basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2017

MARKS: 150

TIME: 3 hours

This question paper consists of 8 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs et cetera that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + 9x + 14 = 0$ (3)

1.1.2 $4x^2 + 9x - 3 = 0$ (correct to TWO decimal places) (4)

1.1.3 $\sqrt{x^2 - 5} = 2\sqrt{x}$ (4)

1.2 Solve for x and y if:

$3x - y = 4$ and $x^2 + 2xy - y^2 = -2$ (6)

1.3 Given: $f(x) = x^2 + 8x + 16$

1.3.1 Solve for x if $f(x) > 0$. (3)

1.3.2 For which values of p will $f(x) = p$ have TWO unequal negative roots? (4)

[24]

QUESTION 2

2.1 Given the following quadratic number pattern: 5 ; –4 ; –19 ; –40 ; …

2.1.1 Determine the constant second difference of the sequence. (2)

2.1.2 Determine the n^{th} term (T_n) of the pattern. (4)

2.1.3 Which term of the pattern will be equal to $-25\,939$? (3)

2.2 The first three terms of an arithmetic sequence are $2k - 7$; $k + 8$ and $2k - 1$.

2.2.1 Calculate the value of the 15^{th} term of the sequence. (5)

2.2.2 Calculate the sum of the first 30 even terms of the sequence. (4)

[18]

QUESTION 3

A convergent geometric series consisting of only positive terms has first term a, constant ratio r and n^{th} term, T_n, such that $\sum_{n=3}^{\infty} T_n = \frac{1}{4}$.

3.1 If $T_1 + T_2 = 2$, write down an expression for a in terms of r. (2)

3.2 Calculate the values of a and r. (6)

[8]

QUESTION 4

Given: $f(x) = -ax^2 + bx + 6$

4.1 The gradient of the tangent to the graph of f at the point $\left(-1 ; \frac{7}{2}\right)$ is 3.

Show that $a = \frac{1}{2}$ and $b = 2$. (5)

4.2 Calculate the x-intercepts of f. (3)

4.3 Calculate the coordinates of the turning point of f. (3)

4.4 Sketch the graph of f. Clearly indicate ALL intercepts with the axes and the turning point. (4)

4.5 Use the graph to determine the values of x for which $f(x) > 6$. (3)

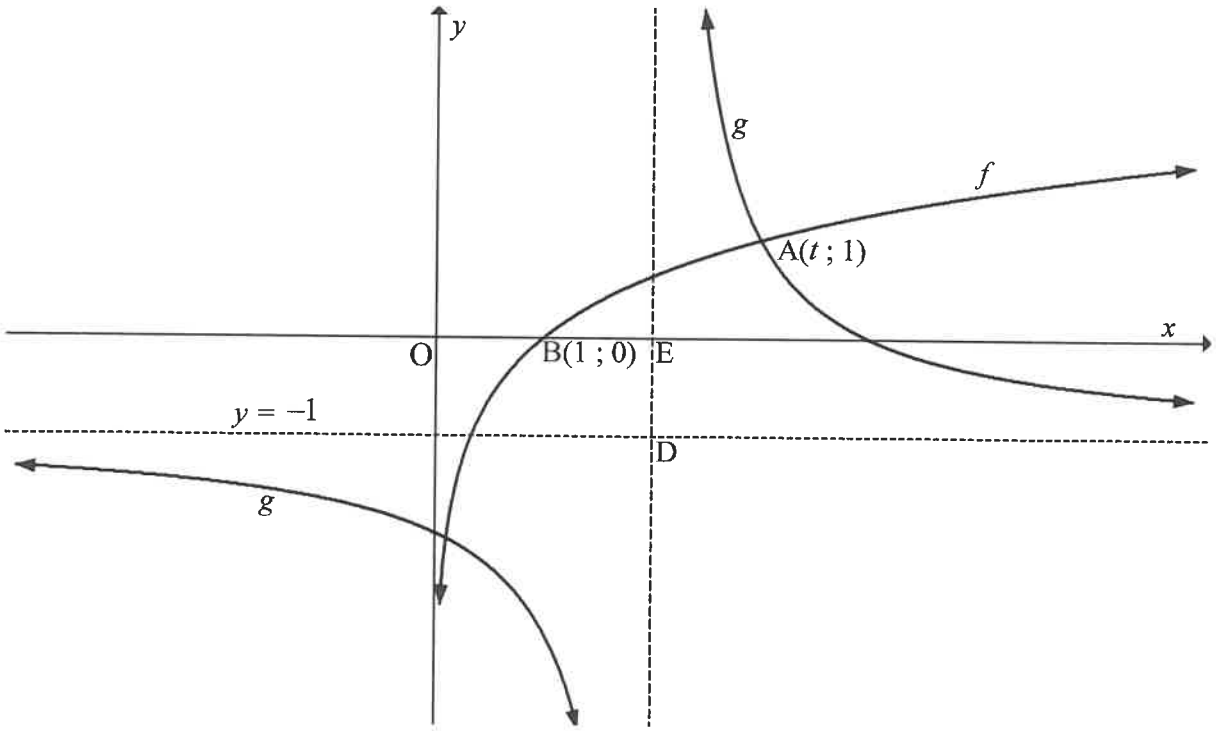
4.6 Sketch the graph of $g(x) = -x - 1$ on the same set of axes as f. Clearly indicate ALL intercepts with the axes. (2)

4.7 Write down the values of x for which $f(x).g(x) \leq 0$. (3)
[23]

QUESTION 5

The diagram below shows the graphs of $g(x)=\frac{2}{x+p}+q$ and $f(x)=\log_3 x$.

- $y=-1$ is the horizontal asymptote of g.
- B(1 ; 0) is the x-intercept of f.
- A(t ; 1) is a point of intersection between f and g.
- The vertical asymptote of g intersects the x-axis at E and the horizontal asymptote at D.
- OB = BE.



