basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NASIONALE
SENIOR SERTIFIKAAT

GRAAD 12

WISKUNDE V1

NOVEMBER 2017

PUNTE: 150

TYD: 3 uur

Hierdie vraestel bestaan uit 8 bladsye en 1 inligtingsblad.

INSTRUKSIES EN INLIGTING

Lees die volgende instruksies aandagtig deur voordat jy die vrae beantwoord.

1. Hierdie vraestel bestaan uit 11 vrae.

2. Beantwoord AL die vrae.

3. Nommer die antwoorde korrek volgens die nommeringstelsel wat in hierdie vraestel gebruik is.

4. Dui ALLE berekeninge, diagramme, grafieke ensovoorts wat jy gebruik het om jou antwoorde te bepaal, duidelik aan.

5. Volpunte sal nie noodwendig aan slegs antwoorde toegeken word nie.

6. Jy mag 'n goedgekeurde, wetenskaplike sakrekenaar (nieprogrammeerbaar en niegrafies) gebruik, tensy anders vermeld.

7. Indien nodig, rond antwoorde tot TWEE desimale plekke af, tensy anders vermeld.

8. Diagramme is NIE noodwendig volgens skaal geteken NIE.

9. 'n Inligtingsblad met formules is aan die einde van die vraestel ingesluit.

10. Skryf netjies en leesbaar.

VRAAG 1

1.1   Los op vir x:

       1.1.1 $x^2 + 9x + 14 = 0$ (3)

       1.1.2 $4x^2 + 9x - 3 = 0$ (korrek tot TWEE desimale plekke)   (4)

       1.1.3 $\sqrt{x^2 - 5} = 2\sqrt{x}$ (4)

1.2   Los op vir x en y indien:

     $3x - y = 4$ en $x^2 + 2xy - y^2 = -2$ (6)

1.3   Gegee: $f(x) = x^2 + 8x + 16$

       1.3.1   Los op vir x indien $f(x) > 0$.   (3)

       1.3.2   Vir watter waardes van p sal $f(x) = p$ TWEE ongelyke negatiewe wortels hê?   (4)

       [24]

VRAAG 2

2.1   Gegee die volgende kwadratiese getalpatroon: 5 ; –4 ; –19 ; –40 ; ...

       2.1.1   Bepaal die konstante tweede verskil van die ry.   (2)

       2.1.2   Bepaal die n^{de} term (T_n) van die patroon.   (4)

       2.1.3   Watter term van die patroon sal aan $-25\,939$ gelyk wees?   (3)

2.2   Die eerste drie terme van 'n rekenkundige ry is $2k - 7$; $k + 8$ en $2k - 1$.

       2.2.1   Bereken die waarde van die 15de term van die ry.   (5)

       2.2.2   Bereken die som van die eerste 30 ewe terme van die ry.   (4)

       [18]

VRAAG 3

'n Konvergente meetkundige reeks wat slegs uit positiewe terme bestaan, het eerste term a, konstante verhouding r en n^{de} term, T_n, sodat $\sum\limits_{n=3}^{\infty} T_n = \dfrac{1}{4}$.

3.1   Indien $T_1 + T_2 = 2$, skryf 'n uitdrukking vir a in terme van r neer.   (2)

3.2   Bereken die waardes van a en r.   (6)

       [8]

VRAAG 4
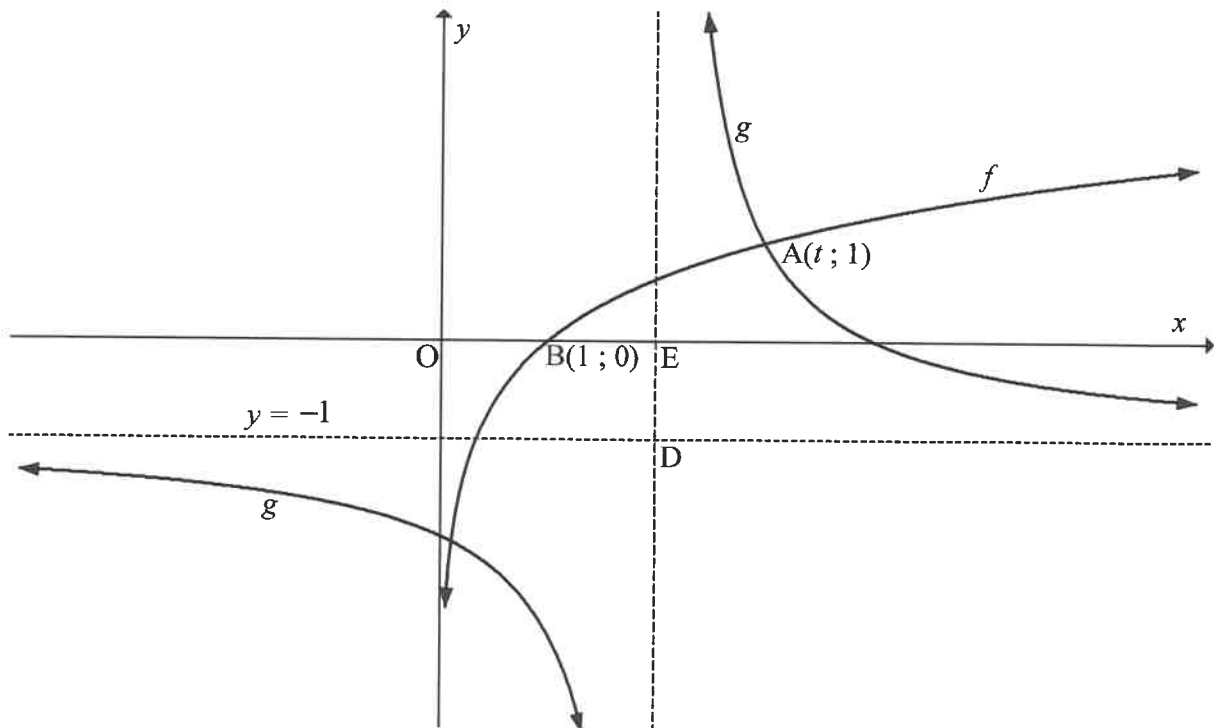
Gegee: $f(x) = -ax^2 + bx + 6$

4.1 Die gradiënt van die raaklyn aan die grafiek van f by die punt $\left(-1 ; \dfrac{7}{2}\right)$ is 3.

Toon dat $a = \dfrac{1}{2}$ en $b = 2$. (5)

4.2 Bereken die x-afsnitte van f. (3)

4.3 Bereken die koördinate van die draaipunt van f. (3)

4.4 Skets die grafiek van f. Dui ALLE afsnitte met die asse en die draaipunt duidelik aan. (4)

4.5 Gebruik die grafiek om die waardes van x te bepaal waarvoor $f(x) > 6$. (3)

4.6 Skets die grafiek van $g(x) = -x - 1$ op dieselfde assestelsel as f. Dui ALLE afsnitte met die asse duidelik aan. (2)

4.7 Skryf die waardes van x neer waarvoor $f(x).g(x) \leq 0$. (3)

[23]

VRAAG 5

Die diagram hieronder toon die grafieke van $g(x) = \dfrac{2}{x+p} + q$ en $f(x) = \log_3 x$.

