= 1,47 \end{aligned}$ <p style="text-align: center;">OR/OF</p> $\begin{aligned} BF &= \sqrt{\left(\frac{15}{8} - 2\right)^2 + \left(0 - \frac{1}{2}\right)^2} \\ &= \frac{\sqrt{17}}{8} \end{aligned}$ $\begin{aligned} \text{Area FOAB} &= \text{Area } \Delta AOF + \text{Area } \Delta ABF \\ &= \frac{1}{2} \times 1 \times \frac{15}{8} + \frac{1}{2} \times \frac{\sqrt{17}}{2} \times \frac{\sqrt{17}}{8} \\ &= 1,47 \end{aligned}$	✓ substitution / <i>vervanging</i> ✓ BF ✓ BC ✓ substitution / <i>vervanging</i> ✓ answer / <i>antwoord</i> ✓ substitution / <i>vervanging</i> ✓ BF ✓ OF ✓ substitution / <i>vervanging</i> ✓ answer / <i>antwoord</i> (5)
3.7	$r = \frac{17}{16}$	✓ answer / <i>antwoord</i> (1)
3.8	$\left(x - \frac{47}{16}\right)^2 + y^2 = \frac{289}{256}$	✓ r^2 ✓ equation / <i>vergelyking</i> (2)
		[22]

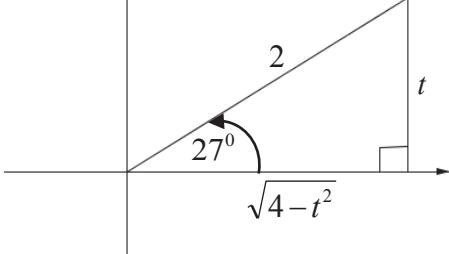
QUESTION/VRAAG 4



4.1	$\begin{aligned}x^2 + 6x + 9 + y^2 - 8y + 16 &= -5 + 9 + 16 \\(x + 3)^2 + (y - 4)^2 &= 26 \\ \therefore M(-3; 4)\end{aligned}$	✓ completing the square <i>kwadraatsvoltooiing</i> ✓ <i>x</i> -value/ <i>waarde</i> ✓ <i>y</i> -value/ <i>waarde</i> (3)
4.2	$r = \sqrt{26}$	✓ answer/ <i>antwoord</i> (1)
4.3	$m_{AS} = 5$ $[SA \perp QB]$ $y - 4 = 5(x + 3)$ $y = 5x + 19$	✓ $m_{AS} = 5$ ✓ subst. /verv. m_{AS} & $(-3; 4)$ ✓ equation/ <i>vergelyking</i> (3)

4.4	$x^2 + 6x + 9 + (5x + 19 - 4)^2 = 26$ $x^2 + 6x + 9 + 25x^2 + 150x + 225 - 26 = 0$ $26x^2 + 156x + 208 = 0$ $x^2 + 6x + 8 = 0$ $(x + 4)(x + 2) = 0$ $x = -4 \text{ or } x \neq -2$ $\therefore y = 5(-4) + 19$ $= -1$ $Q(-4; -1)$	<ul style="list-style-type: none"> ✓ substitution/vervanging ✓ simplification / vereenvoudiging ✓ standard form / standaardvorm ✓ factors / faktore ✓ choosing correct x-value korrekte keuse van x-waarde ✓ y-value/waarde (6)
4.5	$-1 = -\frac{1}{5}(-4) + k$ $\therefore k = -\frac{9}{5}$	<ul style="list-style-type: none"> ✓ substitute/vervang $(-4; -1)$ ✓ answer / antwoord (2)
4.6	$\tan \theta = -\frac{1}{5}$ $\therefore \theta = 169^\circ$ $\hat{\angle}OQP = 11^\circ [\angle s \text{ on a str line}] / [\angle e \text{ op 'n reguitlyn}]$ $\therefore \hat{\angle}OQP = \hat{\angle}RC$ <p>$\therefore RCPQ$ is a cyclic quad [converse \angles same seg]</p> <p>$RCPQ$ is 'n koordevierhoek [omgekeerde \anglee in dieselfde seg]</p>	<ul style="list-style-type: none"> ✓ $\tan \theta = -\frac{1}{5}$ ✓ size of θ / grootte van θ ✓ size of $\hat{\angle}OQP$ / grootte van $\hat{\angle}OQP$ <p style="text-align: right;">✓ R</p> (4)
		[19]