5.1 Write down the range of g. (2)

5.2 Determine the equation of g. (2)

5.3 Calculate the value of t. (3)

5.4 Write down the equation of f^{-1}, the inverse of f, in the form $y=\ldots$ (2)

5.5 For which values of x will $f^{-1}(x)<3$? (2)

5.6 Determine the point of intersection of the graphs of f and the axis of symmetry of g that has a negative gradient. (3)

[14]

QUESTION 6

6.1 Mbali invested R10 000 for 3 years at an interest rate of r % p.a., compounded monthly. At the end of this period, she received R12 146,72. Calculate r, correct to ONE decimal place. (5)

6.2 Piet takes a loan from a bank to buy a car for R235 000. He agrees to repay the loan over a period of 54 months. The first instalment will be paid one month after the loan is granted. The bank charges interest at 11% p.a., compounded monthly.

6.2.1 Calculate Piet's monthly instalment. (4)

6.2.2 Calculate the total amount of interest that Piet will pay during the first year of the repayment of the loan. (6)

[15]

QUESTION 7

7.1 Given: $f(x)=2x^2-x$

Determine $f'(x)$ from first principles. (6)

7.2 Determine:

7.2.1 $D_x[(x+1)(3x-7)]$ (2)

7.2.2 $\frac{dy}{dx}$ if $y=\sqrt{x^3}-\frac{5}{x}+\frac{1}{2}\pi$ (4)

[12]

QUESTION 8

Given: $f(x)=x(x-3)^2$ with $f'(1)=f'(3)=0$ and $f(1)=4$

8.1 Show that f has a point of inflection at $x=2$. (5)

8.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4)

8.3 For which values of x will $y=-f(x)$ be concave down? (2)

8.4 Use your graph to answer the following questions:

8.4.1 Determine the coordinates of the local maximum of h if $h(x)=f(x-2)+3$. (2)

8.4.2 Claire claims that $f'(2)=1$.

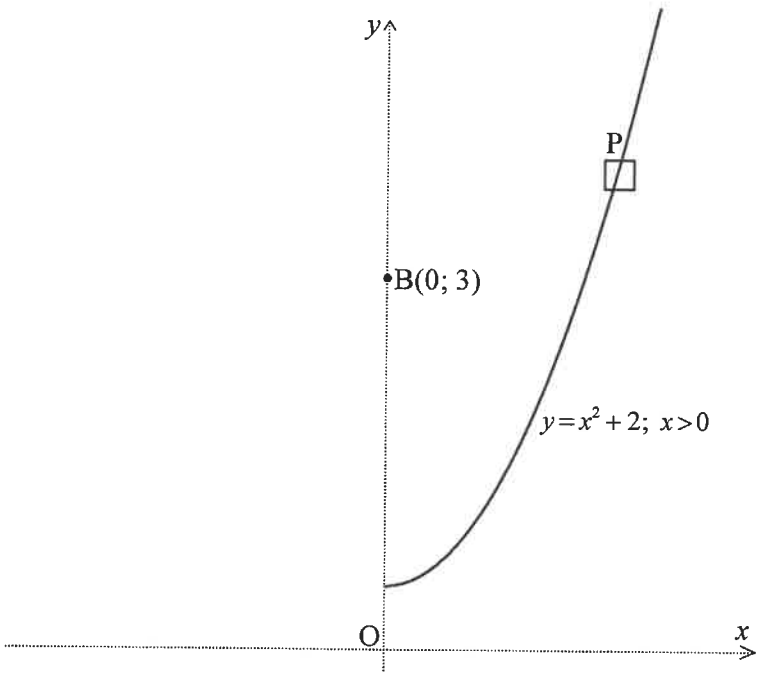
Do you agree with Claire? Justify your answer. (2)

[15]

QUESTION 9

An aerial view of a stretch of road is shown in the diagram below. The road can be described by the function $y=x^2+2$, $x\geq 0$ if the coordinate axes (dotted lines) are chosen as shown in the diagram.

Benny sits at a vantage point B(0 ; 3) and observes a car, P, travelling along the road.



Calculate the distance between Benny and the car, when the car is closest to Benny. **[7]**

QUESTION 10

A survey was conducted among 100 Grade 12 learners about their use of Instagram (I), Twitter (T) and WhatsApp (W) on their cell phones. The survey revealed the following:

- 8 use all three.
- 12 use Instagram and Twitter.
- 5 use Twitter and WhatsApp, but not Instagram.
- x use Instagram and WhatsApp, but not Twitter.
- 61 use Instagram.
- 19 use Twitter.
- 73 use WhatsApp.
- 14 use none of these applications.

10.1 Draw a Venn diagram to illustrate the information above. (4)

10.2 Calculate the value of x. (2)

10.3 Calculate the probability that a learner, chosen randomly, uses only ONE of these applications. (2)
[8]

QUESTION 11

A company uses a coding system to identify its clients. Each code is made up of two letters and a sequence of digits, for example AD108 or RR 45789.

The letters are chosen from A; D; R; S and U. Letters may be repeated in the code.

The digits 0 to 9 are used, but NO digit may be repeated in the code.

11.1 How many different clients can be identified with a coding system that is made up of TWO letters and TWO digits? (3)

11.2 Determine the least number of digits that is required for a company to uniquely identify 700 000 clients using their coding system. (3)
[6]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1} \;;\; r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;;\; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\,\Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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NATIONAL SENIOR CERTIFICATE
NASIONALE SENIOR SERTIFIKAAT

GRADE 12/*GRAAD* 12

MATHEMATICS P1/*WISKUNDE V1*

NOVEMBER 2017

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/*PUNTE:* ***150***

This memorandum consists of 22 pages.
Hierdie memorandum bestaan uit 22 bladsye.

NOTE:

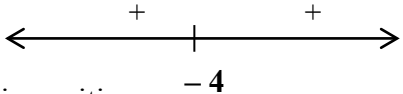
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent accuracy applies in ALL aspects of the marking guidelines.

