**NASIONALE
SENIOR SERTIFIKAAT**

GRAAD 12

JUNIE 2018

WISKUNDE V2

PUNTE: 150

TYD: 3 uur

JMATHA2

Hierdie vraestel bestaan uit 14 bladsye, insluitend 'n 1-bladsy inligtingsblad, en 'n spesiale antwoordeboek.

INSTRUKSIES EN INLIGTING

1. Hierdie vraestel bestaan uit 11 vrae.

2. Antwoord AL die vrae in die SPESIALE ANTWOORDEBOEK voorsien.

3. Toon duidelik ALLE berekeninge, diagramme, grafieke, ensovoorts wat jy gebruik het in die bepaling van jou antwoorde.

4. Antwoorde alleen sal NIE noodwendig volpunte toegeken word NIE.

5. Indien nodig moet jy jou antwoorde tot TWEE desimale plekke afrond, tensy anders vermeld.

6. Diagramme is nie noodwendig volgens skaal geteken nie.

7. Jy mag 'n goedgekeurde wetenskaplike sakrekenaar (nieprogrammeerbaar en niegrafies) gebruik, tensy anders vermeld.

8. 'n Inligtingsblad met formules is aan die einde van die vraestel ingesluit.

9. Skryf netjies en leesbaar.

VRAAG 1

'n Klas van 15 leerders is 'n toets uit 100 punte gegee. Die punte wat die leerders behaal is soos volg:

34   58   26   44   28   29   36   49   54   43   45   59   37   29   48

1.1	Bereken die gemiddelde punt vir hierdie leerders.	(2)

1.2	Bereken die standaardafwyking.	(2)

1.3	Hoeveel leerders het punte behaal wat binne een standaardafwyking van die gemiddelde is?	(3)

| 1.4	Bereken die semi-IKW.	(3)

[10]

VRAAG 2

'n Groep van 64 leerders het 'n Engelse opstel geskryf, en die tyd wat dit geneem het om die taak te voltooi is soos volg opgeteken:

Tyd (in minute)	Frekwensie	Kumulatiewe frekwensie
$5 \leq t < 10$	3	
$10 \leq t < 15$	5	
$15 \leq t < 20$	y	
$20 \leq t < 25$	16	
$25 \leq t < 30$	15	
$30 \leq t < 35$	17	
$35 \leq t < 40$	y	

2.1	Bereken die waarde van y .	(2)

2.2	Voltooi die kumulatiewe frekwensie tabel.	(2)

2.3	Teken 'n ogief (kumulatiewe frekwensie grafiek) om die inligting, op die rooster wat in die ANTWOORDEBOEK voorsien is, voor te stel.	(3)

2.4	Gebruik jou grafiek om te beraam hoeveel leerders die taak na 33 minute voltooi het.	(3)

[10]

VRAAG 3

In die diagram hieronder is die lyn AC geteken, met A en C y- en x- afsnitte onderskeidelik. Die vergelyking van AC is $x + py = p$, $p > 0$. Dit is ook gegee dat $OC = 4OA$. Die reguitlyn EB is geteken. B is die punt op AC sodat $EB \perp AC$ is. EB sny die y-as by $E\left(0; -7\frac{1}{2}\right)$ en die x-as by F.
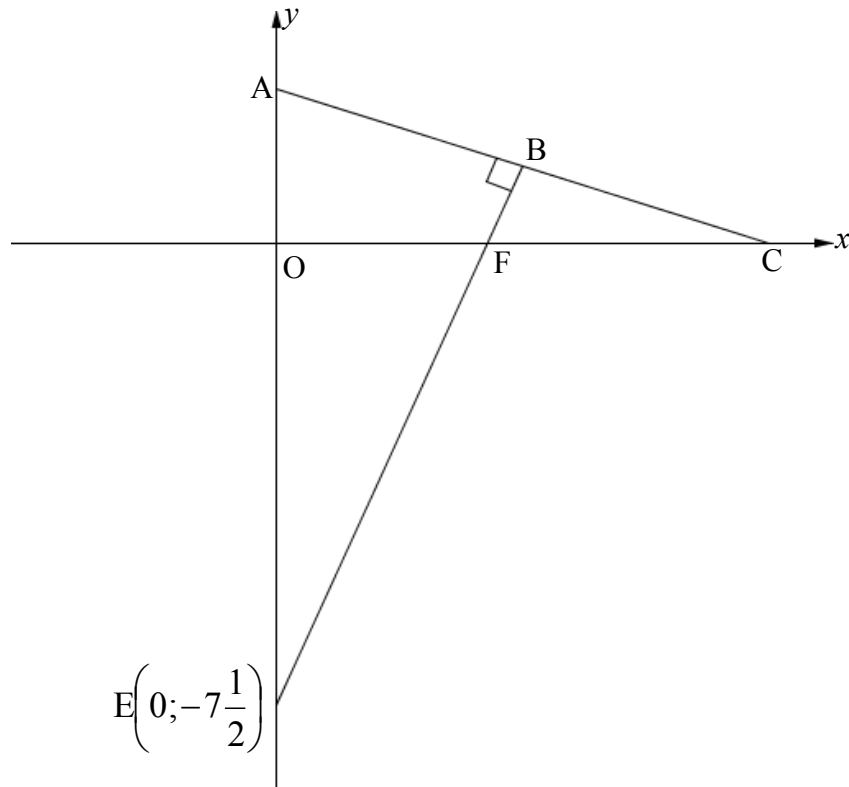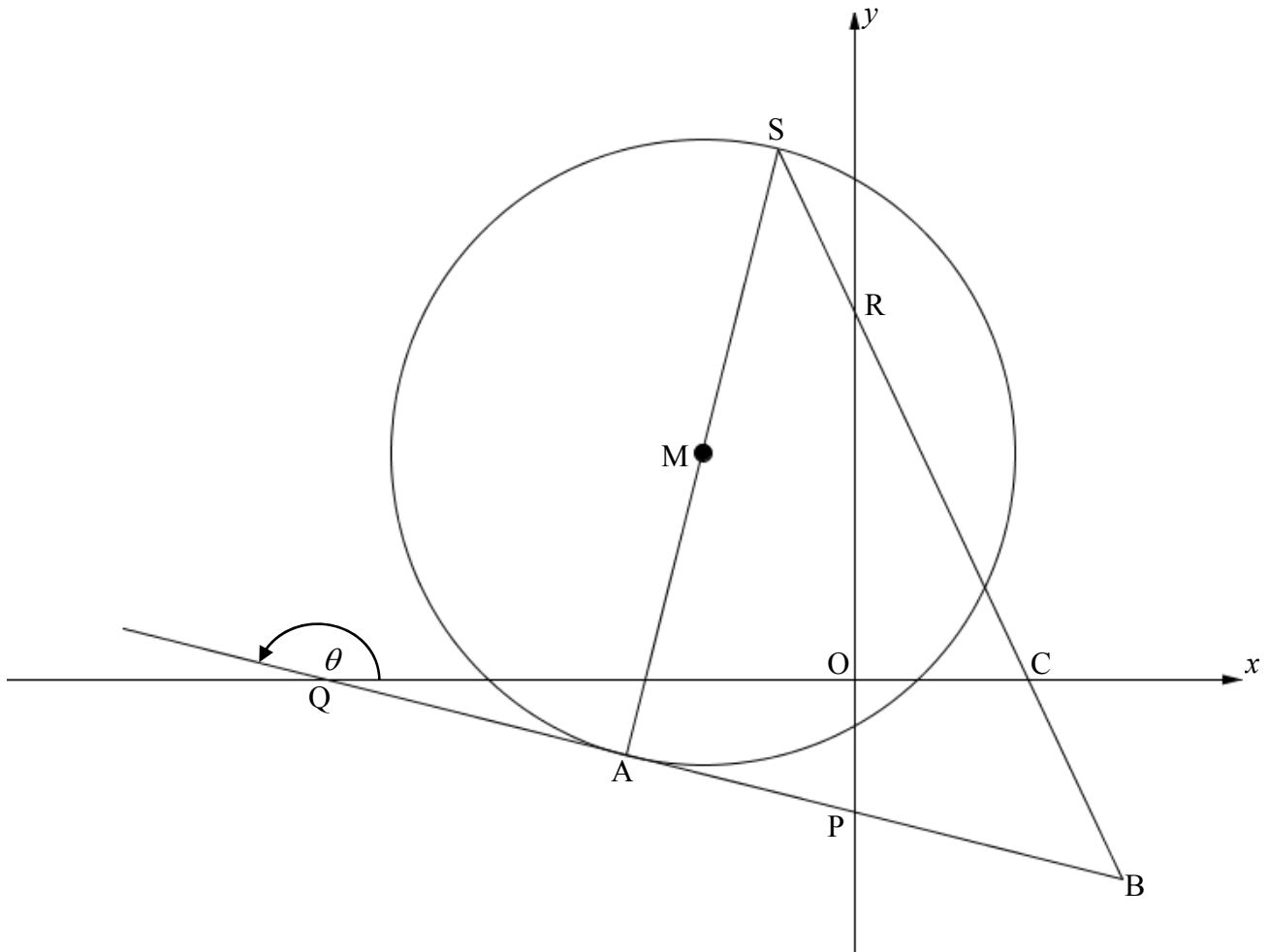