QUESTION/VRAAG 5

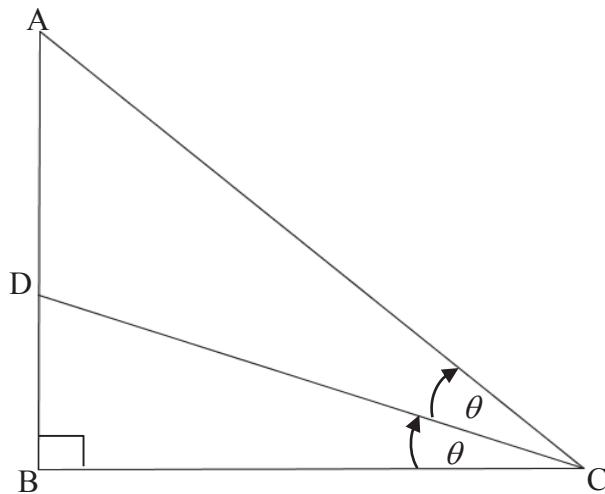
5.1.1	$2 \sin 27^\circ = t$ $\sin 27^\circ = \frac{t}{2}$  $x = \sqrt{4 - t^2}$ $\begin{aligned} \sin 54^\circ &= 2 \sin 27^\circ \cos 27^\circ \\ &= 2 \times \frac{t}{2} \times \frac{\sqrt{4 - t^2}}{2} \\ &= \frac{t\sqrt{4 - t^2}}{2} \end{aligned}$	$\checkmark \sin 27^\circ = \frac{t}{2}$ $\checkmark x = \sqrt{4 - t^2}$ $\checkmark 2 \sin 27^\circ \cos 27^\circ$ $\checkmark \text{substitution/vervanging}$ (4)
5.1.2	$\begin{aligned} \tan 513^\circ \cos 27^\circ &= (-\tan 27^\circ) \cos 27^\circ \\ &= -\frac{\sin 27^\circ}{\cos 27^\circ} \cdot \cos 27^\circ \\ &= -\frac{t}{2} \end{aligned}$	$\checkmark (-\tan 27^\circ)$ $\checkmark -\frac{\sin 27^\circ}{\cos 27^\circ}$ $\checkmark -\frac{t}{2}$ (3)
5.1.3	$\begin{aligned} \cos 87^\circ &= \cos(60^\circ + 27^\circ) \\ &= \cos 60^\circ \cos 27^\circ - \sin 60^\circ \sin 27^\circ \\ &= \frac{1}{2} \cdot \frac{\sqrt{4 - t^2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{t}{2} \\ &= \frac{\sqrt{4 - t^2} - t\sqrt{3}}{4} \end{aligned}$	$\checkmark 60^\circ + 27^\circ$ $\checkmark \text{expansion / uitbreiding}$ $\checkmark \text{subst./verv. } \frac{\sqrt{4 - t^2}}{2} \text{ & } \frac{t}{2}$ $\checkmark \text{subst. / verv. } \frac{1}{2} \text{ & } \frac{\sqrt{3}}{2}$ (4)

5.2	$ \begin{aligned} & \frac{\sin(-2\alpha)\cos(90^\circ + \alpha)}{\sin(-\alpha + 360^\circ)\cos(-\alpha - 180^\circ)} \\ &= \frac{(-\sin 2\alpha)(-\sin \alpha)}{(\sin \alpha)(-\cos \alpha)} \\ &= -\frac{2 \sin \alpha \cos \alpha}{\cos \alpha} \\ &= -2 \sin \alpha \end{aligned} $	✓ $(-\sin 2\alpha)$ ✓ $(-\sin \alpha)$ ✓ $\sin \alpha$ ✓ $(-\cos \alpha)$ ✓ $2 \sin \alpha \cos \alpha$ ✓ $-2 \sin \alpha$ (6)
5.3	$ \begin{aligned} 9 \sin^2 x - 4 \cos^2 x &= 0 \\ (3 \sin x - 2 \cos x)(3 \sin x + 2 \cos x) &= 0 \\ \therefore 3 \sin x &= \pm 2 \cos x \\ \tan x &= \pm \frac{2}{3} \\ x &= 33,69^\circ + 180^\circ \cdot k \text{ or/of } x = 146,31^\circ + 180^\circ \cdot k \quad k \in \mathbb{Z} \end{aligned} $	✓ factors/faktore ✓ both equations <i>beide vergelykings</i> ✓ $\tan x = \pm \frac{2}{3}$ ✓ both / <i>beide</i> $33,69^\circ$ & $146,31^\circ$ / $-33,69^\circ$ ✓ $180^\circ \cdot k$ & $k \in \mathbb{Z}$ (5)
		[22]