LET WEL:

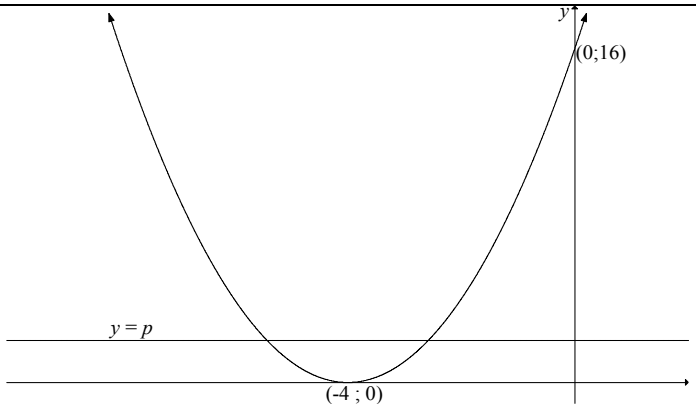
- *Indien 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.*
- *Volgehoue akkuraatheid is op ALLE aspekte van die nasienriglyne van toepassing.*

QUESTION/*VRAAG* 1

1.1.1	$x^2+9x+14=0$ $(x+7)(x+2)=0$ $x=-7$ or $x=-2$	✓ factors ✓ $x=-7$ ✓ $x=-2$ (3)
1.1.2	$4x^2+9x-3=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $=\frac{-9\pm\sqrt{9^2-4(4)(-3)}}{2(4)}$ $=\frac{-9\pm\sqrt{129}}{8}$ $x=0{,}29$ or $x=-2{,}54$ **OR/*OF*** $x^2+\frac{9}{4}x+\frac{81}{64}=\frac{3}{4}+\frac{81}{64}$ $\left(x+\frac{9}{8}\right)^2=\frac{129}{64}$ $x+\frac{9}{8}=\pm\frac{\sqrt{129}}{8}$ $x=\frac{-9\pm\sqrt{129}}{8}$ $x=0{,}29$ or $x=-2{,}54$	✓ substitution ✓ simplification ✓ $x=0{,}29$ ✓ $x=-2{,}54$ **OR/*OF*** ✓ for adding $\frac{81}{64}$ on both sides ✓ simplification ✓ $x=0{,}29$ ✓ $x=-2{,}54$ (4)
1.1.3	$\sqrt{x^2-5}=2\sqrt{x}$ $x^2-5=4x$ $x^2-4x-5=0$ $(x-5)(x+1)=0$ $x=5$ or $x=-1$ $x=5$	✓ $x^2-5=4x$ ✓ standard form ✓ both answers ✓ select $x=5$ (4)

1.2	$3x-y=4$ $y=3x-4$ $x^2+2xy-y^2=-2$ $x^2+2x(3x-4)-(3x-4)^2=-2$ $x^2+6x^2-8x-(9x^2-24x+16)=-2$ $7x^2-8x-9x^2+24x-16=-2$ $-2x^2+16x-14=0$ $x^2-8x+7=0$ $(x-7)(x-1)=0$ $x=1$ or $x=7$ $y=3(1)-4$ $y=3(7)-4$ $y=-1$ or $y=17$ **OR/*OF*** $3x-y=4$ $x=\frac{y+4}{3}$ $x^2+2xy-y^2=-2$ $x^2+2xy-y^2=-2$ $\left(\frac{y+4}{3}\right)^2+2\left(\frac{y+4}{3}\right)y-y^2=-2$ $y^2+8y+16+6y^2+24y-9y^2=-18$ $-2y^2+32y+34=0$ $y^2-16y-17=0$ $(y-17)(y+1)=0$ $y=-1$ or $y=17$ $x=\frac{-1+4}{3}$ $x=\frac{17+4}{3}$ $x=1$ or $x=7$	✓ y subject of formula ✓ substitution ✓ correct standard form ✓ factors ✓x-values ✓y-values **OR/*OF*** ✓x subject of formula ✓ substitution ✓ correct standard form ✓ factors ✓y-values ✓x-values (6)
1.3.1	$x^2+8x+16>0$ $(x+4)(x+4)>0$ $x\in R, x\neq -4$ **or** $x\in(-\infty;-4)$ or $x\in(-4;\infty)$ **or** $x<-4$ or $x>-4$ **OR/*OF*** $x^2+8x+16>0$ $(x+4)(x+4)>0$  The function values remain positive $x\in R, x\neq -4$	✓ $(x+4)(x+4)$ ✓✓ any one of the solutions **OR/*OF*** ✓ $(x+4)(x+4)$ ✓✓ any one of the solutions (3)

1.3.2



For two negative unequal roots:

$0 < p < 16$

✓ 0
✓ 16

✓ ✓ $0 < p < 16$ (4)

OR/*OF*

$x^2 + 8x + 16 = p$

$x^2 + 8x + 16 - p = 0$

$0 < 16 - p < 16$

$-16 < -p < 0$

$0 < p < 16$

✓ 0
✓ 16

✓ ✓ $0 < p < 16$ (4)

OR/*OF*

$x^2 + 8x + 16 - p = 0$

$x = \dfrac{-8 \pm \sqrt{64 - 4(16 - p)}}{2}$

$0 < 64 - 4(16 - p) < 64$

$0 < 4p < 64$

$0 < p < 16$

✓ 0
✓ 16

✓ ✓ $0 < p < 16$ (4)

OR/*OF*

$x^2 + 8x + 16 = p$

$x^2 + 8x + 16 - p = 0$

Roots are real and unequal:

$8^2 - 4(16 - p) > 0$

$4p > 0$

$p > 0$

Roots are: $\dfrac{-8 \pm \sqrt{4p}}{2}$

For both roots to be negative:

$\sqrt{4p} < 8$

$4p < 64$

$p < 16$

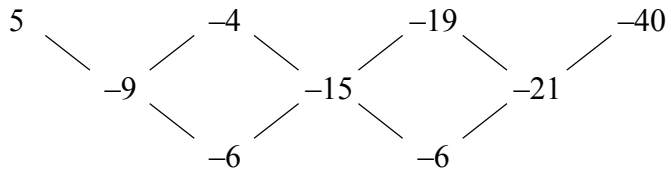
$0 < p < 16$

✓ 0
✓ 16

✓ ✓ $0 < p < 16$ (4)

[24]