- $y = -1$ is die horisontale asimptoot van g.

- B(1 ; 0) is die x-afsnit van f.

- A(t ; 1) is 'n snypunt tussen f en g.

- Die vertikale asimptoot van g sny die x-as by E en die horisontale asimptoot by D.

- OB = BE.

5.1	Skryf die waardeversameling van g neer.	(2)
5.2	Bepaal die vergelyking van g.	(2)
5.3	Bereken die waarde van t.	(3)
5.4	Skryf neer die vergelyking van f^{-1}, die inverse van f, in die vorm $y = \ldots$	(2)
5.5	Vir watter waardes van x sal $f^{-1}(x) < 3$?	(2)
5.6	Bepaal die snypunt van die grafiek van f en die simmetrie-as van g wat 'n negatiewe gradiënt het.	(3)

[14]

VRAAG 6

6.1 Mbali het R10 000 vir 3 jaar belê teen 'n rentekoers van r % p.j., maandeliks saamgestel. Aan die einde van hierdie tydperk het sy R12 146,72 ontvang. Bereken r, korrek tot EEN desimale plek. (5)

6.2 Piet neem 'n lening by 'n bank uit om 'n motor vir R235 000 te koop. Hy stem in om die lening oor 'n tydperk van 54 maande terug te betaal. Die eerste paaiement sal betaal word een maand nadat die lening toegestaan is. Die bank hef rente teen 11% p.j., maandeliks saamgestel.

6.2.1 Bereken Piet se maandelikse paaiement. (4)

6.2.2 Bereken die totale bedrag rente wat Piet in die eerste jaar van die terugbetaling van die lening sal betaal. (6)

[15]

VRAAG 7

7.1 Gegee: $f(x) = 2x^2 - x$

Bepaal $f'(x)$ vanuit eerste beginsels. (6)

7.2 Bepaal:

7.2.1 $D_x[(x+1)(3x-7)]$ (2)

7.2.2 $\dfrac{dy}{dx}$ as $y = \sqrt{x^3} - \dfrac{5}{x} + \dfrac{1}{2}\pi$ (4)
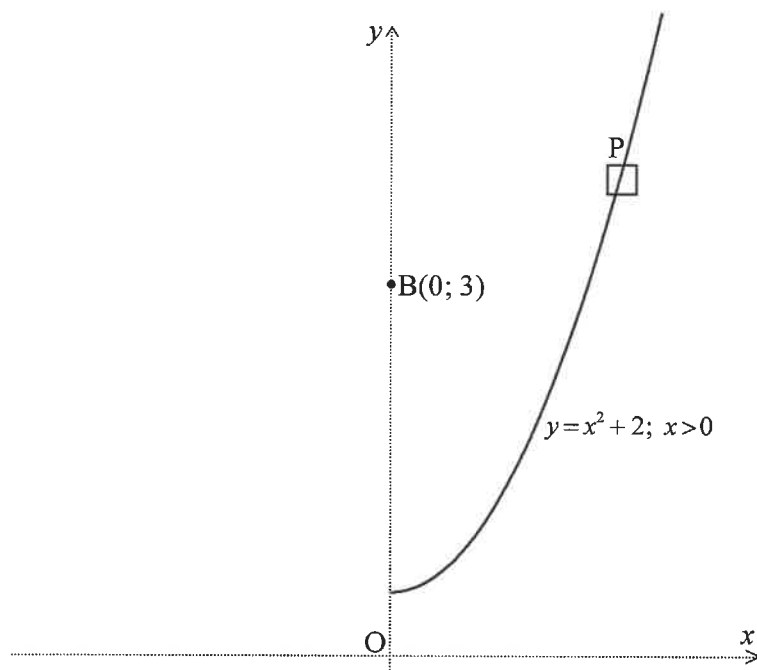
[12]

VRAAG 8

Gegee: $f(x) = x(x-3)^2$ met $f'(1) = f'(3) = 0$ en $f(1) = 4$

8.1 Toon dat f 'n buigpunt by $x = 2$ het. (5)

8.2 Skets die grafiek van f en dui die afsnitte met die asse en die draaipunte duidelik aan. (4)

8.3 Vir watter waardes van x sal $y = -f(x)$ konkaaf na onder wees? (2)

8.4 Gebruik jou grafiek om die volgende vrae te beantwoord:

8.4.1 Bepaal die koördinate van die lokale maksimum van h indien $h(x) = f(x-2) + 3$. (2)

8.4.2 Claire beweer dat $f'(2) = 1$.

Stem jy met Claire saam? Motiveer jou antwoord. (2)

[15]

VRAAG 9

'n Lugaansig van 'n stuk pad word in die diagram hieronder getoon. Die pad kan deur die funksie $y = x^2 + 2$, $x \geq 0$ beskryf word indien die koördinaat-asse (stippellyne) gekies word soos in die diagram getoon.

Benny sit by 'n uitkykpunt B(0 ; 3) en neem 'n motor, P, waar wat op die pad ry.

Bereken die afstand tussen Benny en die motor, wanneer die motor die naaste aan Benny is. **[7]**

VRAAG 10

'n Opname is onder 100 graad 12-leerders oor hulle gebruik van Instagram (I), Twitter (T) en WhatsApp (W) op hulle selfone gedoen. Die opname het die volgende onthul:

- 8 gebruik al drie.

- 12 gebruik Instagram en Twitter.

- 5 gebruik Twitter en WhatsApp, maar nie Instagram nie.

- x gebruik Instagram en WhatsApp, maar nie Twitter nie.

- 61 gebruik Instagram.

- 19 gebruik Twitter.

- 73 gebruik WhatsApp.

- 14 gebruik geeneen van hierdie toepassings nie.

10.1	Teken 'n Venn-diagram om die inligting hierbo te illustreer.	(4)
10.2	Bereken die waarde van x.	(2)
10.3	Bereken die waarskynlikheid dat 'n leerder, wat willekeurig gekies is, slegs EEN van hierdie toepassings gebruik.	(2) **[8]**

VRAAG 11

'n Maatskappy gebruik 'n koderingstelsel om sy kliënte te identifiseer. Elke kode bestaan uit twee letters en 'n ry syfers, byvoorbeeld AD108 of RR 45789.