QUESTION/VRAAG 6

6.1	Amplitude= 2	✓ answer/antwoord	(1)
6.2	$1 \leq y \leq 5$	✓ min & max/maks ✓ notation/notasie	(2)
6.3		✓ both x-intercepts <i>beide x-afsnitte</i> -30° & 150° ✓ max TP & y-intercept ✓ shape / vorm	(3)
6.4	$-30^\circ < x < 0^\circ$	✓ both c.v.s / <i>beide k.w.s</i> ✓ notation/notasie	(2)
6.5	$\begin{aligned} h(x) &= \sin(x + 90^\circ) - 2 \\ &= \cos x - 2 \end{aligned}$	✓✓ $\sin(x + 90^\circ) - 2$ ✓ $\cos x$	(3)
		[11]	

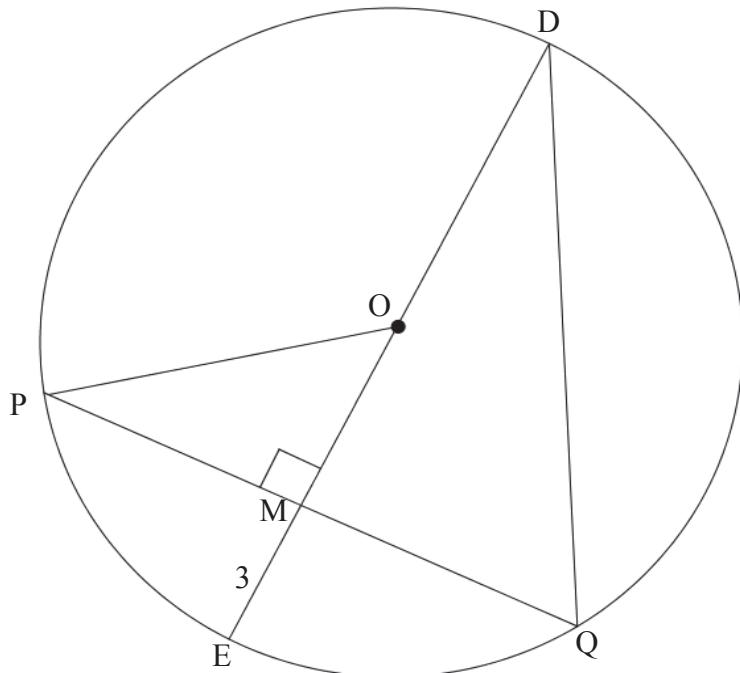
QUESTION/VRAAG 7



7.1	$\hat{A} = 90^\circ - 2\theta$	✓ answer / antwoord	(1)
7.2	$\sin \theta = \frac{DB}{DC}$	✓ answer / antwoord	(1)
7.3	$DC = \frac{DB}{\sin \theta}$ <p>and/en $AD = 2DB = 2DB$</p> $\frac{DC}{\sin(90^\circ - 2\theta)} = \frac{AD}{\sin \theta} \text{ in } \Delta ADC$ $\frac{DB}{\cos 2\theta} = \frac{2DB}{\sin \theta}$ $\frac{DB}{\sin \theta \cdot \cos 2\theta} = \frac{2DB}{\sin \theta}$ $DB \cdot \sin \theta = 2DB \cdot \sin \theta \cdot \cos 2\theta$ $2 \cos 2\theta = 1$ $2 \cos 2\theta - 1 = 0$	✓ AD = 2DB ✓ sine rule/sinusreël in ΔADC ✓ subst. in sine rule / verv. in sinusreël ✓ $\cos 2\theta$ ✓ equation / vergelyking	(5)
			[7]

QUESTION/VRAAG 8

8.1	Perpendicular to the chord / <i>Loodreg op die koord</i>	✓ answer/antwoord	(1)
8.2			



8.2.1	$OM = \frac{2x+3}{2} - 3$ <p>OR</p> $OM = \frac{2x-3}{2}$	✓ answer/antwoord OR/OF ✓ answer/antwoord	(1)
8.2.2	$OQ^2 = OM^2 + QM^2$ $\left(\frac{2x+3}{2}\right)^2 = \left(\frac{2x-3}{2}\right)^2 + 6^2$ $\frac{4x^2 + 12x + 9}{4} = \frac{4x^2 - 12x + 9}{4} + 36$ $24x = 144$ $x = 6$	✓ susbt. in Pyth / verv. in Pyth. ✓ simplification / vereenvoudiging ✓ standard form linear equation <i>Standaardvorm van lineêre vergelyking</i> ✓ x-value/waarde	(4)