QUESTION/*VRAAG* 2

2.1.1	 first differences: –9; –15; –21 second difference = – 6	✓ first differences ✓ – 6 (2)
2.1.2	$T_n = an^2 + bn + c$ $a = \frac{\text{second difference}}{2} = -3$ $3a + b = -9$ $3(-3) + b = -9$ $b = 0$ $a + b + c = 5$ $-3 + 0 + c = 5$ $c = 8$ $T_n = -3n^2 + 8$ **OR/*OF*** $T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)d_2}{2}$ $= 5 + (n-1)(-9) + \frac{(n-1)(n-2)(-6)}{2}$ $= 5 - 9n + 9 - 3n^2 + 9n - 6$ $T_n = -3n^2 + 8$	✓ $a = -3$ ✓ $b = 0$ ✓ $c = 8$ ✓ $T_n = -3n^2 + 8$ **OR/*OF*** ✓ $a = -3$ ✓ $b = 0$ ✓ $c = 8$ ✓ $T_n = -3n^2 + 8$ (4)
2.1.3	$-3n^2 + 8 = -25\,939$ $-3n^2 = -25947$ $n^2 = 8649$ $n = -93 \quad or \quad n = 93$ The 93rd term has a value of –25 939	✓ $T_n = -25\,939$ ✓ $n^2 = 8649$ ✓ answer (3)

2.2.1	$2k-7$; $k+8$ and $2k-1$ $k+8-(2k-7)=2k-1-(k+8)$ $-k+15=k-9$ $2k=24$ $k=12$ $2k-7;k+8$ and $2k-1$ $17\,;20\,;23$..... $d=3$ $T_{15}=17+14(3)$ $=59$	✓ $k+8-(2k-7)=2k-1-(k+8)$ ✓$k=12$ ✓17 ✓ $d=3$ ✓$T_{15}=59$ (5)
2.2.2	Sequence is 17 ; 20 ; 23 ; 26 ; 29 ; 32 ……. Every alternate term of the sequence will be even / *Elke tweede term van die ry sal ewe wees* $20+26+32+$..... $S_{30}=\frac{30}{2}[2(20)+(29)(6)]$ $=15[40+174]$ $=3210$ **OR/*OF*** $T_{30}=20+29(6)$ $=94$ $S_{30}=\frac{30}{2}(20+194)$ $=3210$	✓ $20+26+32+$..... ✓$a=20$ $d=6$ ✓subst into correct formula ✓ answer (4) ✓ $a=20$ $d=6$ ✓ $T_{30}=94$ ✓ $S_{30}=\frac{30}{2}(20+194)$ ✓ answer (4) **[18]**

QUESTION/*VRAAG* 3

3.1	$a+ar=2$ $a(1+r)=2$ $a=\frac{2}{1+r}$ **OR/*OF*** $\frac{a}{1-r}-2=\frac{1}{4}$ $4a-8(1-r)=1-r$ $4a-8+8r=1-r$ $4a=9-9r$ $a=\frac{9-9r}{4}$ **OR/*OF*** $S_n=\frac{a(r^n-1)}{r-1}$ $2=\frac{a(r^2-1)}{r-1}$ $2=\frac{a(r-1)(r+1)}{r-1}$ $2=a(r+1)$ $a=\frac{2}{r+1}$ **OR/*OF*** $\frac{ar^2}{1-r}=\frac{1}{4}$ $a=\frac{1-r}{4r^2}$	✓ $a+ar=2$ ✓ $a=\frac{2}{1+r}$ (2) ✓ $\frac{a}{1-r}-2=\frac{1}{4}$ ✓ $a=\frac{9-9r}{4}$ (2) **OR/*OF*** ✓ $2=\frac{a(r^2-1)}{r-1}$ ✓ $a=\frac{2}{1+r}$ (2) **OR/*OF*** ✓ $\frac{ar^2}{1-r}=\frac{1}{4}$ ✓ $a=\frac{1-r}{4r^2}$ (2)

3.2	$S_\infty = T_1 + T_2 + \sum_{n=3}^{\infty} T_n$	
	$S_\infty = 2 + \frac{1}{4}$	✓ $S_\infty = 2 + \frac{1}{4}$
	$\frac{a}{1-r} = 2 + \frac{1}{4}$	
	$\frac{a}{1-r} = \frac{9}{4}$	✓ $\frac{a}{1-r} = \frac{9}{4}$
	$\left(\frac{2}{1+r}\right) \times \left(\frac{1}{1-r}\right) = \frac{9}{4}$	✓ substitution of a into the correct formula
	$\frac{2}{1-r^2} = \frac{9}{4}$	
	$8 = 9 - 9r^2$	
	$9r^2 = 1$	✓ $9r^2 = 1$
	$r = \frac{1}{3}$	✓ $r = \frac{1}{3}$
	$a = \frac{3}{2}$	✓ $a = \frac{3}{2}$
		(6)
	OR/*OF*	**OR/*OF***
	$S_\infty = T_1 + T_2 + \sum_{n=3}^{\infty} T_n$	
	$S_\infty = 2 + \frac{1}{4}$	✓ $S_\infty = 2 + \frac{1}{4}$
	$\frac{a}{1-r} = 2 + \frac{1}{4}$	
	$\frac{a}{1-r} = \frac{9}{4}$	✓ $\frac{a}{1-r} = \frac{9}{4}$
	$4a = 9 - 9r$	
	$r = \frac{9-4a}{9}$	✓ $r = \frac{9-4a}{9}$
	$a + a\left(\frac{9-4a}{9}\right) = 2$	
	$9a + 9a - 4a^2 = 18$	
	$2a^2 - 9a + 9 = 0$	✓ substitution of a into the correct formula
	$(a-3)(2a-3) = 0$	
	$a = \frac{3}{2}$ or $a = 3$	
	$r = \frac{1}{3}$ or $r = -\frac{1}{3}$	
	N/A	✓ $a = \frac{3}{2}$

OR/*OF*

$r = \frac{2-a}{a}$

$\frac{ar^2}{1-r} = \frac{1}{4}$

$4ar^2 = 1-r$

$4a\left(\frac{2-a}{a}\right)^2 = 1-\frac{2-a}{a}$

$16-16a+4a^2 = 2a+2$

$2a^2-9a+9=0$

$(2a-3)(a-3)=0$

$a = \frac{3}{2}$ $\quad a \neq 3$

$r = \frac{1}{3}$ $\quad r \neq -\frac{1}{3}$

✓ $r = \frac{1}{3}$

(6)

OR/*OF*

✓ $r = \frac{2-a}{a}$

✓ $\frac{ar^2}{1-r} = \frac{1}{4}$

✓ substitution of a

✓ $(2a-3)(a-3)=0$

✓ $a = \frac{3}{2}$

✓ $r = \frac{1}{3}$

(6)