3.1   Bereken die koördinate van A. (2)

3.2   Bereken die waarde van p. (4)

3.3   Bepaal die vergelyking van EB in die vorm $y = mx + c$. (2)

3.4   Bereken die koördinate van B. (4)

3.5   Bereken die koördinate van F. (2)

3.6   Bereken oppervlakte van vierhoek AOFB. (5)

3.7   Skryf die lengte van die radius van die sirkel, wat deur F, B en C gaan, neer. (1)

3.8   Skryf, vervolgens, die vergelyking van die sirkel, wat deur F, B en C gaan, neer indien $\left(\frac{47}{16}; 0\right)$ die koördinate van die middelpunt van FC is.

(2)

[22]

VRAAG 4

In die diagram hieronder, is $x^2 + y^2 + 6x - 8y - 1 = 0$ die vergelyking van die sirkel met middelpunt M. AS is die middellyn van die sirkel. Die vergelyking van die raaklyn aan die sirkel by A is, $y = -\dfrac{1}{5}x + k$. Lyn SRCB is geteken.

4.1 Bepaal die koördinate van M. (3)

4.2 Skryf die lengte van die radius van die sirkel neer. (Laat jou antwoord in eenvoudigste wortelvorm.) (1)

4.3 Bepaal die vergelyking van die middellyn AS. (3)

4.4 Bepaal die koördinate van A. (6)

4.5 Bereken, vervolgens of andersins, die waarde van k. (2)

4.6 As dit verder gegee word dat $\hat{ORC} = 11^0$, bewys dat RCPQ 'n koordevierhoek is. (Rond jou antwoord af tot die naaste grade.) (4)

[19]

VRAAG 5

5.1 Gegee dat $2\sin 27^0 = t$, druk elk van die volgende in terme van t uit:

    5.1.1 $\sin 54^0$ (4)

    5.1.2 $\tan 513^0 . \cos 27^0$ (3)

    5.1.3 $\cos 87^0$ (4)
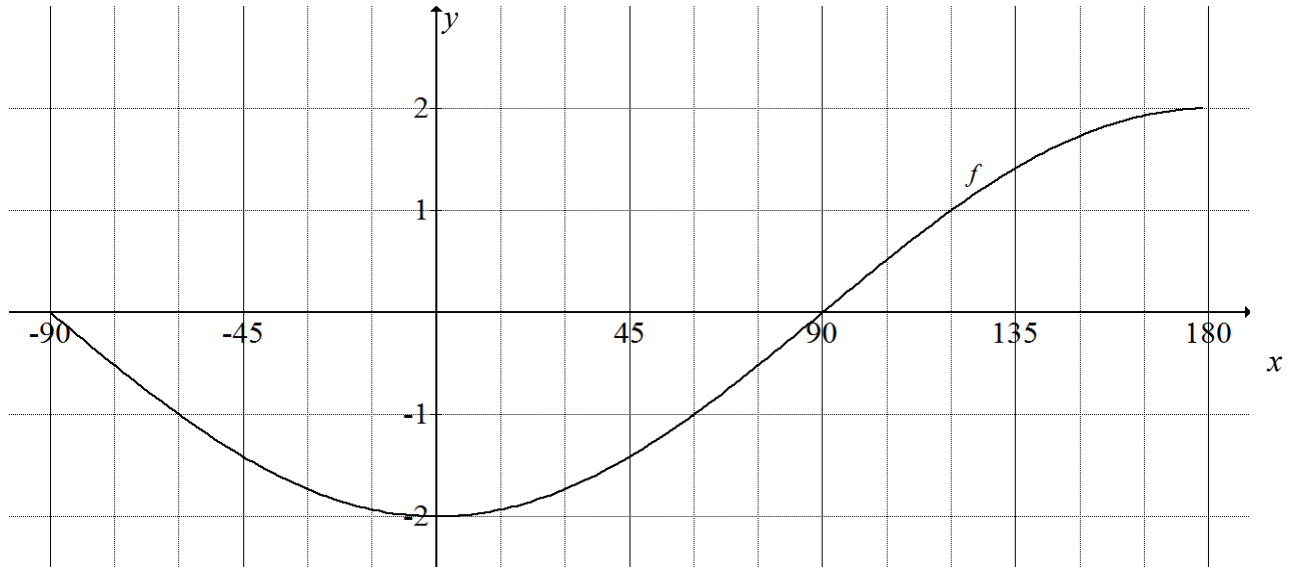
5.2 Vereenvoudig, sonder die gebruik van 'n sakrekenaar:

$$\frac{\sin(-2\alpha)\cos(90^0 + \alpha)}{\sin(-\alpha + 360^0)\cos(-\alpha - 180^0)}$$

    (6)

5.3 Bepaal die algemene oplossing van die vergelyking: $9\sin^2 x - 4\cos^2 x = 0$ (5)

    [22]

VRAAG 6

In die diagram hieronder, is die grafiek van $f(x) = -2\cos x$, vir die interval $-90^0 \le x \le 180^0$ geteken.