Die letters word gekies uit A; D; R; S en U. Letters kan in die kode herhaal word.

Die syfers 0 tot 9 word gebruik, maar GEEN syfer kan in die kode herhaal word NIE.

11.1	Hoeveel verskillende kliënte kan geïdentifiseer word met 'n koderingstelsel wat uit TWEE letters en TWEE syfers bestaan?	(3)
11.2	Bepaal die kleinste getal syfers wat 'n maatskappy nodig het om 700 000 kliënte op unieke wyse met hulle koderingstelsel te identifiseer.	(3) **[6]**

TOTAAL: **150**

INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni) \qquad A = P(1 - ni) \qquad A = P(1 - i)^n \qquad A = P(1 + i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r - 1} \; ; \; r \neq 1 \qquad S_\infty = \frac{a}{1 - r} \; ; \; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

In$\triangle$ABC:
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc.\cos A$$
$$\text{oppervlakte } \triangle ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$
$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum x}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ of } B) = P(A) + P(B) - P(A \text{ en } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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GRADE 12/*GRAAD* 12

MARKS/*PUNTE: 150*

This memorandum consists of 22 pages.
Hierdie memorandum bestaan uit 22 bladsye.

NOTE:
- If a candidate answers a question TWICE, only mark the FIRST attempt.
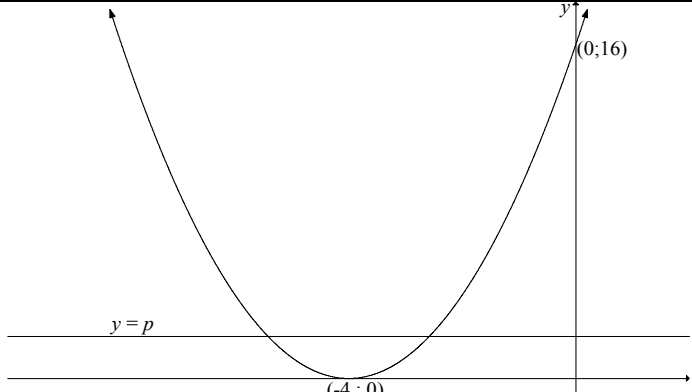- Consistent accuracy applies in ALL aspects of the marking guidelines.

LET WEL:
- *Indien 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.*
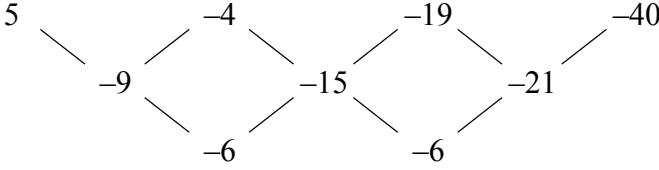- *Volgehoue akkuraatheid is op ALLE aspekte van die nasienriglyne van toepassing.*

QUESTION/*VRAAG* 1

1.1.1	$x^2 + 9x + 14 = 0$ $(x+7)(x+2) = 0$ $\qquad x = -7$ or $x = -2$	✓ factors ✓ $x = -7$ ✓ $x = -2$ (3)
1.1.2	$4x^2 + 9x - 3 = 0$ $\qquad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\qquad = \dfrac{-9 \pm \sqrt{9^2 - 4(4)(-3)}}{2(4)}$ $\qquad = \dfrac{-9 \pm \sqrt{129}}{8}$ $\qquad x = 0{,}29$ or $x = -2{,}54$ **OR/*OF*** $x^2 + \dfrac{9}{4}x + \dfrac{81}{64} = \dfrac{3}{4} + \dfrac{81}{64}$ $\qquad \left(x + \dfrac{9}{8}\right)^2 = \dfrac{129}{64}$ $\qquad x + \dfrac{9}{8} = \pm \dfrac{\sqrt{129}}{8}$ $\qquad x = \dfrac{-9 \pm \sqrt{129}}{8}$ $\qquad x = 0{,}29$ or $x = -2{,}54$	✓ substitution ✓ simplification ✓ $x = 0{,}29$ ✓ $x = -2{,}54$ **OR/*OF*** ✓ for adding $\dfrac{81}{64}$ on 　both sides ✓ simplification ✓ $x = 0{,}29$ ✓ $x = -2{,}54$ (4)
1.1.3	$\sqrt{x^2 - 5} = 2\sqrt{x}$ $x^2 - 5 = 4x$ $x^2 - 4x - 5 = 0$ $(x-5)(x+1) = 0$ $x = 5$ or $x = -1$ $\qquad x = 5$	✓ $x^2 - 5 = 4x$ ✓ standard form ✓ both answers ✓ select $x = 5$ (4)

1.2	$3x - y = 4$ $y = 3x - 4$ $x^2 + 2xy - y^2 = -2$ $x^2 + 2x(3x - 4) - (3x - 4)^2 = -2$ $x^2 + 6x^2 - 8x - (9x^2 - 24x + 16) = -2$ $7x^2 - 8x - 9x^2 + 24x - 16 = -2$ $-2x^2 + 16x - 14 = 0$ $x^2 - 8x + 7 = 0$ $(x - 7)(x - 1) = 0$ $x = 1$ or $x = 7$ $y = 3(1) - 4$ $y = 3(7) - 4$ $y = -1$ or $y = 17$ **OR/*OF*** $3x - y = 4$ $x = \dfrac{y + 4}{3}$ $x^2 + 2xy - y^2 = -2$ $x^2 + 2xy - y^2 = -2$ $\left(\dfrac{y+4}{3}\right)^2 + 2\left(\dfrac{y+4}{3}\right)y - y^2 = -2$ $y^2 + 8y + 16 + 6y^2 + 24y - 9y^2 = -18$ $-2y^2 + 32y + 34 = 0$ $y^2 - 16y - 17 = 0$ $(y - 17)(y + 1) = 0$ $y = -1$ or $y = 17$ $x = \dfrac{-1 + 4}{3}$ $x = \dfrac{17 + 4}{3}$ $x = 1$ or $x = 7$	✓ y subject of formula ✓ substitution ✓ correct standard form ✓ factors ✓x-values ✓y-values **OR/*OF*** ✓x subject of formula ✓ substitution ✓ correct standard form ✓ factors ✓y-values ✓x-values (6)
1.3.1	$x^2 + 8x + 16 > 0$ $(x + 4)(x + 4) > 0$ $x \in R,\ x \neq -4$ **or** $x \in (-\infty; -4)$ or $x \in (-4; \infty)$ **or** $x < -4$ or $x > -4$ **OR/*OF*** $x^2 + 8x + 16 > 0$ $(x + 4)(x + 4) > 0$ The function values remain positive $x \in R,\ x \neq -4$	✓ $(x + 4)(x + 4)$ ✓✓ any one of the solutions **OR/*OF*** ✓ $(x + 4)(x + 4)$ ✓✓ any one of the solutions (3)