OR/*OF*

$S_\infty = T_1 + T_2 + \sum_{n=3}^{\infty} T_n$

$S_\infty = 2 + \frac{1}{4}$

$\frac{a}{1-r} = 2 + \frac{1}{4}$

$\frac{a}{1-r} = \frac{9}{4}$

$\left(\frac{1-r}{4r^2}\right) \times \left(\frac{1}{1-r}\right) = \frac{9}{4}$

$\frac{1}{4r^2} = \frac{9}{4}$

$4 = 36r^2$

$9r^2 = 1$

$r = \frac{1}{3}$

$a = \frac{3}{2}$

OR/*OF*

✓ $S_\infty = 2 + \frac{1}{4}$

✓ $\frac{a}{1-r} = \frac{9}{4}$

✓ substitution of a

✓ $9r^2 = 1$

✓ $r = \frac{1}{3}$

✓ $a = \frac{3}{2}$

(6)

[8]

QUESTION/*VRAAG* 4

4.1	$f(x) = -ax^2 + bx + 6$ $f'(x) = -2ax + b$ $-2ax + b = 3$ at $x = -1$ $2a + b = 3$ [1] $f(-1) = \frac{7}{2}$ $-a - b + 6 = \frac{7}{2}$ $-2a - 2b + 12 = 7$ $2a + 2b = 5$ [2] [2] – [1] $b = 2$ $2a + 2 = 3$ $a = \frac{1}{2}$ **OR/*OF*** $f'(x) = -2ax + b$ $3 = 2a + b$ $b = 3 - 2a$ $\frac{7}{2} = -a(-1)^2 + (3 - 2a)(-1) + 6$ $a + 3 = \frac{7}{2}$ $a = \frac{1}{2}$ $b = 2$	✓$-2ax + b$ ✓✓$2a + b = 3$ ✓$-a - b + 6 = \frac{7}{2}$ ✓solve simultaneously (5) ✓$-2ax + b$ ✓✓$2a + b = 3$ ✓ $\frac{7}{2} = -a(-1)^2 + (3 - 2a)(-1) + 6$ ✓solve simultaneously (5)
4.2	$f(x) = -\frac{1}{2}x^2 + 2x + 6$ x – intercepts: $-\frac{1}{2}x^2 + 2x + 6 = 0$ $-x^2 + 4x + 12 = 0$ $x^2 - 4x - 12 = 0$ $(x - 6)(x + 2) = 0$ $(-2\,;0)$ $(6\,;0)$	✓ $-\frac{1}{2}x^2 + 2x + 6 = 0$ ✓$(-2\,;0)$ ✓ $(6\,;0)$ (3)

4.3

$f(x)=-\frac{1}{2}x^2+2x+6$

$f'(x)=0$ or $x=-\frac{b}{2a}$ or $x=\frac{-2+6}{2}$

$-x+2=0$ $x=-\frac{2}{2.\left(-\frac{1}{2}\right)}$ $x=2$

$x=2$ $x=2$

$y=-\frac{1}{2}(2)^2+2(2)+6$

$=-2+4+6$

$=8$

TP $(2;8)$

OR/OF

$y=-\frac{1}{2}\left(x^2-4x-12\right)$

$=-\frac{1}{2}\left[(x-2)^2-4-12\right]$

$=-\frac{1}{2}(x-2)^2+8$

TP $(2;8)$

✓ $-x+2$ / $-\frac{2}{2.\left(-\frac{1}{2}\right)}$ / $\frac{-2+6}{2}$

✓ $x=2$

✓ $y=8$

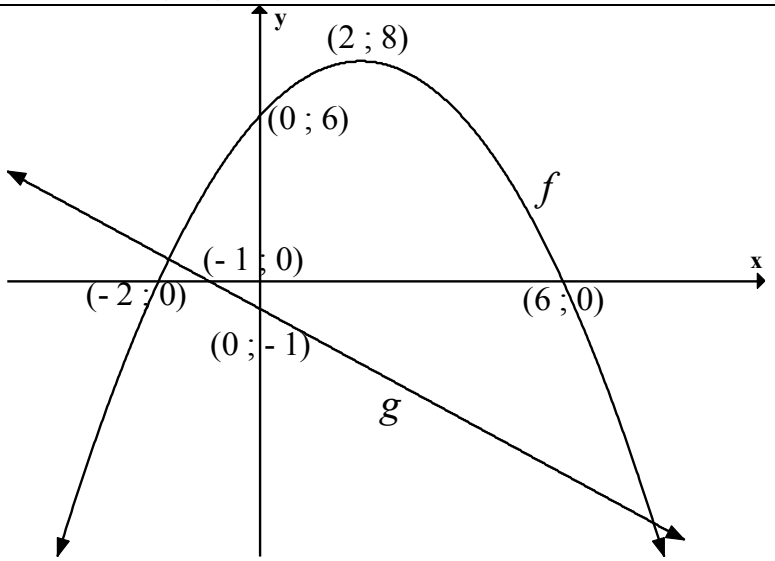
OR/*OF*

✓ $-\frac{1}{2}(x-2)^2+8$

✓ $x=2$
✓ $y=8$

(3)

4.4
4.6



4.4:
f:
✓shape
✓*x*- intercepts
✓*y*- intercept
✓(2 ; 8)

(4)

4.6:
g:

✓ *x*- intercept
✓ *y*- intercept

(2)

4.5

$0 < x < 4$ or $(0 ; 4)$

✓ 4
✓✓ $0 < x < 4$ (3)

4.7

$x \le -2$ or $-1 \le x \le 6$

OR/OF

$(-\infty ; -2]$ or $[-1;6]$

✓ $x \le -2$
✓✓ $-1 \le x \le 6$

(3)
[23]