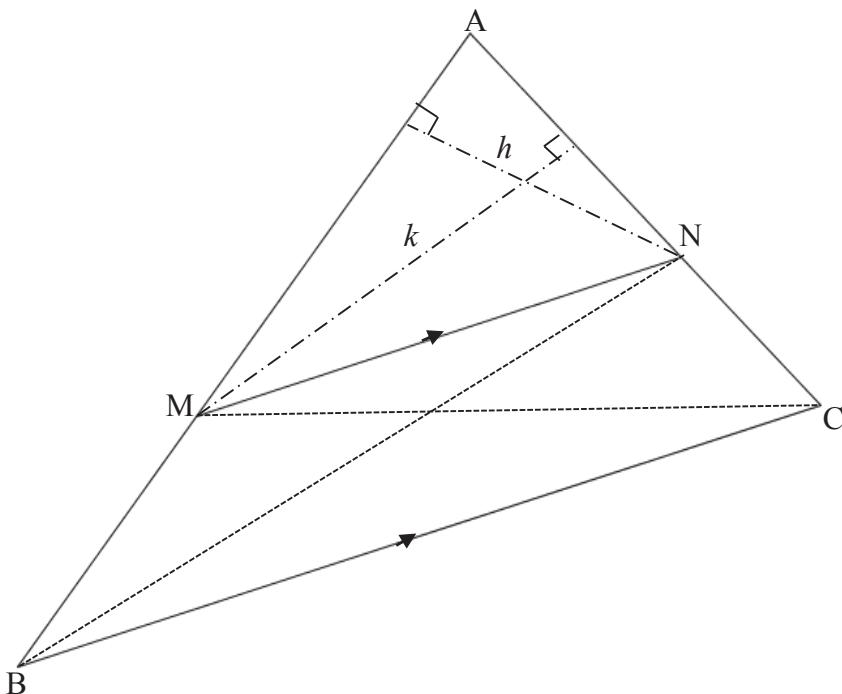
8.2.3	$\begin{aligned} DM &= 12 \\ DQ &= \sqrt{12^2 + 6^2} \\ &= \sqrt{180} \\ &= 6\sqrt{5} \end{aligned}$	<ul style="list-style-type: none">✓ DM✓ subst. in Pyth / verv. in Pyth✓ answer / antwoord	(3)
			[9]

QUESTION/VRAAG 9

9.1	$\hat{A} = x$ [$\angle \text{at centre} = 2 \angle \text{at circumf.} / \text{Middelpunts}\angle = 2 \times \text{Omtreks}\angle$]	<input checked="" type="checkbox"/> S <input checked="" type="checkbox"/> R	
	$\hat{C}_2 = x$ [$\angle \text{s opp. sides} / \angle \text{e teenoor gelyke sye}$]	<input checked="" type="checkbox"/> S <input checked="" type="checkbox"/> R	
	$\hat{B}_1 = x$ [tan chord theorem / raaklyn-koord-stelling]	<input checked="" type="checkbox"/> S <input checked="" type="checkbox"/> R	
	$\hat{T} = x$ [corresp. \angle s, / ooreenk. \angle e : $TF \parallel BC$]	<input checked="" type="checkbox"/> S <input checked="" type="checkbox"/> R	(8)
9.2	$\hat{T} = \hat{A} = x$ $\therefore ATBE \text{ is a cyclic quad}$ [converse \angle s same segment] $ATBE \text{ is 'n koordevierhoek}$ [omgekeerde \angle e in dieselfde seg]	<input checked="" type="checkbox"/> S <input checked="" type="checkbox"/> R	(2)
			[10]

QUESTION/VRAAG 10

10.1



Construction: Join BN and height h from N \perp AM and CM and height k from M \perp AN

Konstruksie : Verbind BN en hoogte, h vanaf N \perp AM

CM en hoogte, k vanaf M \perp AN

$$\frac{\text{Area } \Delta AMN}{\text{Area } \Delta BMN} = \frac{\frac{1}{2} \times AM \times h}{\frac{1}{2} \times BM \times h} \quad [\text{same height/dieselde hoogte}]$$

$$= \frac{AM}{BM}$$

$$\frac{\text{Area } \Delta AMN}{\text{Area } \Delta CMN} = \frac{\frac{1}{2} \times AN \times k}{\frac{1}{2} \times NC \times k} \quad [\text{same height / dieselde hoogte}]$$

$$= \frac{AN}{NC}$$

$$\text{Area } \Delta BMN = \text{Area } \Delta CMN \quad \left[\begin{array}{l} \text{same height, same} \\ \text{base } MN \parallel BC \end{array} \right] / \left[\begin{array}{l} \text{dieselde hoogte, dieselde} \\ \text{basis } MN \parallel BC \end{array} \right]$$

$$\therefore \frac{AM}{BM} = \frac{AN}{NC}$$

✓ constr /
konstr.