1.3.2 For two negative unequal roots: $0 < p < 16$ **OR**/***OF*** $x^2 + 8x + 16 = p$ $x^2 + 8x + 16 - p = 0$ $0 < 16 - p < 16$ $-16 < -p < 0$ $0 < p < 16$ **OR**/***OF*** $x^2 + 8x + 16 - p = 0$ $x = \dfrac{-8 \pm \sqrt{64 - 4(16 - p)}}{2}$ $0 < 64 - 4(16 - p) < 64$ $0 < 4p < 64$ $0 < p < 16$ **OR**/***OF*** $x^2 + 8x + 16 = p$ $x^2 + 8x + 16 - p = 0$ Roots are real and unequal : $8^2 - 4(16 - p) > 0$ $4p > 0$ $p > 0$ Roots are : $\dfrac{-8 \pm \sqrt{4p}}{2}$ For both roots to be negative : $\sqrt{4\mathrm{p}} < 8$ $4p < 64$ $p < 16$ $0 < p < 16$	✓ 0 ✓ 16 ✓ ✓ $0 < p < 16$ (4) **OR**/***OF*** ✓ 0 ✓ 16 ✓ ✓ $0 < p < 16$ (4) ✓ 0 ✓ 16 ✓ ✓ $0 < p < 16$ (4) ✓ 0 ✓ 16 ✓ ✓ $0 < p < 16$ (4) **[24]**

QUESTION/*VRAAG* 2

2.1.1	first differences: -9; -15; -21 second difference $= -6$	✓ first differences ✓ -6 (2)

For 2.1.1:

5 ⟍ ⟋ -4 ⟋ -19 ⟍ -40

-9 -15 -21

-6 -6

2.1.2	$T_n = an^2 + bn + c$ $a = \dfrac{\text{second difference}}{2} = -3$ $3a + b = -9$ $3(-3) + b = -9$ $b = 0$ $a + b + c = 5$ $-3 + 0 + c = 5$ $c = 8$ $T_n = -3n^2 + 8$ **OR/*OF*** $T_n = T_1 + (n-1)d_1 + \dfrac{(n-1)(n-2)d_2}{2}$ $= 5 + (n-1)(-9) + \dfrac{(n-1)(n-2)(-6)}{2}$ $= 5 - 9n + 9 - 3n^2 + 9n - 6$ $T_n = -3n^2 + 8$	✓ $a = -3$ ✓ $b = 0$ ✓ $c = 8$ ✓ $T_n = -3n^2 + 8$ **OR/*OF*** ✓ $a = -3$ ✓ $b = 0$ ✓ $c = 8$ ✓ $T_n = -3n^2 + 8$ (4)

2.1.3	$-3n^2 + 8 = -25\ 939$ $-3n^2 = -25947$ $n^2 = 8649$ $n = -93 \ \ or \ \ n = 93$ The 93ʳᵈ term has a value of $-25\ 939$	✓ $T_n = -25\ 939$ ✓ $n^2 = 8649$ ✓ answer (3)

2.2.1	$2k - 7$; $k + 8$ and $2k - 1$	
	$k + 8 - (2k - 7) = 2k - 1 - (k + 8)$	✓ $k + 8 - (2k - 7) = 2k - 1 - (k + 8)$
	$-k + 15 = k - 9$	
	$2k = 24$	
	$k = 12$	✓ $k = 12$
	$\quad 2k - 7 ; k + 8$ and $2k - 1$	
	$\quad 17 ; 20 ; 23.......$	✓ 17
	$\ d = 3$	✓ $d = 3$
	$T_{15} = 17 + 14(3)$	
	$\quad = 59$	✓ $T_{15} = 59$
		(5)
2.2.2	Sequence is $\ 17 ; \ 20 ; 23 ; \ 26 ; 29 ; 32 \ldots$	
	Every alternate term of the sequence will be even /	
	Elke tweede term van die ry sal ewe wees	
	$\quad 20 + 26 + 32 + ...$	✓ $20 + 26 + 32 + ...$
	$S_{30} = \dfrac{30}{2}\big[2(20) + (29)(6)\big]$	✓ $a = 20 \ d = 6$
		✓ subst into correct formula
	$\quad = 15\big[40 + 174\big]$	
	$\quad = 3210$	✓ answer
		(4)
	OR/*OF*	
	$T_{30} = 20 + 29(6)$	✓ $a = 20 \ d = 6$
	$\quad = 94$	
		✓ $T_{30} = 94$
	$S_{30} = \dfrac{30}{2}(20 + 194)$	✓ $S_{30} = \dfrac{30}{2}(20 + 194)$
	$\quad = 3210$	✓ answer
		(4)
		[18]

QUESTION/*VRAAG* 3

3.1	$a + ar = 2$	$\checkmark\ a + ar = 2$	
	$a(1 + r) = 2$		
	$a = \dfrac{2}{1 + r}$	$\checkmark\ a = \dfrac{2}{1 + r}$	(2)
	OR/*OF*		
	$\dfrac{a}{1 - r} - 2 = \dfrac{1}{4}$	$\checkmark\ \dfrac{a}{1 - r} - 2 = \dfrac{1}{4}$	
	$4a - 8(1 - r) = 1 - r$		
	$4a - 8 + 8r = 1 - r$		
	$4a = 9 - 9r$	$\checkmark\ a = \dfrac{9 - 9r}{4}$	
	$a = \dfrac{9 - 9r}{4}$		(2)
	OR/*OF*	**OR/*OF***	
	$S_n = \dfrac{a(r^n - 1)}{r - 1}$		
	$2 = \dfrac{a(r^2 - 1)}{r - 1}$	$\checkmark\ 2 = \dfrac{a(r^2 - 1)}{r - 1}$	
	$2 = \dfrac{a(r - 1)(r + 1)}{r - 1}$		
	$2 = a(r + 1)$		
	$a = \dfrac{2}{r + 1}$	$\checkmark\ a = \dfrac{2}{1 + r}$	(2)
	OR/*OF*	**OR/*OF***	
	$\dfrac{ar^2}{1 - r} = \dfrac{1}{4}$	$\checkmark\ \dfrac{ar^2}{1 - r} = \dfrac{1}{4}$	
	$a = \dfrac{1 - r}{4r^2}$	$\checkmark\ a = \dfrac{1 - r}{4r^2}$	(2)