QUESTION/*VRAAG* 5

5.1	$y \in R\,;\, y \neq -1$ **OR/*OF*** $y < -1$ or $y > -1$ **OR/*OF*** $y \in (-\infty\,;-1)$ or $y \in (-1\,;\infty)$ **OR/*OF*** $R - \{-1\}$	✓✓ answer (2)
5.2	$\text{D}(2\,;-1)$ $g(x) = \dfrac{2}{x-2} - 1$	✓ $\text{D}(2\,;-1)$ ✓ $\dfrac{2}{x-2} - 1$ (2)
5.3	$f(x) = \log_3 x.$ $\log_3 t = 1$ $t = 3$ **OR/*OF*** $g(x) = \dfrac{2}{x-2} - 1$ $1 = \dfrac{2}{t-2} - 1$ $2 = \dfrac{2}{t-2}$ $2t - 4 = 2$ $t = 3$	✓correct substitution of A ✓✓ $t = 3$ (3)
5.4	$x = \log_3 y$ $y = 3^x$	✓interchange x and y ✓ $y = 3^x$ (2)
5.5	$3^x < 3^1$ $x < 1$ **OR/*OF*** $3^x < 3^1$ $x \in (-\infty\,;1)$	✓$3^x < 3^1$ ✓$x < 1$ (2) ✓$3^x < 3^1$ ✓$x \in (-\infty\,;1)$ (2)
5.6	Equation of the axis of symmetry: $y = -x + 1$ x-intercept of the axis of symmetry is at $x = 1$ f has an x-intercept at B(1 ; 0) which is the same as the x-intercept of the axis of symmetry Point of intersection: B $(1\,;0)$ **OR/*OF*** Since BE = ED = 1 and D lies on the axis of symmetry and the gradient of the axis of symmetry is –1, B will also lie on the axis of symmetry. But B also lies on f. Therefore B(1 ; 0) is the point of intersection between f and the axis of symmetry with a negative gradient./ *Omdat* BE = ED = 1 *en* D *op die simmetrie-as lê en die simmetrie-as se gradiënt* –1 *is, sal* B *ook op die simmetrie-as lê. Maar* B *lê ook op f. Dus is* B(1 ; 0) *die snypunt van f en die simmetrie-as met negatiewe gradiënt.*	✓✓ equation of axis of symmetry ✓B or $(1\,;0)$ **OR/*OF*** ✓✓ BE = ED = 1 ✓ B or $(1\,;0)$ (3) **[14]**

QUESTION/*VRAAG* 6

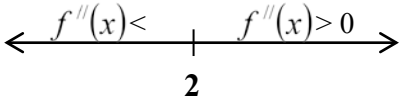
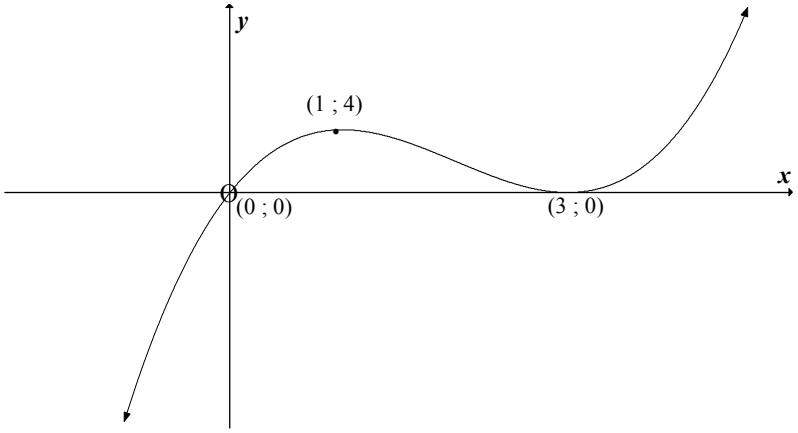
6.1	$A = P(1+i)^n$ $12\ 146{,}72 = 10\ 000\left(1+\frac{r}{12}\right)^{36}$ $\left(1+\frac{r}{12}\right)^{36} = 1{,}214672$ $1+\frac{r}{12} = \sqrt[36]{1{,}214672}$ $= 1{,}005416$ $\frac{r}{12} = 0{,}005416$ $r = 0{,}06500$ $r = 6{,}5\%$	✓ $\frac{r}{12}$ ✓ $n = 36$ ✓ correct substitution into formula ✓ $1+\frac{r}{12} = \sqrt[36]{1{,}214672}$ ✓ 6,5% (5)
6.2.1	$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $235\ 000 = \frac{x\left[1-\left(1+\frac{0{,}11}{12}\right)^{-54}\right]}{\frac{0{,}11}{12}}$ $x = \frac{235\ 000\times\frac{0{,}11}{12}}{\left[1-\left(1+\frac{0{,}11}{12}\right)^{-54}\right]}$ $= \text{R}5\ 536{,}95$ His monthly instalment is R 5 536,95	✓ $i = \frac{0{,}11}{12}$ ✓ $n = 54$ ✓ correct substitution in P ✓ answer (4)
6.2.2	Amount paid for the year : $(5\ 536{,}95\times 12) = \text{R}66\ 443{,}40$ $\text{Balance} = 235\ 000\left(1+\frac{0{,}11}{12}\right)^{12} - \frac{5\ 536{,}95\left[\left(1+\frac{0{,}11}{12}\right)^{12}-1\right]}{\frac{0{,}11}{12}}$ $= 192\ 296{,}17$ $\text{Interest} = (5\ 536{,}95\times 12) - (235\ 000 - 192\ 296{,}17)$ $= 66\ 443{,}40 - 42\ 703{,}83$ $= 23\ 739{,}57$ **OR/*OF***	✓ R66 443,40 ✓ $235\ 000\left(1+\frac{0{,}11}{12}\right)^{12}$ ✓ $\frac{5\ 536{,}95\left[\left(1+\frac{0{,}11}{12}\right)^{12}-1\right]}{\frac{0{,}11}{12}}$ ✓ R192 296,17 ✓ R42 703,83 ✓ R23 739,57 **OR/*OF***