6.1 Skryf die amplitude van f neer. (1)

6.2 Skryf die terrein van $f(x) + 3$ neer. (2)

6.3 Teken, op dieselfde assestelsel, die grafiek van $g(x) = \sin(x + 30^0)$. (3)

6.4 Bepaal die waardes van x, in die interval $-90^0 \le x \le 90^0$, waarvoor $x.g(x) < 0$? (2)

6.5 Skryf die vergelyking van h, waar h gevorm word deur g 60^0 na links en twee eenhede afwaarts te skuif. (Laat jou antwoord in eenvoudigste vorm.) (3)

[11]

VRAAG 7

In die diagram hieronder, is ABC 'n reghoekige driehoek met $\hat{B} = 90^0$. Die reguitlyn CD halveer A$\hat{C}$B en sny AB by D. D$\hat{C}$B $= \theta$.
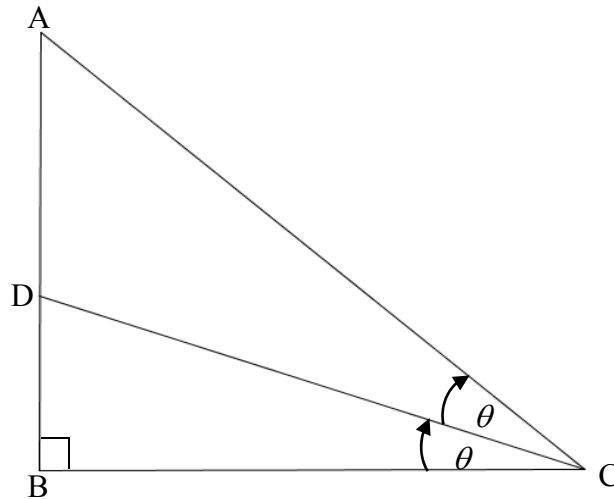

7.1 Skryf die grootte van $\hat{A}$ in terme van θ neer. (1)

7.2 Skryf die verhouding van $\sin\theta$ neer. (1)

7.3 As dit verder gegee word dat $\dfrac{DB}{AD} = \dfrac{1}{2}$ toon, vervolgens of andersins, aan dat

$2\cos 2\theta - 1 = 0$ (5)

[7]

**Gee redes vir jou bewerings in VRAE 8, 9, 10 en 11.**

VRAAG 8

8.1 Voltooi:
'n Lyn getrek vanaf die middelpunt van 'n sirkel na die middelpunt van 'n koord, is … (1)

8.2 In die diagram hieronder geteken, is O die middelpunt van die sirkel. PQ is 'n koord en DE is die middellyn. PQ = 12 eenhede, DM = $2x$ en ME = 3 eenhede.
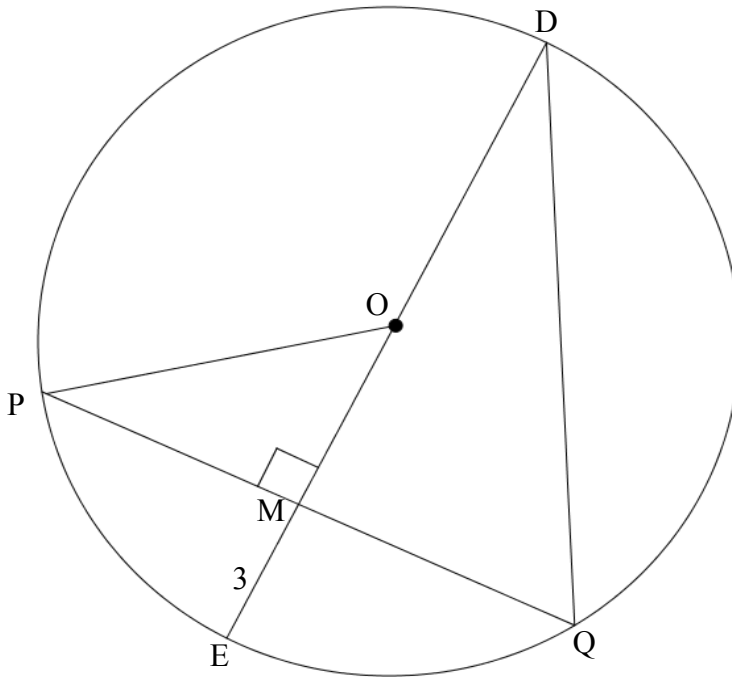
8.2.1 Skryf die lengte van MO in terme van x neer. (1)

8.2.2 Bereken die waarde van x . (4)

8.2.3 Bereken, vervolgens, die lengte van DQ. (Laat jou antwoord in eenvoudigste wortelvorm.) (3)
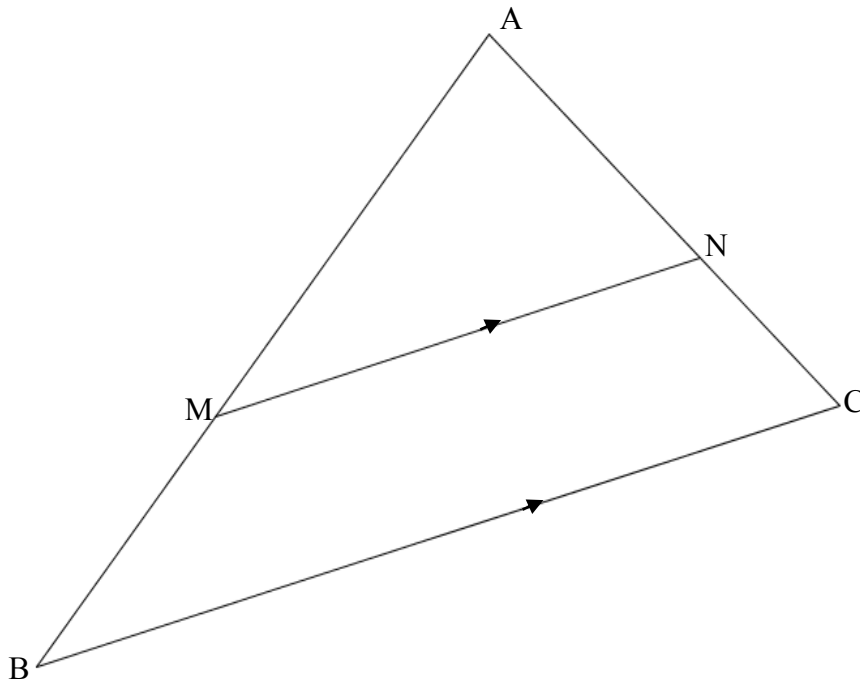
[9]

VRAAG 9

In die diagram hieronder, is F die middelpunt van die sirkel. TBK is 'n raaklyn aan die sirkel by B. E en F is punte op AC en AB onderskeidelik. EF is verleng na T. BC $\parallel$ TE en B$\hat{\text{F}}$C $= 2x$.
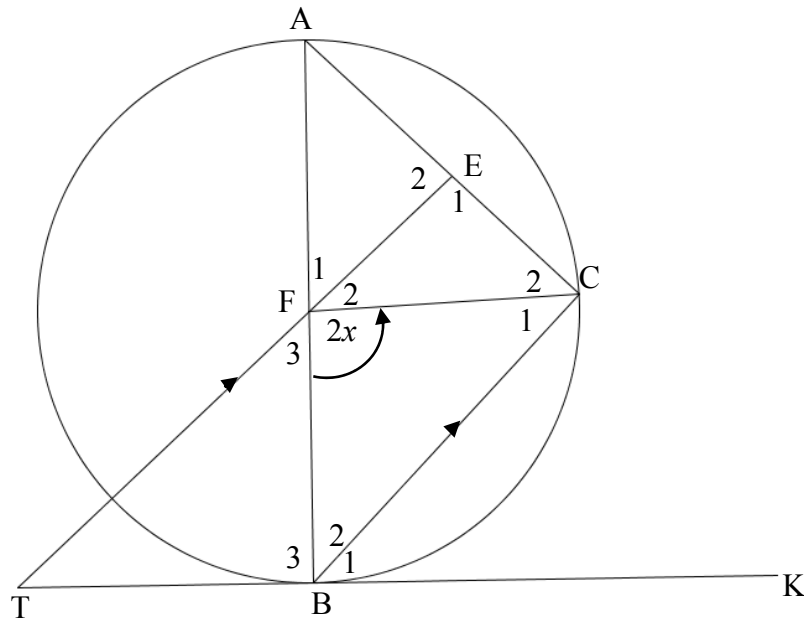
9.1 Noem, met redes, vier hoeke elk gelyk aan x. (8)