3.2		
	$$S_\infty = T_1 + T_2 + \sum_{n=3}^{\infty} T_n$$	
	$$S_\infty = 2 + \frac{1}{4}$$	✓ $S_\infty = 2 + \frac{1}{4}$
	$$\frac{a}{1-r} = 2 + \frac{1}{4}$$	
	$$\frac{a}{1-r} = \frac{9}{4}$$	✓ $\frac{a}{1-r} = \frac{9}{4}$
	$$\left(\frac{2}{1+r}\right) \times \left(\frac{1}{1-r}\right) = \frac{9}{4}$$	✓ substitution of *a* into the correct formula
	$$\frac{2}{1-r^2} = \frac{9}{4}$$	
	$$8 = 9 - 9r^2$$	
	$$9r^2 = 1$$	✓ $9r^2 = 1$
	$$r = \frac{1}{3}$$	✓ $r = \frac{1}{3}$
	$$a = \frac{3}{2}$$	✓ $a = \frac{3}{2}$
		(6)
	OR/*OF*	**OR/*OF***
	$$S_\infty = T_1 + T_2 + \sum_{n=3}^{\infty} T_n$$	
	$$S_\infty = 2 + \frac{1}{4}$$	✓ $S_\infty = 2 + \frac{1}{4}$
	$$\frac{a}{1-r} = 2 + \frac{1}{4}$$	
	$$\frac{a}{1-r} = \frac{9}{4}$$	✓ $\frac{a}{1-r} = \frac{9}{4}$
	$$4a = 9 - 9r$$	
	$$r = \frac{9-4a}{9}$$	✓ $r = \frac{9-4a}{9}$
	$$a + a\left(\frac{9-4a}{9}\right) = 2$$	
	$$9a + 9a - 4a^2 = 18$$	
	$$2a^2 - 9a + 9 = 0$$	✓ substitution of *a* into the correct formula
	$$(a-3)(2a-3) = 0$$	
	$$a = \frac{3}{2} \quad \text{or} \quad a = 3$$	
	$$r = \frac{1}{3} \quad \text{or} \quad r = -\frac{1}{3}$$ N/A	✓ $a = \frac{3}{2}$

OR/*OF*

$$r = \frac{2-a}{a}$$

$$\frac{ar^2}{1-r} = \frac{1}{4}$$

$$4ar^2 = 1-r$$

$$4a\left(\frac{2-a}{a}\right)^2 = 1 - \frac{2-a}{a}$$

$$16 - 16a + 4a^2 = 2a + 2$$

$$2a^2 - 9a + 9 = 0$$

$$(2a-3)(a-3) = 0$$

$$a = \frac{3}{2} \qquad a \neq 3$$

$$r = \frac{1}{3} \qquad r \neq -\frac{1}{3}$$

OR/*OF*

$$S_\infty = T_1 + T_2 + \sum_{n=3}^{\infty} T_n$$

$$S_\infty = 2 + \frac{1}{4}$$

$$\frac{a}{1-r} = 2 + \frac{1}{4}$$

$$\frac{a}{1-r} = \frac{9}{4}$$

$$\left(\frac{1-r}{4r^2}\right) \times \left(\frac{1}{1-r}\right) = \frac{9}{4}$$

$$\frac{1}{4r^2} = \frac{9}{4}$$

$$4 = 36r^2$$

$$9r^2 = 1$$

$$r = \frac{1}{3}$$

$$a = \frac{3}{2}$$

✓ $r = \dfrac{1}{3}$

(6)

OR/*OF*

✓ $r = \dfrac{2-a}{a}$

✓ $\dfrac{ar^2}{1-r} = \dfrac{1}{4}$

✓ substitution of a

✓ $(2a-3)(a-3) = 0$

✓ $a = \dfrac{3}{2}$

✓ $r = \dfrac{1}{3}$

(6)

OR/*OF*

✓ $S_\infty = 2 + \dfrac{1}{4}$

✓ $\dfrac{a}{1-r} = \dfrac{9}{4}$

✓ substitution of a

✓ $9r^2 = 1$

✓ $r = \dfrac{1}{3}$

✓ $a = \dfrac{3}{2}$

(6)

[8]

QUESTION/*VRAAG* 4

| 4.1 | $f(x) = -ax^2 + bx + 6$ $f'(x) = -2ax + b$ $-2ax + b = 3$ at $x = -1$ $2a + b = 3 \qquad [1]$ $f(-1) = \dfrac{7}{2}$ $-a - b + 6 = \dfrac{7}{2}$ $-2a - 2b + 12 = 7$ $2a + 2b = 5 \qquad [2]$ $[2] - [1]$ $b = 2$ $2a + 2 = 3$ $a = \dfrac{1}{2}$ **OR/*OF*** $f'(x) = -2ax + b$ $3 = 2a + b$ $b = 3 - 2a$ $\dfrac{7}{2} = -a(-1)^2 + (3 - 2a)(-1) + 6$ $a + 3 = \dfrac{7}{2}$ $a = \dfrac{1}{2}$ $b = 2$ | $\checkmark -2ax + b$ $\checkmark\checkmark 2a + b = 3$ $\checkmark -a - b + 6 = \dfrac{7}{2}$ $\checkmark$ solve simultaneously (5) $\checkmark -2ax + b$ $\checkmark\checkmark 2a + b = 3$ $\checkmark$ $\dfrac{7}{2} = -a(-1)^2 + (3 - 2a)(-1) + 6$ $\checkmark$ solve simultaneously (5) |
| 4.2 | $f(x) = -\dfrac{1}{2}x^2 + 2x + 6$ $x - \text{intercepts}:$ $-\dfrac{1}{2}x^2 + 2x + 6 = 0$ $-x^2 + 4x + 12 = 0$ $x^2 - 4x - 12 = 0$ $(x - 6)(x + 2) = 0$ $(-2\,;0) \qquad (6\,;0)$ | $\checkmark -\dfrac{1}{2}x^2 + 2x + 6 = 0$ $\checkmark (-2\,;0)$ $\checkmark (6\,;0)$ (3) |