	Total amount paid in first year $= \text{R } 5\,536.95 \times 12$ $= \text{R}66\,443{,}40$ Balance on loan after 1 year = P of remaining installments $\text{P} = \frac{x\left[1-(1+i)^{-n}\right]}{i}$ $= \frac{5\ 536{,}95\left[1-\left(1+\frac{0{,}11}{12}\right)^{-42}\right]}{\frac{0{,}11}{12}}$ $= \text{R}192\,296{,}20$ Amount paid off in the first year: $\text{R}235\,000 - \text{R}192\,296{,}20 = \text{R}42\,703{,}80$ Amount of interest = $\text{R}66\,443{,}40 - \text{R}42\,703{,}80$ $= \text{R}23\,739{,}60$ **OR/*OF*** $P = \frac{5536{,}95\left[1-\left(1+\frac{0{,}11}{12}\right)^{-12}\right]}{\frac{0{,}11}{12}}$ $= R\ 62\,648{,}18$ $235\,000 - 62\,648{,}18 = \text{R}172\,351{,}82$ After 12 months, money owed on house is $172\,351{,}82\left(1+\frac{0{,}11}{12}\right)^{12}$ $= 192\,296{,}17$ Amount paid after 12 months is $5\,536{,}95 \times 12 = \text{R}\ 66\,443{,}40$ Amount of interest paid: R 66 443, 40 – (235 000 – 192 296,17) = R 23 739, 57	✓ R66 443,40 ✓ $n = -42$ ✓ substitution into correct formula ✓ R192 296,20 ✓ R42 703,80 ✓ R23 739,60 (6) **OR/*OF*** ✓ R62 648,18 ✓ R172 351,82 ✓ R192 296,17 ✓ R66 443,40 ✓ 235 000 – 192 296,17 ✓ R23 739,57 (6) **[15]**

QUESTION/*VRAAG* 7

7.1	$f(x+h) = 2(x+h)^2 - (x+h)$ $= 2(x^2 + 2xh + h^2) - x - h$ $= 2x^2 + 4xh + 2h^2 - x - h$ $f(x+h) - f(x) = 2x^2 + 4xh + 2h^2 - x - h - 2x^2 + x$ $= 4xh + 2h^2 - h$ $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h\to 0} \frac{4xh + 2h^2 - h}{h}$ $= \lim_{h\to 0} \frac{h(4x + 2h - 1)}{h}$ $= \lim_{h\to 0} (4x + 2h - 1)$ $= 4x - 1$ **OR/*OF*** $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h\to 0} \frac{2(x+h)^2 - (x+h) - (2x^2 - x)}{h}$ $= \lim_{h\to 0} \frac{2x^2 + 4xh + 2h^2 - x - h - 2x^2 + x}{h}$ $= \lim_{h\to 0} \frac{4xh + 2h^2 - h}{h}$ $= \lim_{h\to 0} \frac{h(4x + 2h - 1)}{h}$ $= \lim_{h\to 0} (4x + 2h - 1)$ $= 4x - 1$	✓ $2x^2 + 4xh + 2h^2 - x - h$ ✓ $4xh + 2h^2 - h$ ✓ $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ ✓subst. into formula ✓ $\lim_{h\to 0}(4x + 2h - 1)$ ✓$4x - 1$ **OR/*OF*** ✓ $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ ✓subst. into formula ✓ $2x^2 + 4xh + 2h^2 - x - h$ ✓ $4xh + 2h^2 - h$ ✓ $\lim_{h\to 0}(4x + 2h - 1)$ ✓ $4x - 1$ (6)
7.2.1	$D_x[(x+1)(3x-7)]$ $= D_x(3x^2 - 4x - 7)$ $= 6x - 4$	✓ $3x^2 - 4x - 7$ ✓ $6x - 4$ (2)
7.2.2	$y = \sqrt{x^3} - \frac{5}{x} + \frac{1}{2}\pi$ $y = x^{\frac{3}{2}} - 5x^{-1} + \frac{1}{2}\pi$ $\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + 5x^{-2}$	✓ $x^{\frac{3}{2}} - 5x^{-1}$ ✓ $\frac{3}{2}x^{\frac{1}{2}}$ ✓ $+5x^{-2}$ ✓ derivative of $\frac{1}{2}\pi$ is 0 (4) **[12]**

QUESTION/*VRAAG* 8

8.1	$f(x) = x^3 - 6x^2 + 9x$ $f'(x) = 3x^2 - 12x + 9$ $f''(x) = 6x - 12 = 0$ $x = 2$ $f''(0) = 6(0) - 12$ $= -12$ $f''(3) = 6(3) - 12$ $= 6$  Point of inflection at $x = 2$	✓ $x^3 - 6x^2 + 9x$ ✓ $3x^2 - 12x + 9$ ✓ $6x - 12$ ✓ $6x - 12 = 0$ ✓ explanation (5)
8.2	 	✓shape ✓ (0 ; 0) ✓ (3 ; 0) as TP ✓(1 ; 4) (4)
8.3	f concave up for $x > 2$ $y = -f(x)$ will be concave down for $x > 2$	✓✓ $x > 2$ (2)
8.4.1	$(3;7)$	✓ 3 ✓7 (2)
8.4.2	Do not agree with Claire as her statement is incorrect. Between $x = 1$ and $x = 3$ the graph of f is decreasing. Therefore at $x = 2$ the gradient will have a negative value. *Stem nie saam met Claire nie, want haar stelling in verkeerd.* *Die grafiek van f is dalend/afnemend tussen $x = 1$ en $x = 3$.* *By $x = 2$ moet die gradiënt dus 'n negatiewe waarde hê.* **OR/*OF*** $f'(2) = 3(2)^2 - 12(2) + 9$ $= -3$ $\neq 1$	✓no ✓justification (2) **[15]**

QUESTION/*VRAAG* 9

	$y = x^2 + 2$ $\text{P}\left(x\,;x^2+2\right)$ $\text{B}(0\,;3)$	
	$\text{PB}^2 = (x-0)^2 + \left(x^2+2-3\right)^2$	✓ $(x-0)^2 + \left(x^2+2-3\right)^2$
	$= x^2 + x^4 - 2x^2 + 1$	
	$= x^4 - x^2 + 1$	✓ $x^4 - x^2 + 1$
	PB will be a minimum if PB^2 is a minimum	
	$\frac{d\left(\text{PB}^2\right)}{dx} = 4x^3 - 2x$	✓ $4x^3 - 2x$
	$4x^3 - 2x = 0$	✓ $\frac{d\left(\text{PB}^2\right)}{dx} = 0$
	$x\left(2x^2 - 1\right) = 0$	
	$x = 0$ or $x^2 = \frac{1}{2}$	
	$x = \frac{1}{\sqrt{2}}$	✓ $x = \frac{1}{\sqrt{2}}$
	$\text{PB}^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1$	✓ $\text{PB}^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1$
	$= \frac{1}{4} - \frac{1}{2} + 1$	
	$= \frac{3}{4}$	
	$\text{PB} = \frac{\sqrt{3}}{2} = 0{,}87$	✓ answer
	OR/*OF*	**OR/*OF***

Gradient of tangent to curve $= 2x$ ✓ $= 2x$

Gradient of line joining B and the curve $= \dfrac{x^2+2-3}{x-0}$

$= \dfrac{x^2-1}{x}$ ✓ $= \dfrac{x^2-1}{x}$

Shortest distance will be where tangent to curve is perpendicular to the line joining P and the curve.