9.2 Bewys dat ATBE 'n koordevierhoek is. (2)

[10]

VRAAG 10

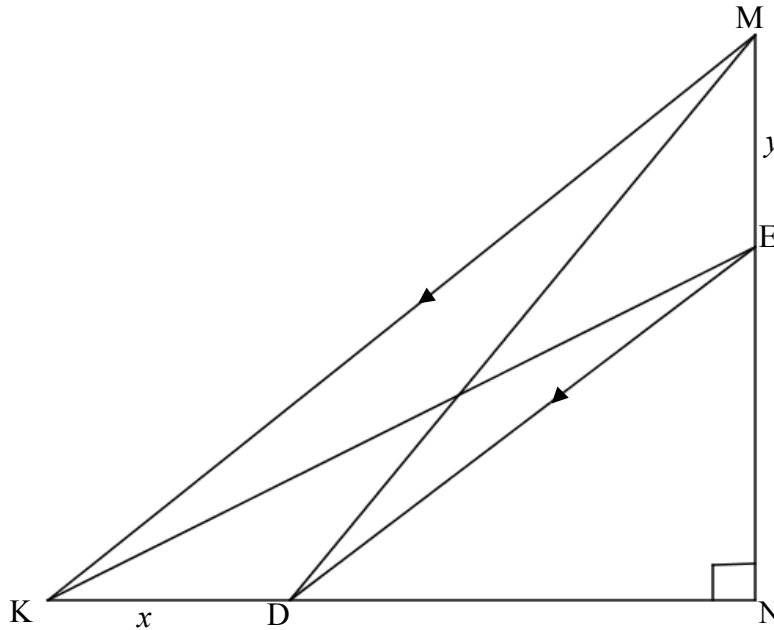10.1 ΔABC is geteken sodat MN ewewydig aan BC is.

Bewys dat: $\dfrac{AM}{MB} = \dfrac{AN}{NC}$

(5)

10.2 In $\triangle MNK$; is $\hat{N} = 90^0$ en D is 'n punt op KN en E is 'n punt op MN sodat $DE \| KM$. $ND : DK = 2 : 1$, $ME = y$ en $KD = x$.
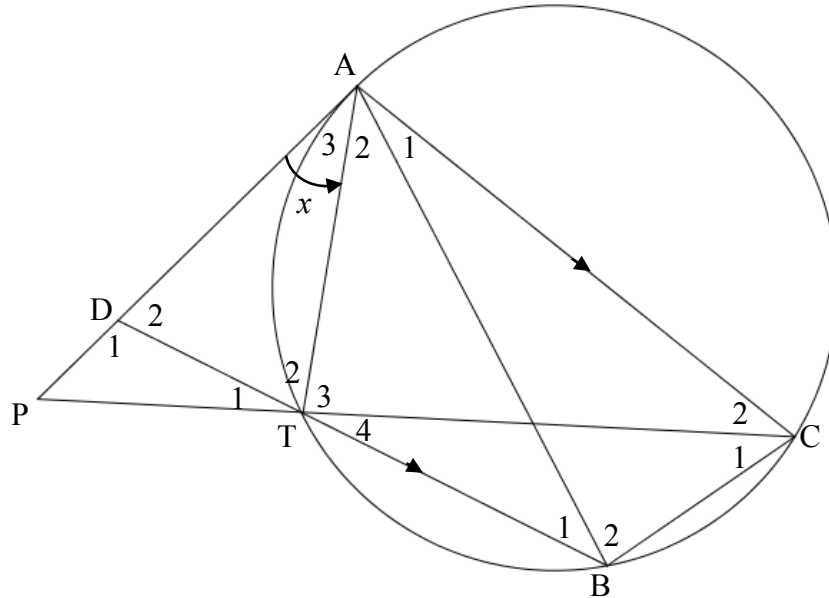

10.2.1 Bepaal die lengte van KM^2 in terme van x en y . (4)

10.2.2 Toon aan dat: $\dfrac{DM^2 + KE^2}{KM^2} = \dfrac{13}{9}$ (4)

[13]

VRAAG 11

In die diagram is PA 'n raaklyn aan sirkel ACBT by A. CT en AD is verleng en ontmoet by P.
BT is verleng en sny PA by D. AC, CB, AB en AT is verbind. AC∥BD .

11.1   Bewys dat $\triangle ABC \vert\vert\vert \triangle ADT$.   (6)

11.2   Bewys dat PT 'n raaklyn aan die sirkel ADT by T is.   (4)

11.3   Bewys dat $\triangle APT \vert\vert\vert \triangle TPD$.   (3)

11.4   As $AD = \dfrac{2}{3} AP$, toon aan dat $AP^2 = 3PT^2$.

                                                                                          (4)
                                                                                        [17]

                                                              TOTAAL: 150

INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r - 1} \; ; \; r \neq 1 \qquad S_\infty = \frac{a}{1 - r} \; ; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In $\triangle ABC$:
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc.\cos A$$
$$area\triangle ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

**NATIONAL
SENIOR CERTIFICATE/**
*NASIONALE
SENIOR SERTIFIKAAT*

GRADE/*GRAAD* 12

JUNE/*JUNIE* 2018

**MATHEMATICS P2/*WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN***

MARKS /*PUNTE*: 150

This marking guideline consists of 20 pages./
Hierdie nasienriglyn bestaan uit 20 bladsye.

NOTE:
- If a candidate answers a question **TWICE**, only mark the **FIRST** attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in **ALL** aspects of the marking guideline.
- Assuming answers/ values in order to solve a problem is **NOT** acceptable.