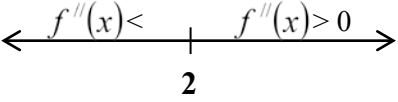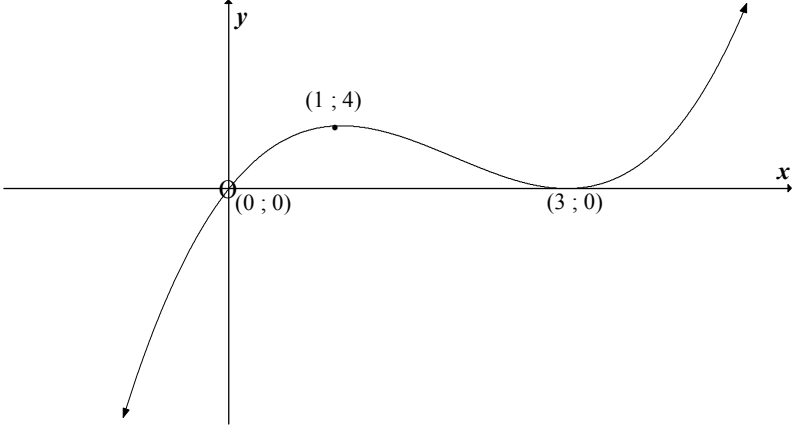| 4.3	$f(x) = -\frac{1}{2}x^2 + 2x + 6$	$\checkmark -x+2$ / $-\frac{2}{2.\left(-\frac{1}{2}\right)}$ /
	$f'(x) = 0 \qquad$ or $\quad x = -\frac{b}{2a} \qquad$ or $\quad x = \frac{-2+6}{2}$	$\frac{-2+6}{2}$
	$-x+2 = 0 \qquad\qquad x = -\frac{2}{2.\left(-\frac{1}{2}\right)} \qquad x = 2$	
	$x = 2 \qquad\qquad x = 2$	$\checkmark x = 2$
	$y = -\frac{1}{2}(2)^2 + 2(2) + 6$	$\checkmark \; y = 8$
	$\quad = -2 + 4 + 6$	
	$\quad = 8$	
	$\text{TP}(2\,;8)$	
	OR/OF	**OR/*OF***
	$y = -\frac{1}{2}(x^2 - 4x - 12)$	$\checkmark -\frac{1}{2}(x-2)^2 + 8$
	$\quad = -\frac{1}{2}\big[(x-2)^2 - 4 - 12\big]$	$\checkmark x = 2$
	$\quad = -\frac{1}{2}(x-2)^2 + 8$	$\checkmark \; y = 8$
	$\text{TP}(2\,;8)$	(3)
4.4 4.6		4.4: *f*: $\checkmark$shape $\checkmark x$- intercepts $\checkmark y$- intercept $\checkmark(2\,;8)$ (4) 4.6: *g*: $\checkmark \; x$- intercept $\checkmark \; y$- intercept (2)
4.5	$0 < x < 4$ or $(0\,;4)$	$\checkmark$ 4 $\checkmark\checkmark \; 0 < x < 4$ (3)
4.7	$x \le -2$ or $-1 \le x \le 6$ **OR/OF** $(-\infty\,;-2]$ or $[-1;6]$	$\checkmark \; x \le -2$ $\checkmark\checkmark \; -1 \le x \le 6$ (3) **[23]**

QUESTION/*VRAAG* **5**

5.1	$y \in R \; ; \; y \neq -1$ **OR/*OF*** $y < -1$ or $y > -1$ **OR/*OF*** $y \in (-\infty; -1)$ or $y \in (-1; \infty)$ **OR/*OF*** $R - \{-1\}$	✓✓ answer (2)
5.2	$D(2 \; ; -1)$ $g(x) = \dfrac{2}{x-2} - 1$	✓ $D(2 \; ; -1)$ ✓ $\dfrac{2}{x-2} - 1$ (2)
5.3	$f(x) = \log_3 x.$ $\log_3 t = 1$ **OR/*OF*** $g(x) = \dfrac{2}{x-2} - 1$ $t = 3$ $1 = \dfrac{2}{t-2} - 1$ $2 = \dfrac{2}{t-2}$ $2t - 4 = 2$ $t = 3$	✓ correct substitution of A ✓✓ $t = 3$ (3)
5.4	$x = \log_3 y$ $y = 3^x$	✓ interchange x and y ✓ $y = 3^x$ (2)
5.5	$3^x < 3^1$ $x < 1$ **OR/*OF*** $3^x < 3^1$ $x \in (-\infty \; ; 1)$	✓ $3^x < 3^1$ ✓ $x < 1$ (2) ✓ $3^x < 3^1$ ✓ $x \in (-\infty \; ; 1)$ (2)
5.6	Equation of the axis of symmetry: $y = -x + 1$ x-intercept of the axis of symmetry is at $x = 1$ f has an x-intercept at B(1 ; 0) which is the same as the x-intercept of the axis of symmetry Point of intersection: B $(1 \; ; 0)$ **OR/*OF*** Since BE = ED = 1 and D lies on the axis of symmetry and the gradient of the axis of symmetry is –1, B will also lie on the axis of symmetry. But B also lies on *f*. Therefore B(1 ; 0) is the point of intersection between f and the axis of symmetry with a negative gradient./ *Omdat* BE = ED = 1 *en* D *op die simmetrie-as lê en die simmetrie-as se gradiënt* –1 *is, sal* B *ook op die simmetrie-as lê. Maar* B *lê ook op f. Dus is* B(1 ; 0) *die snypunt van f en die simmetrie-as met negatiewe gradiënt.*	✓✓ equation of axis of symmetry ✓B or $(1 \; ; 0)$ **OR/*OF*** ✓✓ BE = ED = 1 ✓ B or $(1 \; ; 0)$ (3) **[14]**

QUESTION/*VRAAG* 6

6.1	$A = P(1+i)^n$	✓ $\dfrac{r}{12}$
	$12\ 146{,}72 = 10\ 000\left(1+\dfrac{r}{12}\right)^{36}$	✓ $n = 36$
	$\left(1+\dfrac{r}{12}\right)^{36} = 1{,}214672$	✓ correct substitution into formula
	$1+\dfrac{r}{12} = \sqrt[36]{1{,}214672}$	
	$\qquad\quad = 1{,}005416$	✓ $1+\dfrac{r}{12} = \sqrt[36]{1{,}214672}$
	$\dfrac{r}{12} = 0{,}005416$	
	$r = 0{,}06500$	
	$r = 6{,}5\%$	✓ 6,5%
		(5)
6.2.1	$P = \dfrac{x\left[1-(1+i)^{-n}\right]}{i}$	
	$235\ 000 = \dfrac{x\left[1-\left(1+\dfrac{0{,}11}{12}\right)^{-54}\right]}{\dfrac{0{,}11}{12}}$	✓ $i = \dfrac{0{,}11}{12}$ ✓ $n = 54$ ✓ correct substitution in P
	$x = \dfrac{235\ 000 \times \dfrac{0{,}11}{12}}{\left[1-\left(1+\dfrac{0{,}11}{12}\right)^{-54}\right]}$	
	$\quad = R5\ 536{,}95$	✓ answer
	His monthly instalment is R 5 536,95	(4)
6.2.2	Amount paid for the year : $(5\ 536{,}95 \times 12) = R66\ 443{,}40$	✓ R66 443,40
	Balance $= 235\ 000\left(1+\dfrac{0{,}11}{12}\right)^{12} - \dfrac{5\ 536{,}95\left[\left(1+\dfrac{0{,}11}{12}\right)^{12}-1\right]}{\dfrac{0{,}11}{12}}$	✓ $235\ 000\left(1+\dfrac{0{,}11}{12}\right)^{12}$ ✓ $\dfrac{5\ 536{,}95\left[\left(1+\dfrac{0{,}11}{12}\right)^{12}-1\right]}{\dfrac{0{,}11}{12}}$
	$\qquad\quad = 192\ 296{,}17$	✓ R192 296,17
	Interest $= (5\ 536{,}95 \times 12) - (235\ 000 - 192\ 296{,}17)$	✓ R42 703,83
	$\qquad\quad = 66\ 443{,}40\ -\ 42\ 703{,}83$	✓ R23 739,57
	$\qquad\quad = 23\ 739{,}57$	
	OR/*OF*	**OR/*OF***