$\dfrac{x^2-1}{x} = -\dfrac{1}{2x}$ ✓ $\dfrac{x^2-1}{x} = -\dfrac{1}{2x}$

$2x\left(x^2-1\right) = -x$

$2x^3 - 2x = 0$ ✓ $2x^3 - 2x = 0$

$x\left(2x^2-1\right) = 0$

$x = 0 \quad \text{or} \quad x^2 = \dfrac{1}{2}$

$x = \dfrac{1}{\sqrt{2}}$ ✓ $x = \dfrac{1}{\sqrt{2}}$

$\text{PB}^2 = \left(\dfrac{1}{\sqrt{2}}\right)^4 - \left(\dfrac{1}{\sqrt{2}}\right)^2 + 1$ ✓ $\text{PB}^2 = \left(\dfrac{1}{\sqrt{2}}\right)^4 - \left(\dfrac{1}{\sqrt{2}}\right)^2 + 1$

$= \dfrac{1}{4} - \dfrac{1}{2} + 1$

$= \dfrac{3}{4}$

$\text{PB} = \dfrac{\sqrt{3}}{2} = 0{,}87$ ✓ answer

OR/*OF*

OR/*OF*

$\text{P}\left(k; k^2+2\right)$ and B(0; 3) ✓ $\text{P}\left(k; k^2+2\right)$

$\text{BP} \perp$ tangent passing through $y = x^2 + 2$ at P.

$m_{\text{tangent at P}} = 2k$ ✓ $m_{\text{tangent at P}} = 2k$

$m_{\text{BP}} = -\dfrac{1}{2k}$ ✓ $m_{BP} = -\dfrac{1}{2k}$

Equation of BP: $y = \left(-\dfrac{1}{2k}\right)x + 3$ ✓ $y = \left(-\dfrac{1}{2k}\right)x + 3$

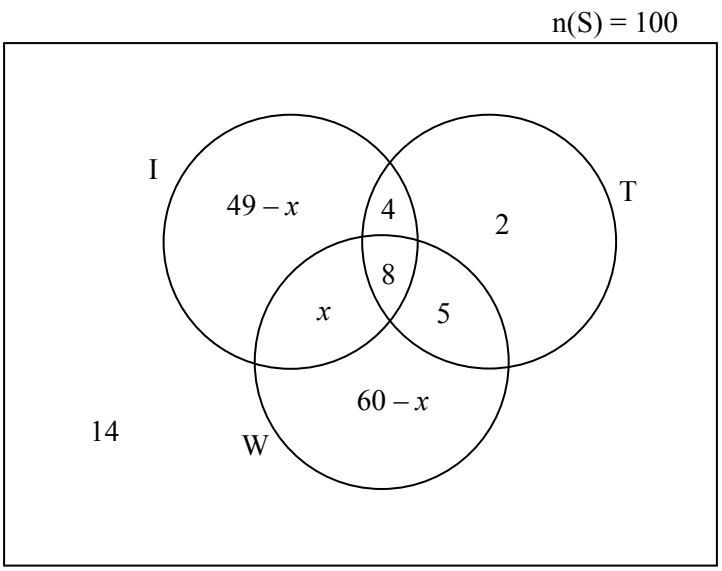
$y_{\text{P}} = \left(-\dfrac{1}{2k}\right)(k) + 3 = 2{,}5$ ✓ value of y at P

$\Rightarrow k^2 + 2 = 2{,}5$ and so $k = \sqrt{0{,}5}$ and $\text{P}\left(\sqrt{0{,}5}; 2{,}5\right)$ ✓ value of k

$\text{BP} = \sqrt{\left(\sqrt{0{,}5}-0\right)^2 + (2{,}5-3)^2} = \sqrt{\dfrac{3}{4}} = \dfrac{\sqrt{3}}{2} = 0{,}87$ ✓ answer

[7]

QUESTION/*VRAAG* 10

10.1		8 values need to be placed in correct position: 2 or 3 correct: 1 mark 4 or 5 correct: 2 marks 6 or 7 correct: 3 marks 8 correct: 4 marks (4)
10.2	$(49-x)+x+8+4+5+2+(60-x)+14=100$ $-x+142=100$ $x=42$	✓ setting up equation ✓ answer (2)
10.3	P (use only one application) $=\frac{7+2+18}{100}$ $=\frac{27}{100}$ or 27%	✓ $\frac{7+2+18}{100}$ ✓answer (2) **[8]**

QUESTION/*VRAAG* 11

11.1	5 x 5 x 10 x 9 = 2250	✓ 5 x 5 ✓ 10 x 9 ✓ 2250 (3)
11.2	<table><tr><th>No of digits used</th><th>Letters</th><th>Digits</th><th>Total</th></tr><tr><td>1</td><td>5 x 5</td><td>10</td><td>250</td></tr><tr><td>2</td><td>5 x 5</td><td>10 x 9</td><td>2 250</td></tr><tr><td>3</td><td>5 x 5</td><td>10 x 9 x 8</td><td>18 000</td></tr><tr><td>4</td><td>5 x 5</td><td>10 x 9 x 8 x7</td><td>126 000</td></tr><tr><td>5</td><td>5 x 5</td><td>10 x 9 x 8 x7 x 6</td><td>756 000</td></tr></table> Codes of two letters and five digits will ensure unique numbers for 700 000 clients.	✓ 5 x 5 x 10 x 9 x 8 x 7 x 6 ✓✓ five digits (3) **[6]**

TOTAL/*TOTAAL*: 150