NEEM KENNIS:
- *As 'n kandidaat 'n vraag **TWEEKEER** beantwoord, merk slegs die **EERSTE** poging*
- *As 'n kandidaat 'n poging om 'n vraag te beantwoord deurgehaal het en nie weer probeer het nie, merk die deurgehaalde poging.*
- *Volgehoue akkuraatheid is van toepassing op **ALLE** aspekte van die nasienriglyn.*
- *Aanvaarding van antwoorde/waardes om sodoende probleme op te los is **NIE** aanvaarbaar **NIE**.*
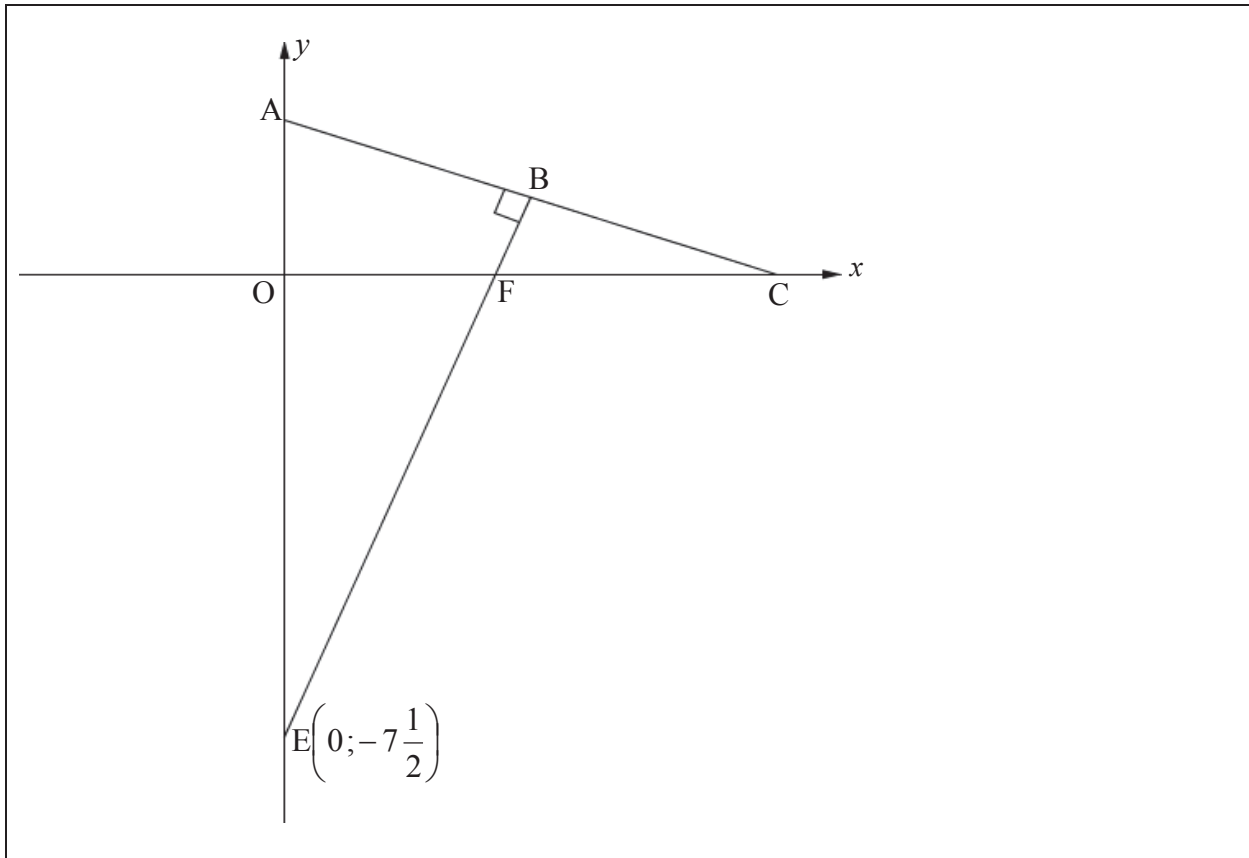
	GEOMETRY/*MEETKUNDE*
S	**A mark for a correct statement.** **(A statement mark is independent of a reason.)**
	'n Punt vir korrekte stelling/bewering. *('n Stelling/bewering punt is onafhanklik van die rede.)*
R	**A mark for a correct reason.** **(A reason mark may only be awarded if the statement is correct.)**
	'n Punt vir korrekte rede. *('n Rede punt mag slegs toegeken word indien die stelling/bewering korrek is.)*
S/R	**Award a mark if statement and reason are both correct.**
	Ken 'n punt toe indien die stelling/bewering sowel as die rede korrek is.

QUESTION/VRAAG 1		
1.1	$\bar{x} = \dfrac{\sum x}{n}$ $= \dfrac{619}{15}$ $= 41,27$ **Answer only full marks/** *Slegs antwoord volpunte*	$\checkmark 619$ $\checkmark 41,27$ (2)
1.2	$\sigma = 10,63$	$\checkmark\checkmark$ Answer / *antwoord* (2)
1.3	$41,27 - 10,63 \leq x \leq 41,27 + 10,63$ $30,64 \leq x \leq 51,90$ $\therefore 8$ learners/leerders	$\checkmark$ both c.v^s / *beide k.w^s* $\checkmark$ notation / *notasie* $\checkmark$ 8 learners/*leerders* (3)
1.4	$Q_1 = 29$ $Q_3 = 49$ $\therefore$ Semi $-$ IQR/IKW $= \dfrac{49-29}{2}$ $= 10$	$\checkmark Q_1$ $\checkmark Q_3$ $\checkmark$ answer/*antwoord* (3)
		[10]

QUESTION/VRAAG 2		
2.1	$56 + 2y = 64$ $2y = 8$ **Answer only 1 mark** *Slegs antwoord 1 punt* $y = 4$	$\checkmark$ correct equation *korrekte vergelyking* $\checkmark$ y-value/*waarde* (2)

2.2	Time (in minutes) *Tyd (in minute)*	Frequency / *Frekwensie*	Cumulative frequency/ *Kumulatiewe frekwensie*	
	$5 \leq t < 10$	3	3	$\checkmark$ 8, 12 and/*en 28*
	$10 \leq t < 15$	5	8	
	$15 \leq t < 20$	4	12	
	$20 \leq t < 25$	16	28	$\checkmark$ 43, 60 and/*en 64*
	$25 \leq t < 30$	15	43	
	$30 \leq t < 35$	17	60	
	$35 \leq t < 40$	4	64	
				(2)

2.3		
	Ogive/*Ogief* Time in minutes/ *Tyd in minute*	✓ grounding/*anker* ✓ plotting against the upper limit/ *afsteek teen die boonste limiet* ✓ shape / *vorm*
		(3)
2.4	Number of learners / *Aantal leerders* $\approx 64 - 54 \approx 10$ Accept / *Aanvaar* $[9 - 11]$	✓ method/*metode* ✓ 54 ✓ answer/*antwoord*
		(3)
		[10]

QUESTION/*VRAAG 3*

3.1	$x + py = p$	✓ y-subject of formula
	$y = -\dfrac{x}{p} + 1$	y-onderwerp van formule
	$\therefore A(0;1)$	✓ coordinates of A
		koördinate van A
		(2)
3.2	$OA = 1$	✓ OC
	$\therefore OC = 4(1) = 4$	✓ $-\dfrac{1}{p} = \dfrac{1-0}{0-4}$
	$\therefore C(4;0)$	✓
	$m_{AC} = -\dfrac{1}{p} = \dfrac{1-0}{0-4}$	simplification/*vereenvoudiging*
	$p = 4$	✓ p-value/*waarde* (4)
3.3	$m_{EB} = 4$ $EB \perp AC$	✓ $m_{EB} = 4$
	$y = 4x - 7\dfrac{1}{2}$	✓ equation/*vergelyking*
		(2)

3.4	$-\dfrac{x}{4}+1 = 4x - 7\dfrac{1}{2}$ $-x + 4 = 16x - 30$ $17x = 34$ $x = 2$ $y = \dfrac{1}{2}$	✓ equating / *gelykstel* ✓ simplification / *vereenvoudiging* ✓ x-value/*waarde* ✓ y-value/*waarde* (4)
3.5	$4x - 7\dfrac{1}{2} = 0$ $4x = \dfrac{15}{2}$ $x = \dfrac{15}{8}$ $\text{F}\left(\dfrac{15}{8};0\right)$	✓ $y = 0$ ✓ x-value/*waarde* (2)
3.6	$\text{BF} = \sqrt{\left(\dfrac{15}{8}-2\right)^2 + \left(0-\dfrac{1}{2}\right)^2}$ $= \dfrac{\sqrt{17}}{8}$ $\therefore \text{Area FOAB} = \text{Area }\triangle\text{AOC} - \text{Area }\triangle\text{BFC}$ $= \dfrac{1}{2}\times 4\times 1 - \dfrac{1}{2}\left(\dfrac{\sqrt{17}}{2}\right)\dfrac{\sqrt{17}}{8}$ $= 1{,}47$ **OR/*OF*** $\text{BF} = \sqrt{\left(\dfrac{15}{8}-2\right)^2 + \left(0-\dfrac{1}{2}\right)^2}$ $= \dfrac{\sqrt{17}}{8}$ $\text{Area FOAB} = \text{Area }\triangle\text{AOF} + \text{Area }\triangle\text{ABF}$ $= \dfrac{1}{2}\times 1\times\dfrac{15}{8} + \dfrac{1}{2}\times\dfrac{\sqrt{17}}{2}\times\dfrac{\sqrt{17}}{8}$ $= 1{,}47$	✓ substitution / *vervanging* ✓ BF ✓ BC ✓ substitution / *vervanging* ✓ answer / *antwoord* ✓ substitution / *vervanging* ✓ BF ✓ OF ✓ substitution / *vervanging* ✓ answer / *antwoord* (5)
3.7	$r = \dfrac{17}{16}$	✓ answer / *antwoord* (1)
3.8	$\left(x - \dfrac{47}{16}\right)^2 + y^2 = \dfrac{289}{256}$	✓ r^2 ✓ equation / *vergelyking* (2)
		[22]
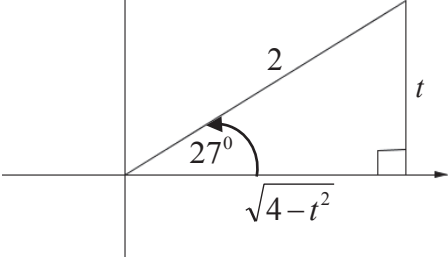
QUESTION/*VRAAG 4*