Total amount paid in first year $= \text{R } 5\,536.95 \times 12$

$\qquad\qquad\qquad\qquad\qquad = \text{R } 66\,443{,}40$

Balance on loan after 1 year $= \text{P}$ of remaining installments

$$P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$$

$$= \frac{5\,536{,}95\left[1-\left(1+\dfrac{0{,}11}{12}\right)^{-42}\right]}{\dfrac{0{,}11}{12}}$$

$\qquad = \text{R} 192\,296{,}20$

Amount paid off in the first year:

$\text{R} 235\,000 - \text{R} 192\,296{,}20 = \text{R} 42\,703{,}80$

Amount of interest $= \text{R} 66\,443{,}40 - \text{R} 42\,703{,}80$

$\qquad\qquad\qquad\quad = \text{R} 23\,739{,}60$

OR/*OF*

$$P = \frac{5536{,}95\left[1-\left(1+\dfrac{0{,}11}{12}\right)^{-12}\right]}{\dfrac{0{,}11}{12}}$$

$\quad = R\ 62\,648{,}18$

$235\,000 - 62\,648{,}18 = \text{R} 172\,351{,}82$

After 12 months, money owed on house is

$172\,351{,}82\left(1+\dfrac{0{,}11}{12}\right)^{12}$

$= 192\,296{,}17$

Amount paid after 12 months is

$5\,536{,}95 \times 12 = \text{R } 66\,443{,}40$

Amount of interest paid:

$\text{R } 66\,443{,}40 - (235\,000 - 192\,296{,}17)$

$= \text{R } 23\,739{,}57$

✓ R66 443,40	
✓ $n = -42$	
✓ substitution into correct formula	
✓ R192 296,20	
✓ R42 703,80	
✓ R23 739,60	(6)
OR/*OF*	
✓ R62 648,18	
✓ R172 351,82	
✓ R192 296,17	
✓ R66 443,40	
✓ 235 000 – 192 296,17	
✓ R23 739,57	(6)
	[15]

QUESTION/*VRAAG*7

7.1	$f(x+h) = 2(x+h)^2 - (x+h)$	
	$\qquad = 2(x^2 + 2xh + h^2) - x - h$	$\checkmark\, 2x^2 + 4xh + 2h^2 - x - h$
	$\qquad = 2x^2 + 4xh + 2h^2 - x - h$	
	$f(x+h) - f(x) = 2x^2 + 4xh + 2h^2 - x - h - 2x^2 + x$	$\checkmark\, 4xh + 2h^2 - h$
	$\qquad = 4xh + 2h^2 - h$	
	$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$	$\checkmark\, f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$
	$\qquad = \lim\limits_{h \to 0} \dfrac{4xh + 2h^2 - h}{h}$	$\checkmark$ subst. into formula
	$\qquad = \lim\limits_{h \to 0} \dfrac{h(4x + 2h - 1)}{h}$	$\checkmark\, \lim\limits_{h \to 0}(4x + 2h - 1)$
	$\qquad = \lim\limits_{h \to 0}(4x + 2h - 1)$	
	$\qquad = 4x - 1$	$\checkmark\, 4x - 1$
	OR/*OF*	**OR/*OF***
	$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$	$\checkmark\, f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$
	$\qquad = \lim\limits_{h \to 0} \dfrac{2(x+h)^2 - (x+h) - (2x^2 - x)}{h}$	$\checkmark$ subst. into formula
	$\qquad = \lim\limits_{h \to 0} \dfrac{2x^2 + 4xh + 2h^2 - x - h - 2x^2 + x}{h}$	$\checkmark\, 2x^2 + 4xh + 2h^2 - x - h$
	$\qquad = \lim\limits_{h \to 0} \dfrac{4xh + 2h^2 - h}{h}$	$\checkmark\, 4xh + 2h^2 - h$
	$\qquad = \lim\limits_{h \to 0} \dfrac{h(4x + 2h - 1)}{h}$	$\checkmark\, \lim\limits_{h \to 0}(4x + 2h - 1)$
	$\qquad = \lim\limits_{h \to 0}(4x + 2h - 1)$	
	$\qquad = 4x - 1$	$\checkmark\, 4x - 1$
		(6)
7.2.1	$D_x[(x+1)(3x-7)]$	
	$= D_x(3x^2 - 4x - 7)$	$\checkmark\, 3x^2 - 4x - 7$
	$= 6x - 4$	$\checkmark\, 6x - 4$ $\qquad$ (2)
7.2.2	$y = \sqrt{x^3} - \dfrac{5}{x} + \dfrac{1}{2}\pi$	$\checkmark\, x^{\frac{3}{2}} - 5x^{-1}$
	$y = x^{\frac{3}{2}} - 5x^{-1} + \dfrac{1}{2}\pi$	$\checkmark\, \dfrac{3}{2}x^{\frac{1}{2}}$
	$\dfrac{dy}{dx} = \dfrac{3}{2}x^{\frac{1}{2}} + 5x^{-2}$	$\checkmark\, + 5x^{-2}$
		$\checkmark$ derivative of $\dfrac{1}{2}\pi$ is 0
		(4)
		[12]

QUESTION/*VRAAG* 8

8.1	$f(x) = x^3 - 6x^2 + 9x$ $f'(x) = 3x^2 - 12x + 9$ $f''(x) = 6x - 12 = 0$ $\quad x = 2$ $f''(0) = 6(0) - 12$ $\quad = -12$ $f''(3) = 6(3) - 12$ $\quad = 6$ Point of inflection at $x = 2$	✓ $x^3 - 6x^2 + 9x$ ✓ $3x^2 - 12x + 9$ ✓ $6x - 12$ ✓ $6x - 12 = 0$ ✓ explanation (5)
	$\xleftarrow{\quad f''(x)<\quad}\underset{\mathbf{2}}{\mid}\xrightarrow{\quad f''(x)>0\quad}$	
8.2		✓ shape ✓ (0 ; 0) ✓ (3 ; 0) as TP ✓ (1 ; 4) (4)
8.3	f concave up for $x > 2$ $y = -f(x)$ will be concave down for $x > 2$	✓✓ $x > 2$ (2)
8.4.1	$(3 ; 7)$	✓ 3 ✓ 7 (2)
8.4.2	Do not agree with Claire as her statement is incorrect. Between $x = 1$ and $x = 3$ the graph of f is decreasing. Therefore at $x = 2$ the gradient will have a negative value. *Stem nie saam met Claire nie, want haar stelling in verkeerd. Die grafiek van f is dalend/afnemend tussen $x = 1$ en $x = 3$. By $x = 2$ moet die gradiënt dus 'n negatiewe waarde hê.* **OR/*OF*** $f'(2) = 3(2)^2 - 12(2) + 9$ $\quad = -3$ $\quad \neq 1$	✓ no ✓ justification (2) **[15]**