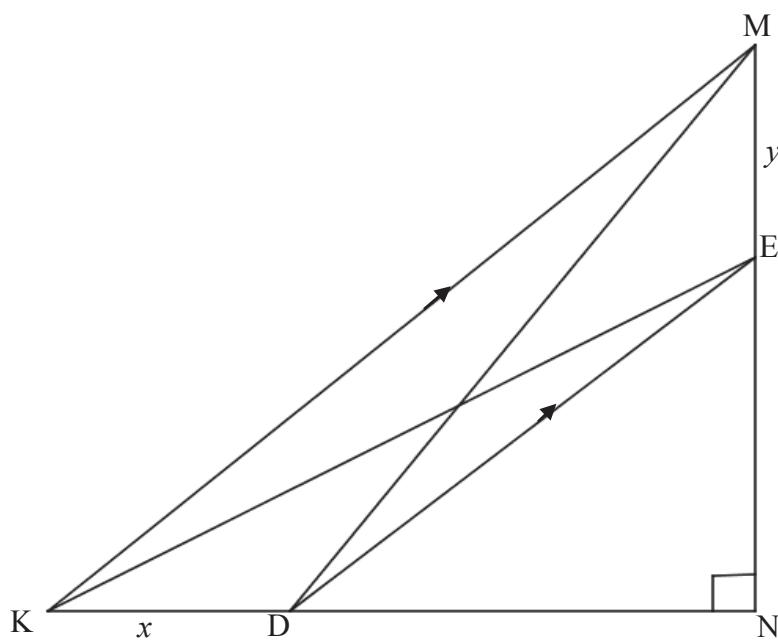
✓ S ✓ R

✓ S

✓ R

(5)