4.1	$x^2 + 6x + 9 + y^2 - 8y + 16 = -5 + 9 + 16$ $(x + 3)^2 + (y - 4)^2 = 26$ $\therefore M(-3;4)$	✓ completing the square *kwadraatsvoltooiing* ✓ x-value/*waarde* ✓ y-value/*waarde* (3)
4.2	$r = \sqrt{26}$	✓ answer/*antwoord* (1)
4.3	$m_{AS} = 5$ [SA $\perp$ QB] $y - 4 = 5(x + 3)$ $y = 5x + 19$	✓ $m_{AS} = 5$ ✓ subst. /*verv.* m_{AS} & $(-3;4)$ ✓ equation/*vergelyking* (3)

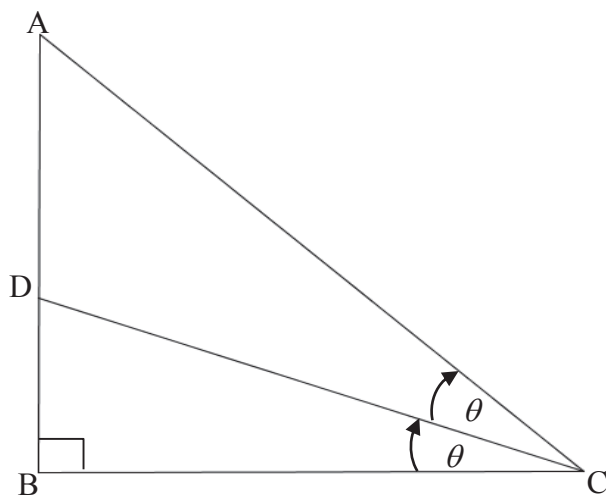| 4.4	$x^2 + 6x + 9 + (5x + 19 - 4)^2 = 26$ $x^2 + 6x + 9 + 25x^2 + 150x + 225 - 26 = 0$ $26x^2 + 156x + 208 = 0$ $x^2 + 6x + 8 = 0$ $(x + 4)(x + 2) = 0$ $x = -4 \ or \ x \neq -2$ $\therefore y = 5(-4) + 19$ $= -1$ $Q(-4; -1)$	✓ substitution/*vervanging* ✓ simplification / *vereenvoudiging* ✓ standard form / *standaardvorm* ✓ factors / *faktore* ✓ choosing correct *x-value korrekte keuse van x-waarde* ✓ *y*-value/*waarde* (6)
4.5	$-1 = -\dfrac{1}{5}(-4) + k$ $\therefore k = -\dfrac{9}{5}$	✓ substitute/*vervang* $(-4; -1)$ ✓ answer / *antwoord* (2)
4.6	$\tan\theta = -\dfrac{1}{5}$ $\therefore \theta = 169^0$ $O\hat{Q}P = 11^0 \ [\angle s \text{ on a str line}] / [\angle e \text{ op 'n reguitlyn}]$ $\therefore O\hat{Q}P = O\hat{R}C$ $\therefore \text{RCPQ is a cyclic quad} \quad [\text{converse } \angle s \text{ same seg}]$ $\textit{RCPQ is 'n koordevierhoek} [\textit{omgekeerde } \angle e \textit{ in dieselfde seg}]$	✓ $\tan\theta = -\dfrac{1}{5}$ ✓ size of θ / *grootte van* θ ✓ size of $O\hat{Q}P$ / *grootte van* $O\hat{Q}P$ ✓ R (4)
		[19]

QUESTION/*VRAAG* 5

5.1.1	$2\sin 27^0 = t$ $\sin 27^0 = \dfrac{t}{2}$ $x = \sqrt{4 - t^2}$ $\sin 54^0 = 2\sin 27^0 \cos 27^0$ $= 2 \times \dfrac{t}{2} \times \dfrac{\sqrt{4-t^2}}{2}$ $= \dfrac{t\sqrt{4-t^2}}{2}$	✓ $\sin 27^0 = \dfrac{t}{2}$ ✓ $x = \sqrt{4-t^2}$ ✓ $2\sin 27^0 \cos 27^0$ ✓ substitution/*vervanging* (4)
5.1.2	$\tan 513^0 \cdot \cos 27^0 = (-\tan 27^0)\cos 27^0$ $\qquad = -\dfrac{\sin 27^0}{\cos 27^0}\cdot \cos 27^0$ $\qquad = -\dfrac{t}{2}$	✓ $\left(-\tan 27^0\right)$ ✓ $-\dfrac{\sin 27^0}{\cos 27^0}$ ✓ $-\dfrac{t}{2}$ (3)
5.1.3	$\cos 87^0 = \cos\left(60^0 + 27^0\right)$ $\qquad = \cos 60^0 \cdot \cos 27^0 - \sin 60^0 \cdot \sin 27^0$ $\qquad = \dfrac{1}{2}\cdot\dfrac{\sqrt{4-t^2}}{2} - \dfrac{\sqrt{3}}{2}\cdot\dfrac{t}{2}$ $\qquad = \dfrac{\sqrt{4-t^2} - t\sqrt{3}}{4}$	✓ $60^0 + 27^0$ ✓ expansion / *uitbreiding* ✓ subst./*verv.* $\dfrac{\sqrt{4-t^2}}{2}$ & $\dfrac{t}{2}$ ✓ subst. / *verv.* $\dfrac{1}{2}$ & $\dfrac{\sqrt{3}}{2}$ (4)

5.2	$\dfrac{\sin(-2\alpha)\cos(90^0+\alpha)}{\sin(-\alpha+360^0).\cos(-\alpha-180^0)}$	✓ $(-\sin 2\alpha)$
	$=\dfrac{(-\sin 2\alpha)(-\sin\alpha)}{(\sin\alpha)(-\cos\alpha)}$	✓ $(-\sin\alpha)$ ✓ $\sin\alpha$ ✓ $(-\cos\alpha)$
	$=-\dfrac{2\sin\alpha\cos\alpha}{\cos\alpha}$	✓ $2\sin\alpha\cos\alpha$
	$=-2\sin\alpha$	✓ $-2\sin\alpha$ (6)
5.3	$9\sin^2 x-4\cos^2 x=0$	
	$(3\sin x-2\cos x)(3\sin x+2\cos x)=0$	✓ factors/*faktore*
	$\therefore 3\sin x=\pm 2\cos x$	✓ both equations *beide vergelykings*
	$\tan x=\pm\dfrac{2}{3}$	✓ $\tan x=\pm\dfrac{2}{3}$
	$x=33,69^0+180^0.k\text{ or/of }x=146,31^0+180^0.k\;\;k\in Z$	✓ both / *beide* $33,69^0\,\&\,146,31^0/-33,69^0$ ✓ $180^0.k\,\&\,k\in Z$ (5)
		[22]