QUESTION/*VRAAG* 9

$$y = x^2 + 2$$
$$P\left(x\,;x^2 + 2\right)$$
$$B\left(0\,;3\right)$$
$$PB^2 = \left(x - 0\right)^2 + \left(x^2 + 2 - 3\right)^2 \qquad \checkmark \left(x - 0\right)^2 + \left(x^2 + 2 - 3\right)^2$$
$$= x^2 + x^4 - 2x^2 + 1$$
$$= x^4 - x^2 + 1 \qquad\qquad\qquad \checkmark\ x^4 - x^2 + 1$$

PB will be a minimum if PB^2 is a minimum

$$\frac{d\left(PB^2\right)}{dx} = 4x^3 - 2x \qquad\qquad \checkmark\ 4x^3 - 2x$$
$$4x^3 - 2x = 0 \qquad\qquad\qquad \checkmark\ \frac{d\left(PB^2\right)}{dx} = 0$$
$$x\left(2x^2 - 1\right) = 0$$

$$x = 0 \quad\text{or}\quad x^2 = \frac{1}{2}$$

$$x = \frac{1}{\sqrt{2}} \qquad\qquad\qquad \checkmark\ x = \frac{1}{\sqrt{2}}$$

$$PB^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1 \qquad \checkmark\ PB^2 = \left(\frac{1}{\sqrt{2}}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^2 + 1$$

$$= \frac{1}{4} - \frac{1}{2} + 1$$

$$= \frac{3}{4}$$

$$PB = \frac{\sqrt{3}}{2} = 0{,}87 \qquad\qquad \checkmark\ \text{answer}$$

OR/*OF*   **OR/*OF***

Gradient of tangent to curve $= 2x$	✓ $= 2x$
Gradient of line joining B and the curve $= \dfrac{x^2 + 2 - 3}{x - 0}$	
$\qquad\qquad\qquad\qquad = \dfrac{x^2 - 1}{x}$	✓ $= \dfrac{x^2 - 1}{x}$
Shortest distance will be where tangent to curve is perpendicular to the line joining P and the curve.	
$\dfrac{x^2 - 1}{x} = -\dfrac{1}{2x}$	✓ $\dfrac{x^2 - 1}{x} = -\dfrac{1}{2x}$
$2x(x^2 - 1) = -x$	
$2x^3 - 2x = 0$	✓ $2x^3 - 2x = 0$
$x(2x^2 - 1) = 0$	
$x = 0 \quad \text{or} \quad x^2 = \dfrac{1}{2}$	
$x = \dfrac{1}{\sqrt{2}}$	✓ $x = \dfrac{1}{\sqrt{2}}$
$PB^2 = \left(\dfrac{1}{\sqrt{2}}\right)^4 - \left(\dfrac{1}{\sqrt{2}}\right)^2 + 1$	✓ $PB^2 = \left(\dfrac{1}{\sqrt{2}}\right)^4 - \left(\dfrac{1}{\sqrt{2}}\right)^2 + 1$
$\qquad = \dfrac{1}{4} - \dfrac{1}{2} + 1$	
$\qquad = \dfrac{3}{4}$	
$PB = \dfrac{\sqrt{3}}{2} = 0{,}87$	✓ answer
OR/*OF*	**OR/*OF***
$P\left(k; k^2 + 2\right)$ and $B(0; 3)$	✓ $P\left(k; k^2 + 2\right)$
$BP \perp$ tangent passing through $y = x^2 + 2$ at P.	
$m_{\text{tangent at P}} = 2k$	✓ $m_{\text{tangent at P}} = 2k$
$m_{BP} = -\dfrac{1}{2k}$	✓ $m_{BP} = -\dfrac{1}{2k}$
Equation of BP: $y = \left(-\dfrac{1}{2k}\right)x + 3$	✓ $y = \left(-\dfrac{1}{2k}\right)x + 3$
$y_P = \left(-\dfrac{1}{2k}\right)(k) + 3 = 2{,}5$	✓ value of y at P
$\Rightarrow k^2 + 2 = 2{,}5$ and so $k = \sqrt{0{,}5}$ and $P\left(\sqrt{0{,}5}; 2{,}5\right)$	✓ value of k
$BP = \sqrt{\left(\sqrt{0{,}5} - 0\right)^2 + (2{,}5 - 3)^2} = \sqrt{\dfrac{3}{4}} = \dfrac{\sqrt{3}}{2} = 0{,}87$	✓ answer
	[7]

QUESTION/*VRAAG* 10

10.1	n(S) = 100	8 values need to be placed in correct position: 2 or 3 correct: 1 mark 4 or 5 correct: 2 marks 6 or 7 correct: 3 marks 8 correct: 4 marks (4)
10.2	$$(49 - x) + x + 8 + 4 + 5 + 2 + (60 - x) + 14 = 100$$ $$- x + 142 = 100$$ $$x = 42$$	✓ setting up equation ✓ answer (2)
10.3	P (use only one application) $= \dfrac{7 + 2 + 18}{100}$ $$= \dfrac{27}{100} \text{ or } 27\%$$	✓ $\dfrac{7 + 2 + 18}{100}$ ✓ answer (2) **[8]**

QUESTION/*VRAAG* 11

11.1	5 x 5 x 10 x 9 = 2250	✓ 5 x 5 ✓ 10 x 9 ✓ 2250 (3)

11.2			

No of digits used	Letters	Digits	Total
1	5 x 5	10	250
2	5 x 5	10 x 9	2 250
3	5 x 5	10 x 9 x 8	18 000
4	5 x 5	10 x 9 x 8 x7	126 000
5	5 x 5	10 x 9 x 8 x7 x 6	756 000

Codes of two letters and five digits will ensure unique numbers for 700 000 clients.

✓ 5 x 5 x 10 x 9 x 8 x 7 x 6
✓✓ five digits   (3)
[6]

TOTAL/*TOTAAL*:   150