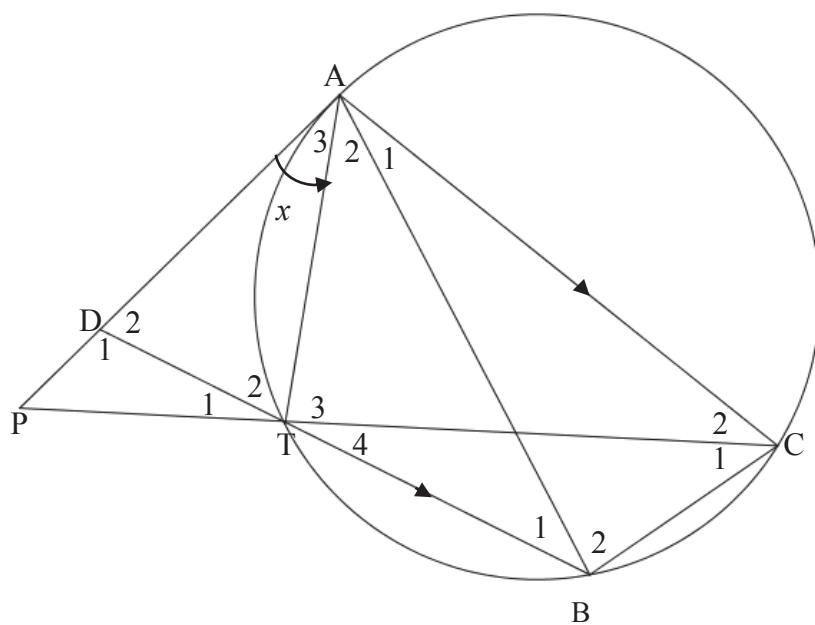
10.2



10.2.1	$\frac{ND}{x} = \frac{2}{1}$ [given/gegee] $ND = 2x$ and/en $\frac{NE}{y} = \frac{2}{1}$ [prop theorem $DE \parallel KM$ or line drawn \parallel to one side of a Δ] / [$\frac{\text{verhouding stelling } DE \parallel KM \text{ of}}{\text{lyn } \text{getrek aan een sy van 'n } \Delta}$] $\therefore NE = 2y$ $KM^2 = KN^2 + MN^2$ [Pyth theorem/stelling] $= (3x)^2 + (3y)^2$ $= 9x^2 + 9y^2$	 ✓S ✓R ✓subst in Pyth theo verv. in Pyth stel ✓simplification/ vereenvoudiging (4)
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10.2.2	$\begin{aligned} DM^2 &= DN^2 + MN^2 & [\text{Pyth}] \\ &= (2x)^2 + (3y)^2 \\ KE^2 &= KN^2 + NE^2 & [\text{Pyth}] \\ &= (3x)^2 + (2y)^2 \\ DM^2 + KE^2 &= 4x^2 + 9y^2 + 9x^2 + 4y^2 & \\ &= 13(x^2 + y^2) \\ \frac{DM^2 + KE^2}{KM^2} &= \frac{13(x^2 + y^2)}{9(x^2 + y^2)} \\ &= \frac{13}{9} \end{aligned}$	✓ subst in Pyth / verv. in Pyth ✓ subst in Pyth / verv. in Pyth ✓ value of / waarde van $DM^2 + KE^2$ ✓ $\frac{13(x^2 + y^2)}{9(x^2 + y^2)}$ (4)	[13]

QUESTION/VRAAG 11



11.1	$\hat{A}_1 = \hat{B}_1$	$[\text{alt } \angle s, AC \parallel DB] / [\text{verw. } \angle e, AC \parallel DB]$	$\checkmark S/R$
	$\hat{A}_3 = \hat{B}_1$	$[\text{tan chord}] / [\text{Raaklyn-koord}]$	$\checkmark S \quad \checkmark R$
	$\therefore \hat{A}_1 = \hat{A}_3$		
	$\hat{T}_2 = \hat{A}CB$	$[\text{ext } \angle \text{ of a cyclic quad}] / [\text{buite } \angle \text{ van koordev}]$	$\checkmark S \quad \checkmark R$
	$\hat{D}_2 = \hat{B}_2$	$[\text{3rd } \angle s] / [3\text{de } \angle e]$	$\checkmark S$
OR/OF		OR/OF	
$\hat{A}_1 = \hat{B}_1$		$\checkmark S/R$	
$\hat{A}_3 = \hat{B}_1$		$\checkmark S \quad \checkmark R$	
$\therefore \hat{A}_1 = \hat{A}_3$			
$\hat{T}_2 = \hat{A}CB$		$\checkmark S \quad \checkmark R$	
$\therefore \Delta ABC \parallel \Delta ADT$		$\checkmark R$	(6)

11.2	$\hat{T}_4 = \hat{T}_1$ [vert opp \angle s]/[regoorst. \angle e] $\hat{A}_1 = \hat{T}_4$ [\angle s in same seg]/[\angle e in dieselfde segment] $A_1 = \hat{A}_3$ [proven]/[Bewys] $\therefore \hat{T}_1 = \hat{A}_3$ \therefore PT is a tangent to circle ADT [converse tan chord] <i>PT is 'n raaklyn aan sirkel ADT [omgekeerde raaklyn – koord]</i>	✓ S/R ✓ S ✓ R ✓ R (4)
11.3	$\hat{A}_3 = \hat{T}_1$ [proven]/[bewys] $\hat{P} = \hat{P}$ [common]/[gemeen] $P\hat{T}A = \hat{D}_1$ [3rd \angle s]/[3de \angle e] $\Delta APT \parallel \Delta TPD$ [$\angle\angle\angle$]	✓ S ✓ S ✓ R
	OR/OF $\hat{A}_3 = \hat{T}_1$ [proven]/[Bewys] $\hat{P} = \hat{P}$ [common]/[gemeen] $P\hat{T}A = \hat{D}_1$ [3rd \angle s]/[3de \angle e]	✓ S ✓ S ✓ S (3)
11.4	$\frac{AP}{TP} = \frac{PT}{PD}$ $AP \cdot PD = PT^2$ $AP(AP - AD) = PT^2$ $AP\left(AP - \frac{2}{3}AP\right) = PT^2$ $AP \cdot \frac{AP}{3} = PT^2$ $AP^2 = 3PT^2$	✓ S/R ✓ simplification <i>vereenvoudiging</i> ✓ PD i.t.o AP and AD <i>PD i.t.v AP en AD</i> ✓ subst in AD <i>verv. in AD</i> (4)

[17]

TOTAL/TOTAAL: 150