QUESTION/*VRAAG 6*
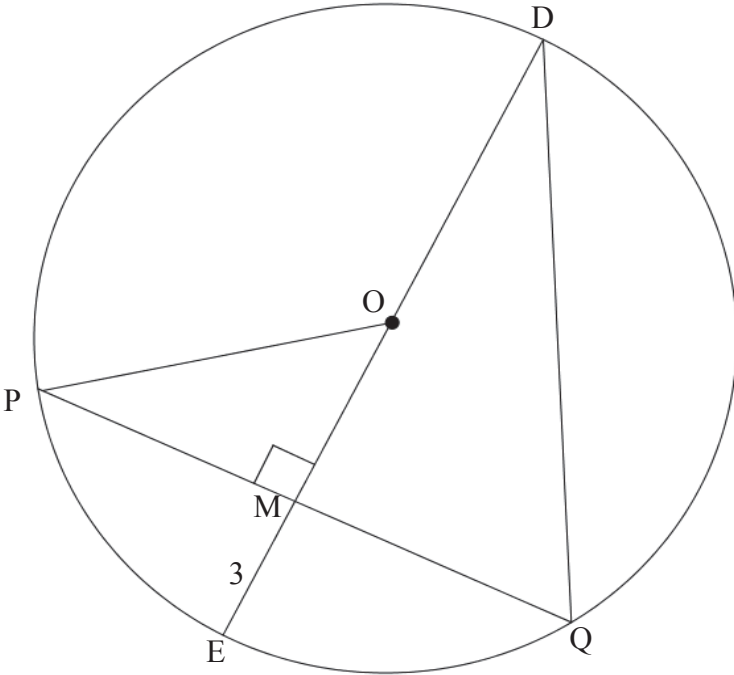
6.1	Amplitude $= 2$	✓ answer/*antwoord*	(1)
6.2	$1 \leq y \leq 5$	✓ min & max/*maks* ✓ notation/*notasie*	(2)
6.3		✓ both x-intercepts *beide x-afsnitte* $-30^0 \ \& 150^0$ ✓ max TP & y-intercept ✓ shape / *vorm*	(3)
6.4	$-30^0 < x < 0^0$	✓ both c.v.^{s} / *beide k.w^s* ✓ notation/*notasie*	(2)
6.5	$h(x) = \sin(x + 90^0) - 2$ $\quad = \cos x - 2$	✓✓ $\sin(x + 90^0) - 2$ ✓ $\cos x$	(3)
			[11]

QUESTION/VRAAG 7

7.1	$\hat{A} = 90^0 - 2\theta$	✓ answer / antwoord	(1)
7.2	$\sin\theta = \dfrac{DB}{DC}$	✓ answer / antwoord	(1)
7.3	$DC = \dfrac{DB}{\sin\theta}$ and/en $AD = 2DB = 2DB$ $\dfrac{DC}{\sin(90^0 - 2\theta)} = \dfrac{AD}{\sin\theta}$ in $\triangle ADC$ $\dfrac{\dfrac{DB}{\sin\theta}}{\cos 2\theta} = \dfrac{2DB}{\sin\theta}$ $\dfrac{DB}{\sin\theta.\cos 2\theta} = \dfrac{2DB}{\sin\theta}$ $DB.\sin\theta = 2DB.\sin\theta.\cos 2\theta$ $2\cos 2\theta = 1$ $2\cos 2\theta - 1 = 0$	✓ AD = 2DB ✓ sine rule/sinusreël in $\triangle ADC$ ✓ subst. in sine rule / verv. in sinusreël ✓ cos2θ ✓ equation / vergelyking	(5)
			[7]

QUESTION/*VRAAG 8*

8.1	**Perpendicular** to the chord / ***Loodreg*** *op die koord*	✓ answer/*antwoord*	(1)

8.2			

8.2.1	$OM = \dfrac{2x+3}{2} - 3$ OR $OM = \dfrac{2x-3}{2}$	✓ answer/*antwoord* OR/*OF* ✓ answer/*antwoord*	(1)

8.2.2	$OQ^2 = OM^2 + QM^2$ $\left(\dfrac{2x+3}{2}\right)^2 = \left(\dfrac{2x-3}{2}\right)^2 + 6^2$ $\dfrac{4x^2 + 12x + 9}{4} = \dfrac{4x^2 - 12x + 9}{4} + 36$ $24x = 144$ $x = 6$	✓ susbt. in Pyth / *verv. in Pyth.* ✓ simplification / *vereenvoudiging* ✓ standard form linear equation *Standaardvorm van lineêre vergelyking* ✓ *x*-value/*waarde*	(4)

| 8.2.3 | $DM = 12$ | $\checkmark$ DM | |
	$DQ = \sqrt{12^2 + 6^2}$	$\checkmark$ subst. in Pyth / *verv. in Pyth*	
	$= \sqrt{180}$	$\checkmark$ answer / *antwoord*	
	$= 6\sqrt{5}$		(3)
			[9]

QUESTION/VRAAG 9

9.1	$\hat{A} = x$ [$\angle$ at centre $= 2\angle$ at circumf. / Middelpunts$\angle = 2 \times$ Omtreks$\angle$]	✓ S ✓R	
	$\hat{C}_2 = x$ [$\angle s$ opp. $=$ sides / $\angle$e teenoor gelyke sye]	✓ S ✓R	
	$\hat{B}_1 = x$ [tan chord theorem / raaklyn-koord-stelling]	✓ S ✓R	
	$\hat{T} = x$ [corresp. $\angle$s, / ooreenk. $\angle$e : TF$\parallel$BC]	✓ S ✓R	(8)
9.2	$\hat{T} = \hat{A} = x$	✓ S	
	$\therefore$ ATBE is a cyclic quad [converse $\angle$s same segment]	✓ R	
	ATBE is 'n koordevierhoek [*omgekeerde $\angle$e in dieselfde seg*]		(2)
			[10]

QUESTION/*VRAAG* 10

10.1
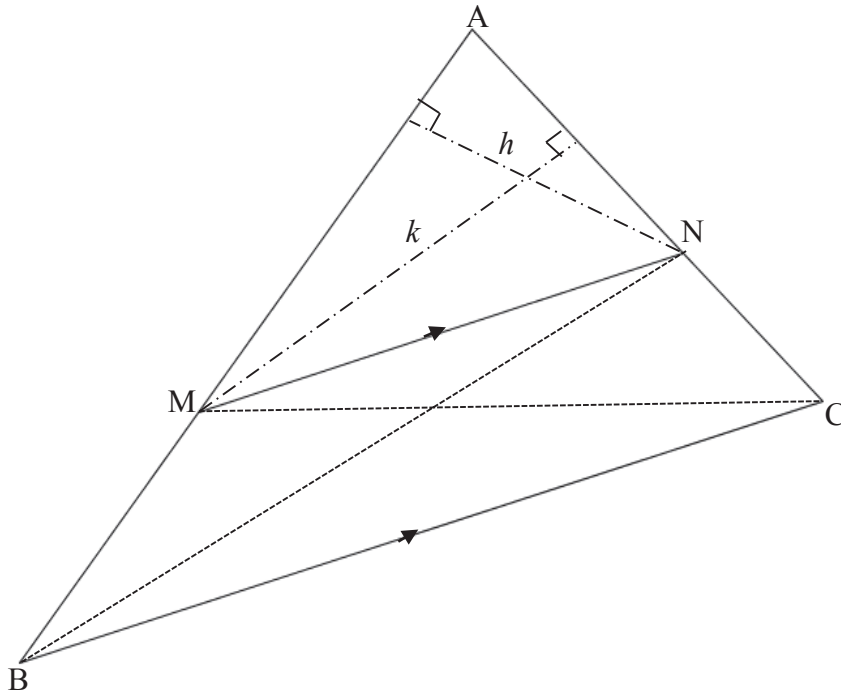
Contruction:Join BN and height h from N $\perp$ AM and CM and height k from M $\perp$ AN
 	
Konstruksie :Verbind BN en hoogte, h vanaf N $\perp$ AM
                    CM en hoogte, k vanaf M $\perp$ AN
	✓ constr /
	  konstr.

$\dfrac{\text{Area} \triangle \text{AMN}}{\text{Area} \triangle \text{BMN}} = \dfrac{\frac{1}{2} \times \text{AM} \times h}{\frac{1}{2} \times \text{BM} \times h}$ [same height/dieselfde hoogte] $= \dfrac{\text{AM}}{\text{BM}}$	✓ S ✓R
$\dfrac{\text{Area} \triangle \text{AMN}}{\text{Area} \triangle \text{CMN}} = \dfrac{\frac{1}{2} \times \text{AN} \times k}{\frac{1}{2} \times \text{NC} \times k}$ [same height / dieselfde hoogte] $= \dfrac{\text{AN}}{\text{NC}}$	✓ S ✓ R
Area $\triangle$BMN = Area $\triangle$ CMN $\begin{bmatrix}\text{same height, same} \\ \text{base MN}\|\text{BC}\end{bmatrix} / \begin{bmatrix}\textit{dieselfde hoogte, dieselfde} \\ \textit{basis MN} \| \textit{BC}\end{bmatrix}$ $\therefore \dfrac{\text{AM}}{\text{BM}} = \dfrac{\text{AN}}{\text{NC}}$	(5)

10.2

| 10.2.1 | $\dfrac{\text{ND}}{x} = \dfrac{2}{1}$ [given/gegee]

 $\text{ND} = 2x$

 and/en

 $\dfrac{\text{NE}}{y} = \dfrac{2}{1}$ $\left[\begin{array}{l}\text{prop theorem DE}\|\text{KM or}\\ \text{line drawn}\|\text{to one side of a }\Delta\end{array}\right] / \left[\begin{array}{l}\textit{verhouding stelling DE}\|\textit{KM of}\\ \textit{lyn}\|\textit{getrek aan een sy van'n }\Delta\end{array}\right]$

 $\therefore \text{NE} = 2y$

 $\text{KM}^2 = \text{KN}^2 + \text{MN}^2$ [Pyth theorem/stelling]

 $= (3x)^2 + (3y)^2$

 $= 9x^2 + 9y^2$ | ✓S   ✓R

 ✓subst in Pyth theo
 verv. in Pyth stel

 ✓simplification/
 vereenvoudiging   (4) |

10.2.2	$\begin{aligned} DM^2 &= DN^2 + MN^2 \qquad [\text{Pyth}] \\ &= (2x)^2 + (3y)^2 \\ KE^2 &= KN^2 + NE^2 \qquad [\text{Pyth}] \\ &= (3x)^2 + (2y)^2 \\ DM^2 + KE^2 &= 4x^2 + 9y^2 + 9x^2 + 4y^2 \\ &= 13(x^2 + y^2) \\ \frac{DM^2 + KE^2}{KM^2} &= \frac{13(x^2 + y^2)}{9(x^2 + y^2)} \\ &= \frac{13}{9} \end{aligned}$	$\checkmark$ subst in Pyth / *verv. in Pyth* $\checkmark$ subst in Pyth / *verv. in Pyth* $\checkmark$ value of / *waarde van* $DM^2 + KE^2$ $\checkmark \dfrac{13(x^2 + y^2)}{9(x^2 + y^2)}$ (4)
		[13]

QUESTION/*VRAAG 11*

11.1	$\hat{A}_1 = \hat{B}_1$	$\left[\text{alt} \angle s, AC \parallel DB\right] / \left[verw.\angle e, AC \parallel DB\right]$	✓ S/R	
	$\hat{A}_3 = \hat{B}_1$	$\left[\text{tan chord}\right] / \left[Raaklyn - koord\right]$	✓S	✓R
	$\therefore \hat{A}_1 = \hat{A}_3$			
	$\hat{T}_2 = A\hat{C}B$	$\left[\text{ext} \angle \text{ of a cyclic quad}\right] / \left[buite \angle van koordev\right]$	✓S	✓R
	$\hat{D}_2 = \hat{B}_2$	$\left[\text{3rd} \angle \text{s}\right] / \left[3de \angle e\right]$	✓S	
	OR/OF		**OR/*OF***	
	$\hat{A}_1 = \hat{B}_1$	$\left[\text{alt} \angle s, AC \parallel DB\right] / \left[verw.\angle e, AC \parallel DB\right]$	✓ S/R	
	$\hat{A}_3 = \hat{B}_1$	$\left[\text{tan chord}\right] / \left[Raaklyn - koord\right]$	✓S	✓R
	$\therefore \hat{A}_1 = \hat{A}_3$			
	$\hat{T}_2 = A\hat{C}B$	$\left[\text{ext} \angle \text{ of a cyclic quad}\right] / \left[buite \angle van koordev\right]$	✓S	✓R
	$\therefore \triangle ABC \parallel\parallel \triangle ADT$	$\left[\angle\angle\angle\right]$	✓R	(6)

11.2	$\hat{T}_4 = \hat{T}_1$ $\quad$ [vert opp $\angle$s]/[_regoorst._$\angle e$]	✓ S/R
	$\hat{A}_1 = \hat{T}_4$ $\quad$ [$\angle$s in same seg]/[$\angle e$ _in dieselfde segment_]	✓S $\quad$ ✓R
	$A_1 = \hat{A}_3$ $\quad$ [proven]/[_Bewys_]	
	$\therefore \hat{T}_1 = \hat{A}_3$	
	$\therefore$ PT is a tangent to circle ADT [converse tan chord]	✓R
	$\quad$ _PT is 'n raaklyn aan sirkel ADT_ [_omgekeerde raaklyn – koord_]	(4)
11.3	$\hat{A}_3 = \hat{T}_1$ $\quad$ [proven]/[_bewys_]	✓S
	$\hat{P} = \hat{P}$ $\quad$ [common]/[_gemeen_]	✓S
	$P\hat{T}A = \hat{D}_1$ $\quad$ [3rd $\angle$s]/[3_de_ $\angle e$]	
	Δ APT $\|\|$ Δ TPD $\quad$ [$\angle\angle\angle$]	✓R
	OR/OF	
	$\hat{A}_3 = \hat{T}_1$ $\quad$ [proven]/[_Bewys_]	✓S
	$\hat{P} = \hat{P}$ $\quad$ [common]/[_gemeen_]	✓S
	$P\hat{T}A = \hat{D}_1$ $\quad$ [3rd $\angle$s]/[3_de_ $\angle e$]	✓S
		(3)
11.4	$\dfrac{AP}{TP} = \dfrac{PT}{PD}$ $\qquad$ [$\|\|$ Δs]	✓S/R
	$AP.PD = PT^2$	✓simplification _vereenvoudiging_
	$AP(AP - AD) = PT^2$	
	$AP\left(AP - \dfrac{2}{3}AP\right) = PT^2$	✓PD i.t.o AP and AD _PD i.t.v AP en AD_
	$AP.\dfrac{AP}{3} = PT^2$	✓ subst in AD _verv. in AD_
	$AP^2 = 3PT^2$	(4)

[17]

TOTAL/_TOTAAL_**:** **150**